Applying the seasonal error correction model to the demand for international reserves in Taiwan

Tai-Hsin Huang\textsuperscript{a}, Chung-Hua Shen\textsuperscript{b,\!*}

\textsuperscript{a}Department of Economics, TamKang University, Tamsui, Taiwan
\textsuperscript{b}Department of Money and Banking, National Chengchi University, Mucha, Taipei 116, Taiwan

Abstract

A dynamic demand function for international reserves based on the seasonal difference is derived. Our model of using seasonal difference distinguishes the present study from previous studies in three aspects. First, the dependent variable is seasonally differenced instead of being first order differenced. Next, the local money market disequilibrium included is also in fourth difference form. Finally, given the existence of stochastic seasonality, a new model, using a seasonal error correction rather than the conventional error correction, is specified and estimated. Based on this new specification, the results yielded are more sensible than those of using a differenced model. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Studies of the demand for international reserves are typically based on one of two theories. The demand-for-reserve theory asserts that reserves are held to finance international transactions and to serve as a buffer stock to meet unexpected payment difficulties. This theory asserts that reserve holdings change in

\footnote{Corresponding author. Tel.: +886 2 9393091 81020; fax: +886 2 9398004; e-mail: chshen@nccu.edu.tw}

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response to the discrepancy between desired reserves and actual reserves, where the desired demand function for reserves depends on three key variables: a measure of the variability in the balance of payments, the scale of the country, and the average propensity to import (see Kenen and Yudin, 1965; Clower and Lipsey, 1968; Heller and Khan, 1978; Frenkel, 1983; Elbadawi, 1990). Once the key determinants are determined, the empirical tests typically rely on a partial adjustment mechanism (PAM) to replace the unobserved desired reserve by its actual counterpart. The adjustment speed and the dynamic process can be inferred from the coefficient on the lagged dependent variable.

Another explanation of the behavior of holding reserves is provided by a simple version of the monetary approach to the balance of payments. This approach focuses on the asset side of the balance sheets of financial institutions, where the money supply is equivalent to the sum of foreign assets and domestic credit. Hence, for given domestic credit, changes in international reserves will be related to changes in the demand for money. The level of international reserves are expected to rise (fall) if there is an excess demand (supply) for money. International reserves are therefore viewed as a residual according to the monetary approach.

A synthesis of the demand for reserves theory and the monetary approach, as claimed by Edwards (1984), can be obtained if there is a stable demand for international reserves. As long as a stable demand for reserve exists, domestic credit cannot be exogenous. Money market disequilibria will also affect reserve holdings. To integrate these two different explanations explicitly, Edwards (1984) incorporates an additional variable, excess money demand, representing an extra adjustment cost between the actual and the desired reserves in the PAM. His underlying assumption is that reserve holdings can be affected by money-market disequilibria only in the short-run, arising possibly from a domestic economic disequilibrium. The resulting reduced form of the demand for reserves equation includes not only the three key determinants, but also the money market disequilibrium.

Though the monetary approach stresses that money-market disequilibria have a significant short-run positive effect on the demand for reserve, Elbadawi (1990) pursued this further by examining whether or not money-market disequilibria influence reserves in the long-run specification. He applied the recent econometric technique of cointegration, together with the error correction mechanism (ECM) to investigate both the short-run dynamics and the long-run equilibrium relationships. Ford and Huang (1994) also employed the ECM to specify the relationship between reserves and their determinants. In contrast to Elbadawi (1990), their money-market disequilibrium appeared only in the short-run dynamic process.

Although Elbadawi (1990) and Ford and Huang (1994) successfully improved the demand function for reserves by explicit consideration of the dynamic process, their models cannot be easily generalized to other countries. The theoretical models underlying Elbadawi (1990) and Ford and Huang (1994) are derived from
optimizing a conventional partial-adjustment type loss function involving two sources of adjustment costs: the gap between desired and actual levels of reserves and the first difference between current and lagged reserves. However, in many developing countries, the data are typically not seasonally adjusted. The official annual growth rate of economic data is typically computed as a log difference over the latest 12-month period, generally referred to as a seasonal difference. Hence, the adjustment costs of reserves in developing countries are better modeled by seasonal differences than first differences. This minor correction, however, causes the subsequent regression specification to be modified substantially. It follows that, the loss functions for these developing countries should incorporate these specification changes.

Furthermore, the models of Elbadawi 1990 and Ford and Huang 1994 are subject to an estimation bias. Prior to their use of the ECM, they adopt the two-step approach of Engle and Granger 1987 to examine whether or not reserves and their determinants are cointegrated. Gonzalo (1994) demonstrated that the Engle and Granger two-step cointegration procedure yields weak and biased results when the model has more than two series. As an alternative, the maximum likelihood approach of Johansen (1988) and Johansen and Juselius (1990) can be utilized, which takes advantages of its robustness in a multivariate setting (see Gonzalo, 1994; Hargreaves, 1994).

This paper generalizes the Elbadawi 1990 and Ford and Huang 1994 demand function for reserves for those countries lacking seasonally adjusted data. The loss function is specified within the framework of seasonal differences, which for quarterly data can be expressed as four seasonal unit roots. The existence of a seasonal difference may cause the conventional cointegration tests, e.g. the two-step procedure of Engle and Granger (1987) and the maximum likelihood method of Johansen (1988), to be inappropriate at the zero frequency. A more appropriate approach is to employ the Lee (1992) maximum likelihood approach to examine the seasonal cointegration relationships among variables. The following seasonal error correction mechanism (SECM), not only correctly integrates the reserve-demand theory with the monetary approach for a country lacking seasonally adjusted data, but also explicitly specifies the entire short-run dynamics and long-run equilibrium of the reserves. Hence, previous approaches, using either PAM or ECM become special cases of our approach.

To exemplify our approach, we select the demand for reserves in Taiwan as an example. Taiwan is chosen due to its unique position of holding a large stock of international reserves, the third largest globally. Also, the data of Taiwan are typically not seasonally adjusted and its official growth rate is calculated by means

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1Engle et al. (1989) show that the conventional cointegration estimate is inconsistent if unit roots of distinct seasonal frequencies are regressed. Moreover, the long-run relationships among seasonal frequencies are also neglected in the conventional cointegration test.
of a seasonal difference. Similar to Elbadawi (1990) and Ford and Huang (1994),
the same key determinants and money market disequilibria are selected in the
desired demand equation for reserves. However, the loss function differs from the
above authors in that it utilizes a seasonally differenced form, thus conforming with
Taiwan's official growth rate. Although Taiwan's data are used as an example, the
generalization to other countries lacking seasonally adjusted data is immediate.

Our results are fruitful. First, we demonstrate that our general specification is
preferable to the conventional Johansen’s modelling in three aspects. The new
model yields a better goodness of fit (higher adjusted $R^2$), a stable functional form
(passing both Chow and the normality tests), and an expected positive structural
coefficient on income. By contrast, the conventional specification has a poorer
model fit, fails to pass the stability test, and yields a wrong negative structural
coefficient on income. Accordingly, those countries without seasonally adjusted
data are encouraged to use our approach to account for movements in their
reserves. Furthermore, even countries with seasonally adjusted data could benefit
from our model. This is because the stochastic seasonality in the raw data cannot
be fully removed by either including seasonal dummies or by utilization of the
official X-11 method.\(^2\) For example, Ghysels et al. (1993); Ghysels (1994) and
Ericsson et al. (1994) have suggested that X-11 adjusted data not be used to fit
models. The ideal procedure, suggested by them, is to use seasonal cointegration
and apply the resulting SECM to investigate the demand for reserves.

In addition to the above results, we find that while a high speed of adjustment
towards equilibrium for the reserve holdings is found in previous literature, a
markedly low speed of adjustment is discovered here. According to the theory of
Clark (1970b), such a low adjustment speed is probably due to the huge stock of
reserve holdings by a country. This is consistent with Taiwan’s situation. Because
Taiwan is currently not an IMF member, has a precarious political entanglement
with mainland China, and has few official diplomatic relations with other countries,
it needs to hold a significant amount of reserves in case of emergency. The low
adjustment rate reflects the reluctance of the authority in Taiwan to adjust actual
reserves to the economically desired level.

Finally, using our generalized approach, we find that a long-run equilibrium
relationship exists at the zero frequency among the reserves, their determinants,
and money. The monetary approach as well as the demand-for-reserve theory are
also found to jointly account for the long- and short-run movements of Taiwan’s
reserves. We also find that the narrow money market (M1B) is a more appropriate
monetary aggregate for disequilibrium as a determinant to the demand for reserves
than is broad money (M2).

\(^2\)The study of Abeyesinghe (1991) demonstrates that inclusion of seasonal dummies cannot eliminate the
stochastic seasonality embedded in most macroeconomic time series. Also, Wallis (1974), Ghysels et al.
(1993), Ghysels (1994), and Ericsson et al. (1994) do not suggest applying the method of X-11 to adjust
raw data.
The remainder of this paper is organized as follows. Section 2 derives a function for the demand for reserves, which will be estimated using Taiwanese data, by minimizing a loss function commonly used in the literature. Section 3 briefly discusses the testing procedures for seasonal unit roots and seasonal cointegration. Section 4 presents and analyzes the empirical results. Our findings are finally summarized in Section 5.

2. The theoretical model

The long-run demand for reserves at time $t$ is similar to that of Elbadawi (1990) with the money market disequilibrium $(m^*_t - m_{t-1})$ being replaced by $(m^*_t - m_{t-4})$.

\[
r^*_t = a_0 + a_1y_t + a_2\sigma_t + a_3im_t + a_4(m^*_t - m_{t-4})
\]

where $r^*_t$ denotes the desired stock of real international reserves, $y_t$ denotes the real GNP, $\sigma_t$ denotes a measure of the variability in the balance of payments, $im_t$ denotes the average propensity to import (being the ratio of real imports to real GNP), and $m^*_t$ and $m_t$ are the desired and actual stocks of money held, respectively. All variables, except for the interest rate, are in the log form.

Our proxy for the scale variable, $y_t$, is expected to have a positive effect on the demand for reserves see Heller and Khan, 1978; Frenkel, 1983; Elbadawi, 1990, and its coefficient, $a_1$, is thus positive. Reserves are expected to be positively influenced by variability $\sigma$ to accommodate fluctuations in the foreign trade factor (see Kenen and Yudin, 1965; Clower and Lipsey, 1968); thus, $a_2$ should also be positive. The impact of $im_t$ on the demand for reserves, however, is uncertain. On the one hand, Frenkel (1974) has stressed that reserves are positively associated with the degree of openness, which can be proxied by $im_t$. On the other hand, Heller (1966), Grubel (1969) and McKinnon and Oates (1966), have derived an inverse relation between $im_t$ and reserve holdings via output adjustments in a Keynesian model. The sign of $a_3$ is therefore theoretically indeterminate.

The monetary disequilibrium $(m^*_t - m_{t-4})$ is used in accordance with the partial adjustment model below. According to the monetary approach to the balance of payments, $(m^*_t - m_{t-4})$, which represents the effect of money market disequilibrium, is anticipated to have a short-run positive effect. In contrast, the traditional demand for reserves theory excludes the role of monetary disequilibrium. It postulates that discrepancies between the desired and the actually held stock of reserves are the main sources of movements in reserves. According to Edwards (1984), the contradiction between the monetary approach and the demand for reserves theory can be reconciled by allowing the sign of $a_4$ to be negative.

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3Edwards (1984), Elbadawi (1990), and Ford and Huang (1994), among others, postulated that the effect of monetary disequilibrium on reserves movements can be captured by the term $(m^*_t - m_{t-1})$.  

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The one period loss function employing a seasonally differenced annual growth rate is thus:

\[
L = d_1(r_t - r_t^*) + d_2(r_t - r_{t-4}) - 2d_3(r_t^* - r_{t-4}^*)(r_t - r_{t-4})
\]  

(2)

where \(d_1\) is the cost of deviation from long-run equilibrium and \(d_2\) is the short-run transaction cost. The cross-product term in Eq. (2) states that the adjustment cost can be reduced, if the desired and actual reserve holdings move in the same direction. The three coefficients are all positive. Hendry and Sternberg (1981) demonstrated that an EC model can be obtained by minimizing Eq. (2) with respect to \(r_t\). Taking the first derivative with respect to \(r_t\) and rearranging terms yields

\[
\Delta_t r_t = r_t - r_{t-4} = \lambda_1(r_{t-4}^* - r_{t-4}) + \lambda_2(r_t^* - r_{t-4}^*)
\]  

(3)

where \(\lambda_1 = d_1/(d_1 + d_2)\) and \(\lambda_2 = (d_1 + d_2)/(d_1 + d_2)\). It is interesting to note that the first term \(r_{t-4}^* - r_{t-4}\) is simply the conventional error correction (EC) term lagged four periods. Hence, \(\lambda_1\) is referred to as the speed of adjustment or ‘loading’ of the disequilibrium of demand for reserves. Substituting Eq. (1) into Eq. (3) yields

\[
\Delta_t r_t = \lambda_1[a_0 + a_1y_{t-4} + a_2\sigma_{t-4} + a_3im_{t-4} - r_{t-4}] + \lambda_1a_4(m_{t-4}^* - m_{t-4})
\]

\[
+ \lambda_2[a_1\Delta_t y_t + a_2\Delta_t \sigma_t + a_3\Delta_t im_t + a_4\Delta_t^2(m_t^* - m_t)]
\]  

(4)

This equation, which is derived from a one-period loss function, coincides with the seasonal error correction model as explained shortly. It will be seen later that diagnostic checks and cointegration tests of this model are also different from previous works, e.g. Eq. (6).

Estimating Eq. (4) requires all variables to be stationary. The first bracket term on the right hand side of Eq. (4) is stationary if variables \(y\), \(\sigma\), \(im\), and \(r\) all have unit roots at the zero frequency and are cointegrated. The second term \((m_{t-4}^* - m_{t-4})\) in Eq. (4) represents the disequilibrium in the money market. Since \(m_t^*\) is unobservable, \((m_t^* - m_{t-4})\) is typically replaced by its linear projection, \(\Delta_t m_t\), which should ordinarily be a stationary process. The third term on the right hand side of the equation is also stationary because each component in the term is in its seasonally differenced form.

Eq. (4) is referred to as a ‘structural’ form equation here, whereas its reduced

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This model is a direct extension of the model of Hendry and Sternberg (1981), which uses the first difference form as

\[
L = d_1(r_t - r_t^*) + d_2(r_t - r_{t-1}) - 2d_3(r_t^* - r_{t-1}^*)(r_t - r_{t-1})
\]
form equation can be written as

$$\Delta r_t = b_0 + b_1 \Delta y_{t-4} + b_2 \Delta q_{t-1} + b_3 \Delta y_t + b_4 \Delta \sigma_t + b_5 \Delta \sigma_{t-1} + b_6 \Delta \sigma_{t-2}$$

(5)

where each reduced form coefficient matches its corresponding combination of structural form coefficients in Eq. (4). Since the matchings are natural, their expressions are suppressed here.

Following similar derivations, the conventional cointegration specification can be expressed as:

$$\Delta r_t = c_0 + c_1 \Delta y_{t-1} + c_2 \Delta \sigma_{t-1} + c_3 \Delta y_t + c_4 \Delta \sigma_t + c_5 \Delta \sigma_d + c_6 \Delta \sigma_{t-2}$$

(6)

Eqs. (5) and (6) will be referred to as the seasonal cointegration specification and conventional cointegration specification, respectively. The performances of these two different specifications are of primary concern to us.

The empirical testing procedure of Eq. (5) includes the following four steps. First, $\Delta y_{t-1}$ and $\Delta \sigma_{t-1}$ are estimated. This can be achieved by applying Lee (1992) seasonal cointegration approach to find appropriate variables that are seasonally cointegrated with the money stock. The subsequent linear projectors of $\Delta y_{t-1}$ and $\Delta \sigma_{t-1}$ act as new proxy variables to be used in Eq. (5). Next, the estimated EC can be obtained by using a cointegration approach again to find the equilibrium relationships among $y$, $\sigma$, $im$, and $r$ at the zero frequency. The subsequent equilibrium error at the zero frequency is the term $EC_{t-4}$ in Eq. (5) lagged four periods. Third, the same four variables should be tested to determine if they are seasonally cointegrated at the 1/2 and 1/4 frequencies. If the cointegrating relationships are found at those frequencies, their equilibrium errors — seasonal error correction terms — must be included in Eq. (5). Eq. (5) can now be called a seasonal error correction model (SECM). Finally, Eq. (5) is estimated by OLS.

3. Seasonal integration and cointegration

Hylleberg et al. (1990) and Engle et al. (1992) have derived seasonal unit root and cointegration tests. Their testing procedures are a direct generalization of the Engle–Granger two-step procedure to the test for seasonal cointegration and, hence, suffer the same difficulties discussed in Gonzalo (1994). Lee (1992) generalizes the maximum likelihood estimation and inference proposed by Johansen to the tests for seasonal cointegration for a non-stationary vector autoregressive (VAR) system. Under the assumptions of no deterministic components (e.g. a constant, linear and/or quadratic trends, and seasonal dummies), Lee presents asymptotic as well as finite sample quantile distributions obtained by Monte Carlo simulations. As the critical values on the seasonal cointegration tests provided by Lee have
dimensions only up to three and are generated under the suppositions that the statistical models do not contain deterministic components, the critical values have limited use. Since the statistical models considered herein have more than three non-stationary variables and include various forms of deterministic components, our own critical values are generated by Monte Carlo simulations.

The preceding section requires the testing of seasonal cointegration. However, prior to this testing procedure, a seasonal unit root test must be first performed.

3.1. HEGY testing for seasonal unit roots

For a seasonally unadjusted series, HEGY introduces a factorization of the seasonal differencing polynomial for quarterly data as follows:

\[ \Delta_4 x_t = (1 - L^4)x_t = (1 - L)(1 + L)(1 + L^2)x_t \]  

where \( x_t \) contains four unit roots, +1, −1, \( i \), and −\( i \), that correspond to zero frequency (\( \omega = 0 \)), 1/2 cycle per quarter (\( \omega = 1/2 \)) or two cycles per year, 1/4 cycle and 3/4 cycle per quarter (\( \omega = 1/4 \)) or one cycle per year. The procedure to test these roots involves estimating the regression:

\[ \varphi(L)\Delta_4 x_t = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-1} + \pi_4 y_{4t-2} + \epsilon_t \]  

where \( y_{1t} = S_1(L)x_t = (1 + L)(1 + L^2)x_t \), \( y_{2t} = S_2(L)x_t = -(1 - L)(1 + L^2)x_t \), \( S_1(L) \) is a seasonal filter that removes unit roots at all seasonal frequencies (i.e. \( \omega = 1/2 \) and 1/4), whereas \( S_2(L) \) eliminate unit roots at seasonal frequencies \( \omega = 1/2 \) and \( \omega = 1/4 \), respectively, as well as at the zero frequency. In estimating Eq. (8), the unit root found at zero frequency in \( x_t \) implies accepting the null that \( \pi_1 \) is zero. Similarly, a finding \( \pi_2 = 0 \) implies a seasonal unit root −1. If both \( \pi_2 \) and \( \pi_4 \) are zero, a seasonal unit root \( \pm i \) exists. Hence, rejection of both a test for \( \pi_2 \) and a joint test for \( \pi_3 \) and \( \pi_4 \) implies the absence of seasonal unit roots. HEGY proposed \( t \)-statistics for \( \pi_1 = 0 \) and \( \pi_2 = 0 \) and an \( F \)-statistic for \( \pi_3 = \pi_4 = 0 \), which are denoted \( t_1 \), \( t_2 \) and \( F_{34} \) respectively. Ghysels et al. (1994) propose \( F \)-statistics to test \( \pi_2 = \pi_3 = \pi_4 = 0 \) and \( \pi_1 = \pi_2 = \pi_3 = \pi_4 = 0 \), which are denoted as \( F_{234} \) and \( F_{1234} \) respectively.

3.2. Lee’s seasonal cointegration

The testing procedure of seasonal cointegration based on the contribution of Lee (1992) is briefly introduced below. We assume that \( \epsilon_t \) are i.i.d. \( n \)-dimensional Gaussian random vectors with zero mean and a variance–covariance matrix \( \Omega \), whereas \( X_t \) is an \( n \)-dimensional vector with components \( (x_{1t}, x_{2t}, \ldots, x_{nt}) \), and \( p \) is the lag length, the estimation model can be expressed as:
\[ \Delta_4 X_t = \Pi_1 Y_{1t-1} + \Pi_2 Y_{2t-1} + \Pi_3 Y_{3t-1} + \Pi_4 Y_{4t-1} + A_1 \Delta_4 X_{t-1} + \cdots + A_1 \Delta_4 X_{t-p+4} + \epsilon_t \]  

(9)

which is similar to the seasonal unit root test of Eq. (8) except that the lower case \( x \) and \( y \) denote univariate processes, whereas the capital letter \( X \) and \( Y \) denote multivariate processes. Namely, \( Y_{1t} \) and \( Y_{2t} \) are equal to \( S_1(L)X_t \) and \( S_2(L)X_t \), respectively. For notational convenience, Lee redefined \( Y_{3t-1} \) as \( L(1-L^3)X_t \) and \( Y_{4t-1} \) as \( (1-L^3)X_t \). As the coefficient matrices \( \Pi_1, \ldots, \Pi_4 \) convey information concerning the long-run behavior of the series, their properties must be investigated in detail. If the matrix \( \Pi_4 \) has full rank, all series considered are stationary at the corresponding frequency. If the rank of \( \Pi_4 \) is zero, seasonal cointegration at that frequency does not exist among the variables. For the intermediate case in which \( 0 < \text{rank}(\Pi_4) = r < n \), a linear combination of non-stationary variables is stationary at the corresponding frequency.

Lee recommended four tests concerning the rank of \( \Pi_4 \) to implement tests of seasonal cointegration. Namely, a cointegration test at frequency \( \omega = 0, \omega = 1/2, \omega = 1/4 \) and the joint test of \( \omega = 1/4 \) and \( \omega = 3/4 \). Since the last two tests are asymptotically equivalent, only the test of \( \omega = 1/4 \) is implemented here.  

4. Empirical results

4.1. Results of the test for seasonal unit roots

Quarterly data covering the period from 1961:1 to 1995:2 are used. Two measures of money aggregates, M1B and M2, are alternatively employed. International reserves \( r \) are defined as the foreign assets of the central bank of Taiwan. The proxy scale variable \( y \) is taken to be real GNP and the exchange rate \( ex \) is represented by the amount of New Taiwan (NT) per US$ dollar. The consumer price index \( cpi \) is used to transform nominal variables into real terms. The interest rate \( r1 \) is proxied by the 1-month time deposit rate of the first commercial bank since a consistent series for money market rates was not available until 1981. The definition of variability in the balance of payments \( s \) is the same as Elbadawi (1990) and Ford and Huang (1994) and, therefore, is not defined here. With the exception of \( r1 \), all variables are in natural logarithms. Fig. 1–7 plots the seven variables with their seasonal transformations.

Table 1 presents the HEGY seasonal unit root tests. The lag length listed, which is designed to remove autocorrelation from the residuals, is based on the Akaike information criterion (AIC). According to the table, the \( t \)-values for the null hypothesis of \( \pi_1 = 0 \) for all variables are smaller than the 5% critical values listed

\[ \text{Hence, } Y_{3t-1} \text{ and } Y_{4t-1} \text{ in (9) correspond to } y_{3t-2} \text{ and } y_{3t-1} \text{ in (8), respectively.} \]

\[ \text{The test of seasonal frequency at } \omega = 1/4, \text{as noted by Lee (1992), is tested on the matrix } \Pi_4 \text{ only on assuming } \Pi_4 = 0 \text{ when cointegration is contemporaneous.} \]
in HEGY regardless of the deterministic components. Thus, the null of the seasonal unit root at zero frequency is accepted by all variables in question. In addition, the $t$-statistic for $\pi_2 = 0$ is rejected for $r$, $ex$, $p$, $r_1$ and $\sigma$, but cannot be rejected for M1B, M2 and $y$. However, for $im$, $\pi_2 = 0$ is rejected when seasonal dummies are excluded; but it is accepted when the same dummies are included. As Osborn et al. (1988) stated, the inclusion of seasonal dummies when testing for seasonal unit roots is important; otherwise, variables tend to display stochastic seasonality even when they contain only deterministic seasonality. Thus, we can conclude that $im$ contains a seasonal unit root at the biannual frequency. The hypothesis of $\pi_3 = \pi_4 = 0$ cannot be rejected only for the cases of $r$ and $y$. Except for $im$ and $y$, the hypothesis of $\pi_3 = \pi_4 = \pi_5 = 0$ and $\pi_3 = \pi_2 = \pi_3 = \pi_4 = 0$ are rejected for all variables. Since $t$-tests display conflicting evidence against $F$-tests, we have to decide which one to use. We prefer to use $t$-tests because their results are consistent with the stylized fact that conventional unit roots exist in the most macro series. However, further research is needed to pursue this issue.

In summary, all series under consideration are non-stationary at zero frequency. Although the logarithm of real GNP ($y$) presents seasonal unit roots at additional 1/2 and 1/4 frequencies, other series are seasonally integrated only at either the
1/2 or 1/4 frequency.

Let notation $x(i, i, i)$ denote whether or not the variable $x$ has unit roots at either zero, 1/2 or 1/4 frequencies, respectively. If $i = 1$, there is a unit root at the corresponding frequency, otherwise, if $i = 0$ it is not. Hence, we can infer the following from Table 1:

1. (C, T): $M1B(1,1,0)$, $M2(1,1,0)$, $r(1,0,0)$, $e(1,0,0)$, $cpi(1,0,0)$, $r1(1,1,0)$, $im(1,0,0)$, $y(1,1,1)$, $s(1,0,0)$

2. (C, SD) and (C, T, SD): except for $im(1,1,0)$, other variables are the same as above.

4.2. The demand for money and seasonal cointegration

Since money market disequilibrium is one of the crucial determinants in the demand for reserves and since it contains seasonal unit root at the 1/2 frequency, its projected values should be obtained on the basis of the SECM specification.

Table 2A,B summarize the seasonal cointegration testing results for $M1B$ and $M2$, respectively. The deterministic components, C, T, and SD, will be included in
the statistical models if they are significant. Since the finite sample quantiles provided by Lee (1992) do not consider any of the deterministic components, the quantile distributions by Monte Carlo simulations are computed here using 30,000 replications with deterministic terms contained in the statistical models.

Table 2A reveals that the five variables, $M_1$, $y$, $r_1$, $e$, and $p$ are all cointegrated at the zero frequency. The corresponding normalized vector of cointegrating coefficients, $\beta$, is \( (1, -4.5800, 0.0929, 0.9793, 0.6870) \). Both signs and magnitudes agree well with existing monetary studies. Namely, income has a positive effect, and the interest rate, exchange rate and inflation rate negatively impact the demand for money.

Since five series are used in the VAR system, there are five error correction terms. The coefficients on these five terms are represented by the vector of $a = (0.0067, 0.0013, 0.0836, 0.0027, 0.0021)$, which are interpreted as the

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7 We have examined whether the residuals of above equations are normal or not. Results show that only $y$ and $r$ pass the normality tests of Bera–Jarque. However, errors with non-Gaussian distributions will not substantially alter the properties owned by the maximum likelihood estimators, as Gonzalo (1994) has pointed out.
average speeds of adjustment towards the long-run equilibrium or the loadings (Johansen and Juselius, 1990). A negative number indicates that when an excess demand or supply occurs, market forces will eliminate the disequilibrium and bring the system back to a steady state. The first element of $\alpha_1$ corresponds to the adjustment speed for the monetary disequilibrium and is roughly $-0.0067$. Thus, the excess demand for money will be at least partially adjusted downward in the next period, so that disequilibrium in the money market would be eliminated eventually.

Since only M1B and $y$ contain seasonal unit roots at the $1/2$ frequency, Lee’s seasonal cointegration test is again applied to these two series only. The two series are discovered to be cointegrated at this frequency with cointegrating vector $\beta_2 = (1, -0.8742)$. This income elasticity is smaller than that at zero frequency, implying that a seasonal increase in income would have a smaller effect on money holding than an increase in income at zero frequency. The adjustment speed of the money market at the $1/2$ frequency, which is equal to $0.0073$, is the first element of $\alpha_2$ in Table 2A. This non-negative number, contrary to the result at the zero frequency, does not imply the adjustment will make the system gradually deviate from the equilibrium. As demonstrated by Shen and Huang (1998), the loadings of
seasonal error correction terms can be positive even if the adjustment is heading toward equilibrium at seasonal frequencies. Lee (1992) provides a similar report. However, the economic meaning of a positive loading is unclear.

In Table 2B, two long-run equilibrium relationships among the five series, M2, y, r1, ex, and p are discovered. Though the elements of the two cointegrating vectors, β1 and β2, differ in magnitudes, they have the same signs thereby corresponding to the predictions of the existing monetary theories. The speed of adjustment of M2 error corrections corresponding to the two long-run equilibrium errors are 0.0056 and 0.0040, respectively. Interestingly, those numbers are smaller than that in M1B, indicating that the adjustment speed towards steady state is slower for the broad money than for the narrow money. This appears to be reasonable since M2 contains both transaction and saving assets and, thus, is less liquid than M1B, which contains mainly transaction assets.

8 Lee (1992) examined the existence of seasonal cointegration relations at different frequencies using quarterly data on unemployment and immigration rates from Canada. The coefficient on the seasonal error correction term at 1/2 frequency was also positive.
The seasonal cointegration test is again applied to M2 and $y$ only at the 1/2 frequency, since only these two series contain a seasonal unit root at the frequency. The (normalized) income elasticity is 0.84 and the loading is $-0.0017$.

Since only $y$ contains unit root at the frequency 1/4, testing the seasonal cointegration at this frequency is not necessary.

Once the seasonal cointegrating vectors are detected for the two measures of money aggregates, the next step is to derive the linear projectors $\Delta y_{t-4}$ and $\Delta y_{t}$. This can be achieved by estimating the SECM with the seasonally differenced money aggregates as dependent variables and the lagged fourth order seasonally differenced explanatory variables in the model. The seasonal error correction terms are also included on the right hand side of the regression equation. The estimated results of the two SECMs are not presented, but are available upon request. Their linear projections then function as explanatory variables and are included in Eq. (5).

4.3. The estimation of demand for international reserves

The term $EC_{t-4}$ in Eq. (5) requires that the series $r$, $y$, $im$ and $\sigma$ (including the
intercept) be cointegrated at the conventional frequency. This conjecture can be tested by means of Johansen’s maximum likelihood procedure. The two test statistics of Johansen’s method, λ-max (the maximal eigenvalue statistic) and the trace statistics, lead to the same conclusion of one cointegrating relationship among the four series.9 The equilibrium error term, which is a stationary process, can be written as

\[ EC_t = r_t - 1.9024y_{t-1} - 0.1742im_{t-1} - 0.2174\sigma_{t-1} + 21.7767 \]  

where the cointegrating coefficients reveal that \( y \), \( im \) and \( \sigma \) positively influence the demand for reserves.

Once the \( EC_{t-1} \) is obtained, it will be treated as an explanatory variable in Eq. (5). Hence, estimating Eq. (5) by OLS is now free of the criticism of spurious regression since all variables are now stationary.

Notably, Eq. (5) may be still misspecified by omitting the seasonal error correction terms (if these exist). More specifically, since seasonal unit roots exist at the

---

8 Results are not reported and are available upon request.
Table 1
HEGY seasonal unit root test: $t$-values

<table>
<thead>
<tr>
<th></th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$F_{34}$</th>
<th>$F_{234}$</th>
<th>$F_{1234}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. M1B (lag = 4, $N = 126$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-3.133</td>
<td>-1.142</td>
<td>20.898$^a$</td>
<td>14.721$^a$</td>
<td>13.266$^a$</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-3.288</td>
<td>-2.981</td>
<td>23.656$^a$</td>
<td>21.393$^a$</td>
<td>18.503$^a$</td>
</tr>
<tr>
<td>2. M2 (lag = 5, $N = 125$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-3.450</td>
<td>-1.527</td>
<td>18.930$^a$</td>
<td>13.381$^a$</td>
<td>15.100$^a$</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-3.258</td>
<td>-2.668</td>
<td>22.485$^a$</td>
<td>17.071$^a$</td>
<td>18.114$^a$</td>
</tr>
<tr>
<td>3. $r$ (lag = 1, $N = 130$)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-2.180</td>
<td>-6.391</td>
<td>40.771$^a$</td>
<td>54.157$^a$</td>
<td>49.435$^a$</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-2.154</td>
<td>-6.335</td>
<td>39.762$^a$</td>
<td>52.991$^a$</td>
<td>48.378$^a$</td>
</tr>
<tr>
<td>4. ex (lag = 0, $N = 133$)</td>
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</tr>
<tr>
<td>C,T</td>
<td>-2.751</td>
<td>-6.391</td>
<td>89.779$^a$</td>
<td>213.75</td>
<td>167.60</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-2.733</td>
<td>-6.335</td>
<td>91.649$^a$</td>
<td>219.21</td>
<td>171.81</td>
</tr>
<tr>
<td>5. $p$ (lag = 0, $N = 133$)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-1.710</td>
<td>-8.015</td>
<td>71.210$^a$</td>
<td>253.94</td>
<td>190.73</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-1.726</td>
<td>-7.730</td>
<td>72.888$^a$</td>
<td>255.84</td>
<td>192.16</td>
</tr>
<tr>
<td>6. $r_1$ (lag = 0, $N = 133$)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-3.142</td>
<td>-9.336</td>
<td>46.465$^a$</td>
<td>361.26</td>
<td>274.60</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-3.143</td>
<td>-9.367</td>
<td>45.146$^a$</td>
<td>362.83</td>
<td>275.77</td>
</tr>
<tr>
<td>7. im (lag = 5, $N = 128$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-1.676</td>
<td>-2.301</td>
<td>7.607$^b$</td>
<td>6.799$^a$</td>
<td>5.883$^a$</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-1.670</td>
<td>-2.475</td>
<td>10.820$^b$</td>
<td>9.151$^a$</td>
<td>7.673$^a$</td>
</tr>
<tr>
<td>8. $y$ (lag = 5, $N = 128$)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-1.200</td>
<td>-1.847</td>
<td>2.750$^b$</td>
<td>2.996$^b$</td>
<td>2.667</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-1.167</td>
<td>-1.452</td>
<td>3.131$^b$</td>
<td>2.794</td>
<td>2.519</td>
</tr>
<tr>
<td>9. $\sigma$ (lag = 0, $N = 117$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C,T</td>
<td>-3.085</td>
<td>-5.711</td>
<td>22.057$^a$</td>
<td>27.904$^a$</td>
<td>22.955$^a$</td>
</tr>
<tr>
<td>C,T,SD</td>
<td>-3.061</td>
<td>-5.665</td>
<td>21.740$^a$</td>
<td>27.486$^a$</td>
<td>22.596$^a$</td>
</tr>
</tbody>
</table>

$^a$Significant at the 1% level.
$^b$Significant at the 5% level.
Critical value (taken from [HEGY] and Ghysels (1994).
5% ($N = 136$ for $t_1$, $t_2$, $F_{34}$ and $N = 100$ for $F_{234}$ and $F_{1234}$)
C, T     - 3.46  - 3.04  2.76  4.26
C, T, SD  - 3.52  - 2.93  6.62  5.99  6.47
1% ($N = 136$)
C, T     - 4.09  - 2.65  4.57  na  na
C, T, SD  - 4.15  - 3.57  8.77  na  na

$1/2$ and/or $1/4$ frequencies for the series $y$, $im$, and $r$, they may be cointegrated at those frequencies. If cointegrating relationships indeed exist at the $1/2$ or $1/4$ frequencies, then the conventional ECM models, such as those used by Elbadawi (1990), Johansen and Juselius (1990), and Ford and Huang (1994) among others, may be subject to ‘the omitted variable problem’.
Table 2
Seasonal cointegration test

### A: M1B Money demand function

\[ \Delta_4 Y_t = C_0 + C_1 T + \Phi_i Y_{1,t-1} + \Phi_i Y_{2,t-1} + \Phi_i Y_{3,t-1} + \Phi_i Y_{4,t-1} + \Phi_i Y_{5,t-1} + \Phi_i Y_{6,t-1} + A(L) \Delta_4 Y_t + \epsilon_t \]

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Rank = 0</th>
<th>Rank = 1</th>
<th>Rank = 2</th>
<th>Rank = 3</th>
<th>Rank = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega = 0 )</td>
<td>96.2935</td>
<td>60.2517</td>
<td>26.4112</td>
<td>7.3341</td>
<td>0.2892</td>
</tr>
<tr>
<td>( \omega = 1/2 )</td>
<td>33.9329</td>
<td>7.3713</td>
<td>1.5232</td>
<td>0.3712</td>
<td>0.1234</td>
</tr>
<tr>
<td>Eigenvectors(( \beta ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero frequency</td>
<td>Error-correction coefficients(( \alpha ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>( \alpha_1 )</td>
<td>( \beta_2 )</td>
<td>( \alpha_2 )</td>
<td>( \beta_3 )</td>
<td>( \alpha_3 )</td>
</tr>
<tr>
<td>M1B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>-4.5800</td>
<td>0.0013</td>
<td>-0.8742</td>
<td>-0.0027</td>
<td></td>
</tr>
<tr>
<td>( r_1 )</td>
<td>0.0929</td>
<td>-0.0856</td>
<td>-0.0046</td>
<td>-0.0021</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>0.9793</td>
<td>0.0227</td>
<td>-0.0020</td>
<td>-0.0028</td>
<td></td>
</tr>
<tr>
<td>( p )</td>
<td>0.6870</td>
<td>-0.0021</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1B critical values (( N = 136 ), generated by authors)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \omega = 0 )</td>
<td>( n - r = 5 )</td>
<td>( n - r = 4 )</td>
<td>( n - r = 3 )</td>
<td>( n - r = 2 )</td>
<td>( n - r = 1 )</td>
</tr>
<tr>
<td>95%</td>
<td>87.442</td>
<td>61.747</td>
<td>40.994</td>
<td>24.349</td>
<td>11.755</td>
</tr>
<tr>
<td>99%</td>
<td>96.339</td>
<td>69.695</td>
<td>47.502</td>
<td>27.694</td>
<td>15.770</td>
</tr>
<tr>
<td>( \omega = 1/2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95%</td>
<td>18.488</td>
<td>8.238</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99%</td>
<td>23.356</td>
<td>12.119</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### B: M2 money demand function

\[ \Delta_4 Y_t = C_0 + C_1 T + \Phi_i Y_{1,t-1} + \Phi_i Y_{2,t-1} + \Phi_i Y_{3,t-1} + \Phi_i Y_{4,t-1} + \Phi_i Y_{5,t-1} + \Phi_i Y_{6,t-1} + A(L) \Delta_4 Y_t + \epsilon_t \]

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Rank = 0</th>
<th>Rank = 1</th>
<th>Rank = 2</th>
<th>Rank = 3</th>
<th>Rank = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega = 0 )</td>
<td>103.9689</td>
<td>62.8085</td>
<td>30.7120</td>
<td>6.0518</td>
<td>0.1510</td>
</tr>
<tr>
<td>( \omega = 1/2 )</td>
<td>21.2934</td>
<td>2.3510</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eigenvectors(( \beta )) and error-correction coefficients (( \alpha )) (( \alpha ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero frequency</td>
<td>1/2 frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_{11} )</td>
<td>( \beta_{12} )</td>
<td>( \alpha_{11} )</td>
<td>( \alpha_{12} )</td>
<td>( \beta_{21} )</td>
<td>( \alpha_{21} )</td>
</tr>
<tr>
<td>M2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )</td>
<td>-5.0895</td>
<td>-1.6585</td>
<td>-0.00033</td>
<td>0.0031</td>
<td>-0.8427</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>0.1685</td>
<td>0.0169</td>
<td>-0.2299</td>
<td>-0.0347</td>
<td>-0.0083</td>
</tr>
<tr>
<td>( c )</td>
<td>1.0525</td>
<td>0.5000</td>
<td>0.0086</td>
<td>-0.0094</td>
<td>-0.0050</td>
</tr>
<tr>
<td>( p )</td>
<td>0.7775</td>
<td>0.6664</td>
<td>-0.0043</td>
<td>0.0020</td>
<td>0.0054</td>
</tr>
</tbody>
</table>

\( a \) Significant at the 1% level.

\( b \) Significant at the 5% level.

The critical values of M2 are close to those of M1B and are available from authors.

Table 3 summarizes the testing results. Similar to Table 2A,B, the finite sample quantile distributions are calculated by Monte Carlo simulations using 30,000 replications. At the 1/2 frequency, \( y \) and \( im \) are found to be cointegrated with the equilibrium error process as:

\[
SEC_{2,1} = S_2(L)(y_{1,1} + 0.2325im_{1,1})
\]
Table 3
Demand for international reserves equations: seasonal cointegration test model:

\[ \Delta_t X_t = C_0 + \psi D_t + \sum \gamma_i \Delta_t Y_{t-i} + \sum \rho_i \Delta_t X_{t-i} + \lambda(L)\Delta_t X_{t-1} + \epsilon_t \]

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Rank = 0</th>
<th>Rank = 1</th>
<th>Rank = 2</th>
<th>Rank = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega = 0)</td>
<td>53.149(^a)</td>
<td>29.026</td>
<td>11.379</td>
<td>1.654</td>
</tr>
<tr>
<td>(\omega = 1/2)</td>
<td>27.396(^a)</td>
<td>4.721</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\omega = 1/4)</td>
<td>19.951</td>
<td>4.497</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Eigenvectors at \(\omega = 0\):

\[ r = 1 \]
\[ y = -1.7279 \]
\[ \sigma = -0.3956 \]
\[ im = 3.4126 \]

Critical values (\(N = 100\), critical values are generated by the authors)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>(n - r = 4)</th>
<th>(n - r = 3)</th>
<th>(n - r = 2)</th>
<th>(n - r = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega = 0)</td>
<td>52.358</td>
<td>33.125</td>
<td>18.350</td>
<td>8.287</td>
</tr>
<tr>
<td>95%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99%</td>
<td>59.886</td>
<td>37.449</td>
<td>23.002</td>
<td>11.885</td>
</tr>
<tr>
<td>(\omega = 1/2)</td>
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<tr>
<td>95%</td>
<td>18.406</td>
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<tr>
<td>99%</td>
<td>23.361</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\omega = 1/4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95%</td>
<td>23.312</td>
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</tr>
<tr>
<td>99%</td>
<td>28.768</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Significant at the 1% level.
\(^b\)Significant at the 5% level.

\(D_t\) is the centered seasonal dummies.
\(\Lambda(L)\) is the lagged seasonal difference, lag 5.

The subscript 2 denotes 1/2 frequency. In contrast, no cointegration relationships are found between \(y\) and \(r\) at the 1/4 frequency. Accordingly, only \(SEC_{2,1}\) is used as an extra variable in Eq. (5).

Table 4 reports the estimation results of the seasonal cointegration specification (5) with the term \(SEC_{2,1}\) being added to the conventional cointegration specification (6). Furthermore, both measures of monetary aggregates are employed. For each specification, all estimated coefficients have the same signs and similar magnitudes, regardless of which monetary aggregate is used. This fact implies that the level of reserve holdings is unaffected by different definitions of monetary disequilibria. While the estimated coefficients for the \(EC_{2,4}\) and \(EC_{1,1}\) have anticipated negative signs; their absolute values are all small, ranging from \(-0.056\) to \(-0.033\). Such a low speed of adjustment is quite different from previous studies in this area. For instance, Elbadawi (1990) obtained a high adjustment coefficient of \(-0.57\) using quarterly data on Sudan. Even higher speeds of adjustment of
Table 4: Estimates of the demand for international reserves

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>SECM</th>
<th>ECM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Δt r_{t-1} (M1B)</td>
<td>Δt r_{t-1} (M2)</td>
</tr>
<tr>
<td>EC_{t-4}</td>
<td>-0.0331 (0.0243)</td>
<td>-0.0501** (0.0260)</td>
</tr>
<tr>
<td>Δ_t \delta r_{t-4}</td>
<td>-0.2229* (0.1198)</td>
<td>-0.2419 (0.1972)</td>
</tr>
<tr>
<td>SEC_{2t-1}</td>
<td>-0.0269** (0.0123)</td>
<td>-0.0230* (0.0130)</td>
</tr>
<tr>
<td>Δ_t y_t</td>
<td>0.5232*** (0.1841)</td>
<td>0.5557*** (0.2831)</td>
</tr>
<tr>
<td>Δ_t m_t</td>
<td>-0.1130 (0.0749)</td>
<td>-0.2098*** (0.0809)</td>
</tr>
<tr>
<td>Δ_t \sigma_t</td>
<td>0.0018 (0.0029)</td>
<td>0.0024 (0.0033)</td>
</tr>
<tr>
<td>Δ_t^2 \delta r_t</td>
<td>-0.2199** (0.0897)</td>
<td>-0.1965 (0.1334)</td>
</tr>
</tbody>
</table>

adj-$R^2$ | 0.9260 | 0.9156 | 0.3906 | 0.4769 |
Ljung–Box (10) | 3.01 | 11.17 | 5.07 | 6.10 |
Ljung–Box (20) | 11.81 | 23.99 | 22.66 | 15.53 |
ARCH (1) | 0.94 | 0.15 | 0.23 | 0.73 |
Jarque–Bera | 2.56 | 2.06 | 13.12*** | 26.85** |
Normality test | 1.20 | 0.96 | 2.53*** | 2.24** |

*: **: ***: significant at the 10, 5 and 1% level, respectively.
Standard errors in parentheses.

Note. The estimated coefficients of lagged dependent variables on the right hand side of the above equations are not reported for the sake of space. The lags are 10, 9, 6, and 6 for the above four equations from left to right, respectively. The Chow test is based on the splitting point 1986:1, see text and the corresponding footnote for details.

-0.81 and -0.52 were derived by Ford and Huang (1994) employing annual narrow and broad monetary aggregates, respectively, from mainland China.

The low adjustment speed of Taiwan’s demand for reserves could be accounted for by the theory of Clark (1970a). As contended by Clark (1970a), a tradeoff arises between the speed of adjustment and the level of reserves held by a country. A country with a lower speed of adjustment heading towards equilibrium requires a higher level of reserves as a buffer stock to finance its payments problems and vice versa. Since Taiwan is well-known for its huge stock of international reserves,10 the

10 The balance-of-payment surplus of Taiwan amounted to $US90 billion at the end of 1993, which was the second largest globally.
low speed of adjustment appears to be reasonable. The coefficient estimates of the term $SEC_{t-1}$ are negative and significant, at least at the 10% level, indicating that the conventional EC model may be inhibited by model misspecification.

While the money market disequilibria are found to have significantly negative effects on the demand for reserves in the seasonal specification, the results are reversed in the conventional EC models. Recall that the effect of money market disequilibrium on the demand for reserves is uncertain. Namely, the monetary approach suggests a positive effect, whereas a reconciliation argument postulates a negative effect. Thus, the negative effect yielded by the seasonal specification favors the latter explanation, whereas the positive effect yielded by the conventional specification supports the former. To clarify which specification is preferable, diagnostic checks can provide valuable insight and will be implemented shortly.

Income $y$ is found to have a significantly positive (as expected) effect on the demand for reserves in the seasonal model, but is found lacking in the conventional model. Both models indicate that variability $\sigma$ has no effect on reserve movements. The coefficients on the average propensity to import $im$ are all negative; nevertheless, all of them are statistically insignificant.

The diagnostic checks are reported in the bottom panel of Table 4. Since these test statistics can be found in the most econometric textbooks, introductions of their definitions are omitted to save space.

Both specifications pass the Ljung–Box $Q(10)$ and $Q(20)$ tests for no residual autocorrelation, where numbers in the parentheses denote the numbers of lagged autocorrelations. Also, no ARCH effect is detected regardless of the specifications. However, the conventional specification fails, but the seasonal specification passes, both the Jarque–Bera normality test and the Chow parameter stability test. Finally, the adjusted $R^2$ is higher for the seasonal model than the conventional model, suggesting a better fit for the former. Thus, on the basis of the above diagnostics, the seasonal specification appears to be preferable to the conventional one.

4.4. The structural parameters

The size of the income elasticity has attracted much attention in previous studies involving demand for reserves. The elasticity, which may be greater than, equal to, or less than unity, indicates the diseconomies, constant economies and economies of scales in holding reserves, respectively. This elasticity may be calculated by recovering the structural parameters in Eq. (4) from the reduced form parameters listed in Table 4. For instance, the structural parameter $\lambda_i$ is simply the absolute

11 Selecting a breaking point for the Chow test is based on preliminary checking of the plots of the cumulative sum (CUSUM) and cumulative sum of squares (CUSUMSQ) test. These two plots, which are not reported here, but are available upon request, display a tendency for breaking at 1977 and 1986. Selecting any breaking point between these two dates (there are 40 splits in total), the Chow test overwhelmingly rejects the null of no structural break for the conventional specification. By contrast, the null cannot be rejected for the seasonal specification. The Chow tests reported in Table 4 uses the breaking point 1986.
Table 5
Reserved structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Seasonal model</th>
<th>Conventional model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta r_t$</td>
<td>$\Delta r_t$</td>
</tr>
<tr>
<td></td>
<td>(M1B)</td>
<td>(M2)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>16.0255***</td>
<td>13.6460**</td>
</tr>
<tr>
<td></td>
<td>(5.6382)</td>
<td>(6.9521)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.0539</td>
<td>0.0593</td>
</tr>
<tr>
<td></td>
<td>(0.0884)</td>
<td>(0.0800)</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-3.4609</td>
<td>-5.1519***</td>
</tr>
<tr>
<td></td>
<td>(2.2953)</td>
<td>(1.9872)</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-6.7357</td>
<td>-4.8258</td>
</tr>
<tr>
<td></td>
<td>(6.5256)</td>
<td>(4.7103)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.0331</td>
<td>0.0501**</td>
</tr>
<tr>
<td></td>
<td>(0.0243)</td>
<td>(0.0260)</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.0326**</td>
<td>0.0407</td>
</tr>
<tr>
<td></td>
<td>(0.0133)</td>
<td>(0.0276)</td>
</tr>
</tbody>
</table>

$, **$, and $***$: significant at the 10, 5 and 1% level, respectively.
Standard error in parenthesis.

value of $b_1$. The structural parameter $\lambda_2$ is obtained by calculating $b_4/a_4$ where $a_4 = b_2/\lambda_1$. The remaining structural parameters can be recovered using the same manner and are reported in Table 5 for both specifications.

Once the structural parameters are recovered, the null hypothesis of a unitary income elasticity can be tested. The null is

$$H_0: a_1 - 1 = b_3/\lambda_2 - 1 = 0$$

Since $a_1$ is 16.03 or 13.65 when M1B or M2 disequilibrium is incorporated in the seasonal specifications, respectively, the null hypothesis is rejected. The demand for international reserves by Taiwan thus faces strong diseconomies of scale. In contrast, $a_1$ has a wrong negative sign in the conventional specification. This finding again suggests that the seasonal model is preferable to the conventional model.

The above diseconomies may be attributed to various regulations imposed on the holding of foreign exchange. Prior to 1986, civilians in Taiwan could buy foreign exchange only with the permission of the monetary authorities for a limited amount annually and for some pre-specified purposes. Firms had to sell their foreign exchange, earned by exporting goods and services, to the central bank. These restrictions have been gradually lifted since 1986 and are now a self-fueled ongoing processes for the Central Bank. Similar strong diseconomies of scale were also obtained by Edwards (1984) for other LDCs.
found the income elasticity to be only 0.687, while Elbadawi (1990) accepted the null of constant returns to scale.

5. Concluding remarks

On the basis of a conventional one-period loss function involving official growth rate computed from seasonal differences, a dynamic demand function for international reserves is derived. This small modification distinguishes the present study from previous studies in three aspects. First, the dependent variable is seasonally differenced instead of being first order differenced. Next, the local money market disequilibrium included is also in fourth difference form. Finally, given the existence of stochastic seasonality, a new model, using a seasonal error correction rather than the conventional error correction, is specified and estimated. The following inferences are drawn from the seasonal specification.

1. Regarding the demand for a money equation, a long-run equilibrium relationship exists at the zero frequency among money, income, the exchange rate, and the price level. Moreover, money and income are seasonally cointegrated at the biannual frequency.

2. With respect to the demand for international reserves, a long-run relationship is discovered among reserves, income, the average propensity to import, and a measure of the variability in the balance of payments at the zero frequency. Also, income and the average propensity to import are seasonally cointegrated at the biannual frequency.

3. The current period's change in income has a significantly positive effect on reserve holdings in the short-run, but is smaller in magnitude than its long-run counterpart.

4. The monetary authorities of Taiwan respond quite slowly to deviations from the desired demand for reserves occurring four periods before. This is most likely due to regulations imposed on private reserve holdings on the one hand, as well as a tradeoff between the speed of adjustment and the stock of reserves a country is willing to hold on the other.

5. While the narrow money market disequilibrium (M1B) displays a short-run effect on the demand for reserves, the broad money market disequilibrium (M2) exerts no significant influences on the reserve holdings in both the short- and long runs.

6. The precautionary motive is found to be an unimportant determinant of the demand for reserves, possibly due to the fact that Taiwan has accumulated a large stock of foreign exchange during the past two decades (relative to its trade flows) and, therefore, the risk of an external payment problems would be very low.

7. Since the equilibrium error from the biannual frequency significantly affects the demand for reserves, it has to be incorporated in the seasonal EC model. This term is neglected in the conventional EC model.
References

