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Alternative errors-in-variables models and their applications in finance research

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ABSTRACT

Specification error and measurement error are two major issues in finance research. The main purpose of this paper is (i) to review and extend existing errors-in-variables (EIV) estimation methods, including classical method, grouping method, instrumental variable method, mathematical programming method, maximum likelihood method, LISREL method, and the Bayesian approach; (ii) to investigate how EIV estimation methods have been used to finance related studies, such as cost of capital, capital structure, investment equation, and test capital asset pricing models; and (iii) to give a more detailed explanation of the methods used by Almeida et al. (2010).

1. Introduction

Specification error and measurement error are two major issues in applying the econometric model to economic and finance research. Studies by Miller and Modigliani (1966) and Roll (1969) are two of the earliest finance related research studies to apply errors-in-variables (EIV) model in their empirical works. Miller and Modigliani (1966) show that, in determining the cost of capital, anticipated average earnings are unobservable and using accounting estimates of earnings as the proxy may result measurement error problems. Roll (1969, 1977) and Lee and Jen (1978) show that the observed market rate returns in terms of stock market index are measured with errors since the stock market index does not include all assets which can be invested by investors. Roll (1969, 1977) argues that testing capital asset pricing model suffers from an EIV problem and concludes that no correct and unambiguous test of the theory can be accomplished. Lee and Jen (1978) have theoretically shown how beta estimates and Jensen performance measures can be affected by both constant and random measurement errors of the market rate of return and risk free rate. Other issues such as the determination of the capital structure and investment functions also suffer EIV problems.[1]

Understanding the existence of measurement error problems on finance related studies, a large extent of the literature subsequently tries to mitigate biased results from measurement errors. For the issue of the estimation of the cost of capital, Miller and Modigliani (1966) use the instrumental variable approach to resolve the measurement error problem and get consistent estimators in determining the cost of capital. Zellner (1970) and Lee and Wu (1989) also uses various estimation methods to deal with potential EIV problems on estimating the cost of capital. For the issue of the

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[1] For the measurement problems related to the determinants of the capital structure, please see Titman and Wessels (1988), Chang et al. (2009), and Yang et al. (2009). For the measurement problems related to the investment function, please see Erickson and Whited (2000, 2002) and Almeida et al. (2010).

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capital asset pricing test, Lee and Jen (1978) argue that both market return and beta coefficient are subjected to measurement error, and show how the beta coefficient can be estimated. Lee (1984) shows that the most generalized beta estimate can be decomposed into three components; bias due to specification error, bias due to measurement error, and interaction bias. Therefore, the evidence of failure in capital asset pricing model or the finding of new risk factors might result from model misspecification error or EIV problem. Gibbons and Ferson (1985), Green (1986), Roll and Ross (1994) and Diacogiannis and Feldman (2011) have argued that market portfolio measure with errors is an inefficient portfolio and show how the inefficient benchmark can affect the theoretical CAPM derivation. For the issue of the determinants of the capital structure, Titman and Wessels (1988), Chang, Lee, and Lee (2009) and Yang, Lee, Gu, and Lee (2009) apply structure equation models to investigate determinants of the capital structure. For the measurement error problems related to Tobin's q in investment function, Erickson and Whited (2000) use generalized method of moments (GMM) to obtain consistent estimators in testing q theory. Most recently, Almeida, Campello, and Galvao (2010) propose an alternative instrumental method to deal with measurement error problems in Tobin's q and support the q theory.

The main purpose of this paper is to study existing EIV estimation methods and to discuss how these estimation methods have been used in finance research. We first show how EIV problems affect estimators in the regression model. We further demonstrate seven alternative estimation methods dealing with EIV problems. Classical method, grouping method, instrumental variable method, mathematical programming method, maxima likelihood method, LISREL method, and Bayesian approach will be discussed. Finally, we conduct a survey on various studies and investigate the effect that resulted from EIV problems associated with cost of capital, capital asset pricing model, capital structure, and investment equation. We also investigate the correction models used in such studies to mitigate the problem raised from measurement errors.

The remainder of this paper is organized as follows. Section 2 shows the classical EIV problems and how they affect estimators of the linear regression model. Section 3 provides seven alternative correction methods in dealing with EIV problems. Section 4 presents the effects of EIV problems on the empirical research of cost of capital, asset pricing, capital structure, and investment decision. Finally, Section 5 presents the conclusion.

2. Effects of errors-in-variables in different cases

2.1. Bivariate normal case

Suppose we have a two variate structural relationship

$$V_i = \alpha + \beta U_i \tag{1}$$

Both V_i and U_i are unobserved, while we can observe $Y_i = V_i + \eta_i$ and $X_i = U_i + \varepsilon_i$. We assume that

(a) $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$ and $\eta_i \sim N(0, \sigma_\eta^2)$.
(b) $E(\varepsilon_i U_i) = 0$, $E(\varepsilon_i V_i) = 0$, $E(\varepsilon_i \eta_i) = 0$, $E(\eta_i U_i) = 0$, and $E(\eta_i V_i) = 0$.
(c) $U_i \sim N(E(X), \sigma_U^2)$ and $V_i \sim N(\alpha + \beta E(X), \beta^2 \sigma_U^2)$.

This results in measurement error on the estimates of α and β implying that the asymptotic biases for β and α are

$$\text{plim}\hat{\beta} - \beta = \frac{-\beta\sigma_1^2}{\sigma_U^2 + \sigma_1^2} \tag{2a}$$

$$\text{plim}\hat{\alpha} - \alpha = \frac{\beta\sigma_1^2}{\sigma_U^2 + \sigma_1^2} E(X). \tag{2b}$$

Eq. (2) implies that $\hat{\beta}$ is downward biased and $\hat{\alpha}$ is upward biased.

2.2. Multivariate case

Suppose we have a trivariate structural relationship

$$W_i = \alpha + \beta U_i + \gamma V_i \tag{3}$$

W_i, U_i, and V_i are unobserved, but we can observe $Z_i = W_i + \tau_i$, $X_i = U_i + \varepsilon_i$, and $Y_i = V_i + \eta_i$. U_i and V_i have a joint normal distribution with variances σ_U^2 and σ_V^2 and correlation coefficient ρ_{UV}. In the observed variables X, Y, and Z, the observed errors ε, η, and τ are independent normal variables with zero means and variance $\sigma_\varepsilon^2, \sigma_\eta^2, \sigma_\tau^2$. X, Y, and Z have a multivariate normal distribution.

The asymptotic biases of $\hat{\beta}$ and $\hat{\gamma}$ can be seen from the following:

$$\text{p}\lim \hat{\beta} - \beta = \frac{\sigma_{VW}\sigma_\eta^2 - \beta(\sigma_U^2\sigma_\eta^2 + \sigma_V^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2\sigma_\eta^2)}{(\sigma_U^2\sigma_V^2 - \sigma_{VW}^2) + \sigma_U^2\sigma_\eta^2 + \sigma_V^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2\sigma_\eta^2} \tag{4a}$$

$$\text{p}\lim \hat{\gamma} - \gamma = \frac{\sigma_{WV}\sigma_\varepsilon^2 - \gamma(\sigma_U^2\sigma_\eta^2 + \sigma_V^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2\sigma_\eta^2)}{(\sigma_U^2\sigma_V^2 - \sigma_{UV}^2) + \sigma_U^2\sigma_\eta^2 + \sigma_V^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2\sigma_\eta^2} \tag{4b}$$

The direction of the biases of $\hat{\beta}$ and $\hat{\gamma}$ can be treated according to different assumptions.[2]

Concerning the coefficient of the reliability, Cochran (1970) shows that measurement errors of both explained and explanatory variables will reduce the multiple correlations and increase the residual variance, and the good prediction formula is more sensitive to measurement errors than the poor one. Moreover, from the analysis of the effects of measurement error on both the simple regression coefficient and residual variance, in general, we can conclude that the t statistic of the simple regression coefficient will be downward biased if variables are measured with errors.

3. Estimation methods when variables are subject to error

In this section, we will discuss alternative EIV estimation methods, classical method, grouping method, instrumental variable method, mathematical method, maxima likelihood method, LISREL method, and the Bayesian approach.

3.1. Classical estimation method

3.1.1. The classical method to a simple regression analysis

In general, the classical method considers three cases: (i) either σ_ε^2 or σ_η^2 is known; (ii) $\lambda = \sigma_\eta^2/\sigma_\varepsilon^2$ is known; and (iii) σ_ε^2 and σ_η^2 are known. We can obtain the estimate for β from Eq. (2) under every possible situation as:

$$\text{(i)}\ \hat{\beta} = \frac{S_{XY}}{S_{XX} - \sigma_\varepsilon^2}, \quad \text{when } \sigma_\varepsilon^2 \text{ is known.} \tag{5}$$

$$\hat{\beta} = \frac{S_{YY} - \sigma_\eta^2}{S_{XY}}, \quad \text{when } \sigma_\eta^2 \text{ is known.} \tag{6}$$

$$\text{(ii)}\ \hat{\beta} = \frac{(S_{YY} - \lambda S_{XX}) + \{(S_{YY} - \lambda S_{XX})^2 + 4\lambda S_{XY}\}^{1/2}}{2S_{XY}},$$

$$\text{when } \lambda = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2} \text{ is known.} \tag{7}$$

[2] Please see Lee (1973) and Chen (2011) for detail.

(iii) When both σ_ε^2 and σ_η^2 are known, Kendall and Stuart (1961) regarded it as an over-identified situation unless a non-zero covariance between U_i and V_i is introduced. Barnett (1967) followed Kiefer's (1964) suggestion and derived a consistent estimator of $\hat{\beta}$ as one of the real roots of Eq. (19).

$$\hat{\beta}^4 - \left(\frac{1}{b_2} - \frac{\lambda}{b_1} - 2\lambda b_2\right)\hat{\beta}^3 - 3\lambda\left(1 - \lambda\frac{b_2}{b_1}\right)\hat{\beta}^2 + \lambda^2\left(\lambda\frac{b_2}{b_1^2} - \frac{1}{b_1} - 2b_2\right)\hat{\beta} - \lambda^3\frac{b_2}{b_1} = 0, \quad (8)$$

where $S_{XX} = (\sum_{i=1}^n (X_i - \bar{X})^2)/n$, $S_{YY} = (\sum_{i=1}^n (Y_i - \bar{Y})^2)/n$, $S_{XY} = (\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y}))/n$, $m_{XX} = Var(X) = \sigma_U^2 + \sigma_\varepsilon^2$, $m_{YY} = Var(Y) = \beta^2\sigma_U^2 + \sigma_\eta^2$, $m_{XY} = Var(Y) = \beta\sigma_U^2$, $b_1 = m_{XY}/m_{XX}$, $b_2 = m_{XY}/m_{YY}$, and $\lambda = \sigma_\eta^2/\sigma_\varepsilon^2$.

The advantage of the knowledge of both σ_ε^2 and σ_η^2 is that one obtains a more efficient estimator of $\hat{\sigma}_\eta^2$.

The analysis of Eqs. (5)–(8) furnish us two important implications. First, if only U or V is subject to measurement error, then we know that the maxima likelihood estimator of β is equivalent to fitting a least square line when using the error-free variable as a regressor. Second, if both U and V are measured with errors, then the estimate of β lies between the values estimated from Eqs. (5) and (6). This situation can be further analyzed. The quadratic equation for Eq. (7) is

$$\hat{\beta}^2 S_{XY} + \hat{\beta}(\lambda S_{XX} - S_{YY}) - \lambda S_{XY} = 0. \quad (9)$$

(a) When either $\sigma_\eta^2 \to 0$ or $\sigma_\varepsilon^2 \to \infty$, Eq. (6) shows $\hat{\beta} = \frac{S_{YY}}{S_{XY}}$.
(b) When $\sigma_\varepsilon^2 \to 0$ or $\sigma_\eta^2 \to \infty$, Eq. (5) shows $\hat{\beta} = \frac{S_{XY}}{S_{XX}}$.

3.1.2. The classical method to a multiple regression analysis

It is clear that U is orthogonal to V if $\rho = 0$. It is well-known that this multiple regression reduces to two simple regression relationships. If $\rho \neq 0$, to identify β and γ, we need to know either the actual values of σ_ε^2, σ_η^2, and σ_τ^2 or the relative ratios among σ_ε^2, σ_η^2, and σ_τ^2. We will investigate the following cases:

(i) $\sigma_\eta^2 = 0$, $\sigma_\varepsilon^2 > 0$, $\sigma_\tau^2 < 0$, and $\sigma_\varepsilon^2 = \lambda\sigma_\tau^2$
(ii) $\sigma_\varepsilon^2 > 0$, $\sigma_\eta^2 > 0$, and $\sigma_\tau^2 > 0$

(a) σ_ε^2 and σ_η^2 are known
(b) $\sigma_\eta^2 = \lambda_1\sigma_\varepsilon^2$ and $\sigma_\tau^2 = \lambda_2\sigma_\varepsilon^2$

(**Case i**) Only Z and X are measured with errors.
The estimator of $\hat{\beta}$ can be one of real root of Eq. (10)

$$k_2\hat{\beta}^2 + k_1\hat{\beta} + k_0 = 0, \quad (10)$$

where

$$k_0 = \left(S_{XZ} - \frac{S_{XY}S_{YZ}}{S_{YY}}\right),\ k_1 = -\left(S_{XX} - \frac{S_{XY}^2}{S_{YY}}\right) + \lambda\left(S_{ZZ} - \frac{S_{YZ}^2}{S_{YY}}\right),$$

$$\text{and}\quad k_2 = -\lambda k_0 = -\lambda\left(S_{XZ} - \frac{S_{XY}S_{YZ}}{S_{YY}}\right).$$

There are three cases to consider:

(a) $\lambda \to 0$ when $\sigma_\varepsilon^2 \to 0$, then in this case from Eq. (10), we know that

$$\hat{\beta} = \frac{-S_{XY}S_{YZ} + S_{XZ}S_{YY}}{S_{XX}S_{YY} - S_{XY}^2}. \quad (11)$$

(b) $\lambda \to \infty$ when $\sigma_\tau^2 \to 0$, in this case from Eq. (11), we know that

$$\hat{\beta} = \frac{-S_{ZZ}S_{YY} - S_{YZ}^2}{S_{XZ}S_{YY} + S_{XY}S_{YZ}}. \quad (12)$$

(c) When both $\sigma_\varepsilon^2 > 0$ and $\sigma_\tau^2 > 0$, then

$$\hat{\beta} = \frac{k_1 \pm \sqrt{k_1^2 + 4\lambda k_O^2}}{2\lambda k_O}. \quad (13)$$

When β is determined, γ can also be estimated. After both β and γ are estimated, then α can be estimated by

$$\hat{\alpha} = \bar{Z} - \hat{\beta}\bar{X} - \hat{\gamma}\bar{Y}. \quad (14)$$

(**Case ii**) When Z, X, and Y are all observed with errors

(a) Both σ_ε^2 and σ_η^2 are known.
We can obtain the two normal equations as follows:

$$S_{XZ} = \beta(S_{XX} - \hat{\sigma}_1^2) + \gamma S_{XY} \quad (15)$$

$$S_{YZ} = \beta S_{XZ} + \gamma(S_{YY} - \hat{\sigma}_\eta^2) \quad (16)$$

Solving Eqs. (15) and (16) by Cramer's rule we have

$$\hat{\beta} = \frac{S_{XZ}S_{YY} - S_{XY}S_{YZ} - S_{XZ}\hat{\sigma}_\eta^2}{S_{XX}S_{YY} - \hat{\sigma}_\varepsilon^2 S_{YY} - \hat{\sigma}_\eta^2 S_{XX} + \hat{\sigma}_\varepsilon^2\hat{\sigma}_\eta^2 - (S_{XY})^2}$$
$$\hat{\gamma} = \frac{S_{YZ}S_{XX} - S_{XZ}S_{YY} - S_{YZ}\hat{\sigma}_\varepsilon^2}{S_{XX}S_{YY} - \hat{\sigma}_\varepsilon^2 S_{YY} - \hat{\sigma}_\eta^2 S_{XX} + \hat{\sigma}_\varepsilon^2\hat{\sigma}_\eta^2 - (S_{XY})^2}. \quad (17)$$

(b) Both $\hat{\sigma}_\eta^2 = \lambda_1\hat{\sigma}_\varepsilon^2$ and $\hat{\sigma}_\tau^2 = \lambda_2\hat{\sigma}_\varepsilon^2$ are known.

We can obtain β estimator as one of real roots of the following cubic equation

$$\hat{\beta}^3 H_3 + \hat{\beta}^2 H_2 + \hat{\beta} H_1 + H_0 = 0, \quad (18)$$

where

$$H_3 = S_{XY}S_{YZ}^2 - MS_{XZ}S_{YZ} - \lambda_1 S_{XY}S_{XZ}^2$$
$$H_2 = -S_{YZ}^2 + 2\lambda_2 S_{XY}^2 S_{YZ} + MTS_{YZ} - M\lambda_2 S_{XY}S_{YZ} - \lambda_1 S_{XZ}^2 S_{YZ} + 2\lambda_1 TS_{XZ}S_{XY}$$
$$H_1 = \lambda_2^2 S_{XY}^3 - 2\lambda_2 S_{XY}S_{YZ}^2 + MT\lambda_2 S_{XY} + \lambda_2 S_{XY}S_{YZ} + \lambda_2 MS_{YZ}S_{XZ} - \lambda_1\lambda_2 S_{XZ}^2 S_{XY} + 2\lambda_1 S_{XY}S_{XZ}^2 - \lambda_1 T^2 S_{XY}$$
$$H_0 = \lambda_2^2 S_{YZ}S_{XY}^2 + \lambda_2^2 S_{XZ}S_{XY} + \lambda_1\lambda_2 S_{XZ}^2 S_{YZ} + T\lambda_1\lambda_2 S_{XZ}S_{YZ} - 2\lambda_1\lambda_2 TS_{XZ}S_{XY}$$

When $\lambda_2 = 0$, Eq. (18) will reduce to a quadratic equation in $\hat{\beta}$.

3.1.3. The constrained classical method

Under the classical case (Case ii), if we only know $\hat{\sigma}_\eta = \hat{\sigma}_\varepsilon\lambda$, then we can identify β and γ by imposing $\beta + \gamma = 1$.

We can obtain a quadratic equation in $\hat{\beta}$

$$\hat{\beta}^2(1 - \lambda)S_{XY} + \hat{\beta}(S_{YZ} - \lambda S_{XZ} + 2\lambda S_{XY} + S_{YY} - \lambda S_{XX}) + \lambda(S_{XZ} - S_{XY}) = 0. \quad (19)$$

When $\lambda = 0$, Eq. (19) will reduce to

$$\hat{\beta} = -\frac{S_{YZ} + S_{YY}}{S_{XY}}. \quad (20)$$

Imposing $\beta + \gamma = 1$, upon a multiple regression will help to identify the regression coefficients, but it should also be realized that the constrained regression technique will bias the estimates of the regression coefficients if the unrestricted estimator fails to satisfy the restriction $\beta + \gamma = 1$. The advantages and disadvantages of the constrained regression technique have been discussed by Theil (1971) in some detail.

3.2. Grouping method

Following the structural relationship described in Eq. (1)

$V_i = \alpha + \beta U_i$.

Both V_i and U_i are unobserved, and only $Y_i = V_i + \eta_i$ and $X_i = U_i + \varepsilon_i$ can be observed. There exists EIV bias when using Y_i and X_i, to investigate the relationship between V_i and U_i.

Wald (1940) proposes a two-portfolio grouping method in dealing with the EIV problem when both dependent and independent variables are subject to measurement errors. He suggests that the measurement error can be reduced by grouping observations into portfolios. In Wald's two-portfolio grouping method, he groups the independent variable either in descending or ascending order, and divides the observations into two equal groups for both dependent and independent variables; therefore, the first-step estimator of the market model, estimated beta risk, can be written as:

$$\hat{\beta} = \frac{(\bar{Y}_1 - \bar{Y}_2)}{(\bar{X}_1 - \bar{X}_2)}, \tag{21}$$

where $\bar{X}_1$ and $\bar{X}_2$ are the arithmetic means of independent variables for the first and the second groups, respectively; and $\bar{Y}_1$ and $\bar{Y}_2$ are the arithmetic means of independent variables for the first and the second groups, respectively.

Grouping method is widely used in finance related research. For example, to minimize the EIV problem in testing the asset pricing model, Black, Jensen, and Scholes (1972), Blume and Friend (1973), Fama and MacBeth (1973) and Litzenberger and Ramaswamy (1979) use two-pass procedure and the k-portfolio grouping method to examine the capital asset pricing model. By combining securities into portfolios, most of the firm-specific component of the returns can be diversified away and the precision of the beta estimates will be enhanced. The grouping method can, therefore, mitigate the problem raised from measurement errors in estimated beta.

However, some limitations affect the grouping method. First, the grouping method shrinks the range of estimators in the first step and reduces statistical power. To mitigate this problem, in two-pass procedure, the grouping method suggests sorting securities on the first-pass estimator first. Then portfolios are formed by grouping securities with same level of first-pass estimators. This sorting procedure is now standard in empirical tests. Second, a trade-off exists between the bias and the variance of the first-pass estimator according to the number of portfolios. Shanken (1992) argues that the grouping method may cause a larger variation in the portfolio beta. As the number of portfolios (N) increases, the magnitude of the bias becomes greater while the variance of the estimator becomes smaller, and vice versa; therefore, an optimal number of portfolios might exist in which a minimum mean squared error can be obtained. More specifically, when risk premium is estimated by the time-series mean of the cross-sectional regression estimates in testing capital asset pricing model, the mean squared error of the risk premium estimate would be dominated by its bias because its variance would monotonically decrease as the testing period becomes longer. Third, the formation of portfolios for the second-pass estimation might cause a loss of valuable information about cross-sectional behavior among individual securities, because the cross-sectional variations would be smoothed out. Fourth, Ahn, Conrad, and Dittmar (2009) argue that the grouping method, although mitigating measurement error, may yield different results by using different portfolio grouping methods.

Although the grouping method suffers from the limitations discussed above, it still has some clear advantages. With the cross-sectional regression in the second pass, interpreting the results in economic terms is straightforward. Examining model misspecification by testing whether firm characteristics, such as firm size and book-to-market ratio, can explain returns across firms is also convenient. Moreover, the grouping method is intuitive and easy to implement with real data. The grouping method is therefore still preferred in many empirical studies.

3.3. Instrumental variable method

Durbin (1954) proposes an instrumental variable method to deal with the EIV problem in a regression model. In the instrumental variable method, the instrumental variable, T_i, is an observable variable known to correlate with V_i and U_i, but is independent of η_i and ε_i. Then β can be estimated by

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(T_i - \bar{T})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(T_i - \bar{T})(X_i - \bar{X})} = \frac{\sum_{i=1}^{n}(T_i - \bar{T})(U_i - \bar{U}) + \sum_{i=1}^{n}(T_i - \bar{T})(\eta_i - \bar{\eta})}{\sum_{i=1}^{n}(T_i - \bar{T})(V_i - \bar{V}) + \sum_{i=1}^{n}(T_i - \bar{T})(\varepsilon_i - \bar{\varepsilon})} \tag{22}$$

If $plim \sum_{i=1}^{n}(T_i - \bar{T})(U_i - \bar{U})$ exists, then $\hat{\beta}\hat{b}$ is a consistent estimator of β because both ε_i and η_i are independent of T_i. Eq. (22) can be written in matrix form as follows:

$$\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = (\mathbf{T}'\mathbf{X})^{-1}\mathbf{T}'\mathbf{Y}, \tag{23}$$

where $\mathbf{T}' = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ T_1 & T_2 & T_3 & \dots & T_n \end{bmatrix}$, $\mathbf{X}' = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ X_1 & X_2 & X_3 & \dots & X_n \end{bmatrix}$, and $\mathbf{Y}' = \begin{bmatrix} Y_1 & Y_2 & Y_3 & \dots & Y_n \end{bmatrix}$.

However, finding an instrumental variable uncorrelated with η_i and ε_i while highly correlated with V_i and U_i is difficult. Durbin (1954) suggests that if the order of U_i is the same as the order of X_i, then a better instrumental variable would be $T_i = i$, where X_i are ordered by magnitude. That is, $T' = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 3 & \dots & n \end{bmatrix}$ and $X' = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ X_1 & X_2 & X_3 & \dots & X_n \end{bmatrix}$. This variable will lead to a more efficient estimate than that of the method of grouping. If we let $T_i = 1$ for X_i greater than its median, $T_i = 0$ for X_i equal to its median, and $T_i = -1$ for X_i smaller than its median, then the estimator of the instrumental variable method will be the same as the estimator of Wald's two-group grouping method. Therefore, Wald's two-group grouping method is a special case of the instrumental variable method. In other words, the instrumental variable method is more generalized than the grouping method.

Griliches and Hausman (1986) propose an instrumental variable approach to reduce the bias resulted from measurement error. In a penal data framework, they show that instrumental variables estimator is consistent if the measurement error $\varepsilon_{i,t}$ is *i.i.d* across i and t and unobserved independent variable is serially correlated. An instrumental variable using the lags or the difference of lags of the unobserved independent variable can result in a consistent estimator when T is finite and N approaches to infinite.

However, Griliches and Hausman's *i.i.d* assumption is too strong. Biorn (2000) further relaxes Griliches and Hausman's *i.i.d* assumption, and, instead, assumes that the regressor has τ period moving average process. Biorn (2000) shows that using the lags of the variables at least $\tau-2$ periods as instruments can clear the memory of the moving average process and obtain the consistent estimator.

Lewbel (1997) show simple functions of the model data can be used as instruments for two staged least squares (TSLS) estimation. Such instruments can be used for identification and estimation when no other instruments are available or improve efficiency.

Given the standard linear regression model with measurement error:

$$Y_i = a + b'W_i + cX_i + e_i, \quad \text{and} \tag{24}$$

$$Z_i = d + X_i + v_i, \tag{25}$$

in which Y_i, W_i, and Z_i are observable for $i = 1, \ldots, n$, while X_i, e_i, and v_i are unobservable. Eqs. (34) and (35) imply that

$$Y_i = \alpha + b'W_i + cZ_i + \varepsilon_i. \tag{26}$$

However, since both Z_i and ε_i depend on v_i, estimators of b and c from OLS regression is inconsistent. Lewbel (1997) shows that the consistent estimators can be obtained by using TSLS with instruments 1, W_i, and q_i, where q_i is some vector of instruments that are correlated with X_i but not correlated with e_i and v_i.

Lewbel (1997) further empirically applies the instrumental variable method to testing elasticity of patent applications with respect to research and development (R&D) expenditures. He finds, using the TSLS instrumental variable model, the estimated elasticity yields very close to one. Therefore, the TSLS instrumental variable model can mitigate the effects of measurement error and confirm the relationship between patent and R&D.

In addition, Erickson and Whited (2000, 2002) propose a two-step generalized method of moments (GMM) estimators that exploit over-identifying information contained in the high-order moments of residuals obtained from perfectly measured regressors. Basing GMM estimation on residual moments of more than second order requires that the GMM covariance matrix be explicitly adjusted to account for the fact that estimated residuals are used instead of true residuals defined by population regressions. Erickson and Whited (2000) show that estimators obtained by using moments up to seventh order perform well in Monte Carlo simulations.

Almeida et al. (2010) use Monte Carlo simulations and empirically test investment models to compare the performance of the instrumental variables approach suggested by Biorn (2000) and generalized method of moments. They find that the instrumental variable method can obtain more consistent and efficient estimators than generalized method of moments when independent variables subject to measurement error.

However, it is difficult to obtain appropriate instrument variables, resulting in weak evidence in empirical research. Lewbel (2012) proposes a new method to deal with measurement error problems in regression model when instrumental variables are not available. Under the assumption of heteroscedastic errors, Lewbel (2012) shows that the regression model with measurement regressors can be identified and estimated by TSLS or GMM.

3.4. Mathematical method

3.4.1. Bivariate case

Deming (1943), York (1966) and Clutton-Brock (1967) have developed a weighted-regression-method-under-iteration approach. Deming (1943) proposed that the best straight line of Eq. (1) can be obtained by minimizing the sum in the following equation:

$$S = \sum_i \{w(X_i)(\hat{U}_i - X_i)^2 + w(Y_i)(\hat{V}_i - Y_i)^2\} \tag{27}$$

$\hat{U}_i$ and $\hat{V}_i$ are the adjusted value of X_i and Y_i which make the sum in Eq. (27) a minimum. Since we require $\hat{U}_i$ and $\hat{V}_i$ to lie on the best straight line, we must have

$$\hat{V}_i = \alpha + \beta\hat{U}_i, \quad (i = 1, \ldots, n) \tag{28}$$

Both $w(X_i)$ and $w(Y_i)$ are the weights of various observations. They are reciprocally proportional to the variance of their measurement error, respectively.

If these values of $\hat{U}_i$, $\hat{V}_i$, α, and β make S a minimum, we have

$$\beta^3 \sum_i \frac{k_i^2 x_i^2}{w(X_i)} - 2\beta^2 \sum_i \frac{k_i^2 x_i y_i}{w(X_i)} - \beta\left\{\sum_i k_i x_i^2 - \sum_i \frac{k_i^2 y_i^2}{w(X_i)}\right\} + \sum_i k_i x_i y_i = 0, \tag{29}$$

where $x_i = X_i - \bar{X}$, $y_i = Y_i - \bar{Y}$, $\bar{X} = \frac{\sum_i k_i X_i}{\sum_i k_i}$, $\bar{Y} = \frac{\sum_i k_i Y_i}{\sum_i k_i}$, and $k_i = \frac{w(X_i)w(Y_i)}{\beta^2 w(Y_i) + w(X_i)}$.

Eq. (29) is the least-square cubic derived by York (1966). To solve Eq. (29), an initial value is assigned to β to estimate k_i. After obtaining the roots of Eq. (29), one of the legitimate solutions is assigned to estimate k_i and obtain new solutions for β again. A similar procedure is employed iteratively until a convergent solution is obtained.

The mathematical approach involves the estimation of the parameters of a function conditional on the maximum likelihood function adjusted for the true values. This method is different from the classical method in three ways. First, variances of measurement errors for every observation are different. Second, a weighted regression method is applied. Third, the iteration procedure is used to obtain a consistent estimator.

It can be proved that the mathematical programming method reduces to the classical method under three certain conditions.

(i) Only Y_i has an EIV problem

We can put more weight on X_i which has no EIV problem, $w(X_i) = \infty$, $k_i = w(Y_i)$. We can therefore solve the least square cubic

$$\beta = \frac{\sum_i w(Y_i) x_i y_i}{\sum_i w(Y_i) x_i^2}, \tag{30}$$

which is the estimated coefficient of weighted regression of Y_i on X_i.

(ii) Only X_i has an EIV problem

In this case, we put more weight on Y_i which has no EIV problem, then $w(Y_i) = \infty$, $k_i = \frac{w(X_i)}{\beta^2}$. We can solve the least square cubic

$$\beta = \frac{\sum_i w(X_i) y_i^2}{\sum_i w(X_i) x_i y_i}, \tag{31}$$

which is the inverse estimated coefficient of weighted regression of Y_i on X_i.

(iii) Both X_i and Y_i have EIV problem, and $w(X_i)/w(Y_i) = c$.

The least square cubic becomes

$$\beta^2 + \beta\frac{\{c\sum_i k_i x_i^2 - \sum_i k_i y_i^2\}}{\sum_i k_i x_i y_i} - c = 0 \tag{32}$$

3.4.2. Multivariate case

Lee (1973) extends the bivariate mathematical programming method, which was developed by Deming (1943), York (1966) and Clutton-Brock (1967), to a trivariate case. We define $w(Z_i)$, $w(Z_i)$, and $w(Y_i)$ which are the weights of the various observations of Z_i, X_i, and Y_i. It is assumed W, U, and V are functionally rather than structurally related. The mathematical programming procedure begins by minimizing[3]

$$S = \sum_i \{w(X_i)(x_i - X_i)^2 + w(Y_i)(y_i - Y_i)^2 + w(Z_i)(z_i - Z_i)^2\} \quad \text{s.t.}\ z_i = \alpha + \beta x_i + \gamma y_i. \tag{33}$$

This extension will reduce to Deming′s (1943) weighted regression results when the quadratic term of equations are omitted, while Lee's (1973) result is more general than Deming's weighted multiple regression analysis.

3.5. Maximum likelihood method

In testing capital asset pricing model with dividend and tax, Litzenberger and Ramaswamy (1979) use maximum likelihood method to reduce the effect of errors-in-variables. Litzenberger and Ramaswamy (1979) show that, assuming that the variance of the measurement error in beta is known, the cross sectional variance of true betas can be replaced by the difference in the variation of the observed betas and the variance of the measurement error. Then the estimator in capital asset pricing model test, under such condition, is consistent by maximum likelihood method.

Kim (1995, 1997, 2010) further provides a maximum likelihood method to correct the EIV problem in testing the asset pricing model. Based upon two-pass capital asset pricing model, Kim (1995) shows that in a multifactor asset pricing model test the EIV leads to an underestimation of the independent variable with a measurement error and an overestimation of the independent variable without measurement error. To correct EIV biases, Kim (1995) extracts additional information about the relation between idiosyncratic error variance which can be obtained from the first step and the measurement error variance, and incorporates such additional information into the second step of the capital asset pricing model test. Assuming the homoscedasticity of the disturbance term of the market model, Kim (1995) shows that the corrected factors for the traditional least squares estimators of the cross-sectional regression coefficients can be obtained by the maximum likelihood method. The closed form estimators of the multifactor asset pricing model test can therefore be obtained. Assuming the first and second steps of the multifactor asset pricing model are

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + e_{i,t}, \tag{34}$$

and

$$R_{i,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{i,t-1} + \gamma_{2,t}V_{i,t-1} + e_{i,t}, \tag{35}$$

where $\beta_{i,t-1}$ is the market risk factor with measurement error, and $V_{i,t-1}$ is a risk factor with no measurement error for security i at time $t-1$. The adjusted estimators in the second step can be written as follows:

$$\begin{aligned} \hat{\gamma}_{1t} &= \frac{M + \left[M^2 + 4\delta_t m_{R\hat{\beta}}^2 (1 - (\hat{\rho}_{RV}\hat{\rho}_{\hat{\beta}V}/\hat{\rho}_{R\hat{\beta}}))^2\right]^{1/2}}{2m_{R\hat{\beta}}(1 - (\hat{\rho}_{RV}\hat{\rho}_{\hat{\beta}V}/\hat{\rho}_{R\hat{\beta}}))} \\ \hat{\gamma}_{2t} &= (m_{RV} - \hat{\gamma}_{1t} m_{\hat{\beta}V})/m_{VV} \\ \hat{\gamma}_{0t} &= \tilde{R}_t - \hat{\gamma}_{1t}\tilde{\hat{\beta}}_{t-1} - \hat{\gamma}_{2t}\tilde{V}_{t-1} \end{aligned} \tag{36}$$

where $M = m_{RR}(1 - \hat{\rho}_{RV}^2) - \delta_t m_{\hat{\beta}\hat{\beta}}(1 - \hat{\rho}_{\hat{\beta}V}^2)$, $m_{xy} = (1/N)\sum_{i=1}^N \sum_{j=1}^N w_{ij}(x_i - \bar{x})(y_i - \bar{y}) / \sum_{i=1}^N \sum_{j=1}^N w_{ij}$, $\bar{x} = \sum_{i=1}^N \sum_{j=1}^N w_{ij} x_i / \sum_{i=1}^N \sum_{j=1}^N w_{ij}$, $\bar{y} = \sum_{i=1}^N \sum_{j=1}^N w_{ij} \bar{x}_i / \sum_{i=1}^N \sum_{j=1}^N w_{ij}$, w_{ij} is the (i, j) element of inverse matrix of residual variance in the first-step, $\hat{\Sigma}_\varepsilon^{-1}$, and $\hat{\rho}_{xy}^2 = m_{xy}/(m_{xx}m_{yy})^{1/2}$.

As a result, the maxima likelihood method can correct the problem on exaggerating the estimated coefficient associated to the variable without measurement error. Moreover, the absolute value of estimated intercept by maxima likelihood method is generally smaller than the absolute value of estimated intercept by traditional least squares.

3.6. LISREL and MIMIC methods

Goldberger (1972) conceptually described the LISREL model as a combination of factor analysis and econometrics model. In addition, Anderson (1963) has shown that factor analysis is a generalized version of errors-in-variables (EIV) methods. In this section, we will review and discuss how LISREL and MIMIC methods can be used to deal with EIV in finance research.

The linear simultaneous equation system is widely used in finance and accounting related research. However, a serious limitation of the simultaneous equation approach is an EIV problem. For example, the theoretical determinants of capital structure in corporate finance can be attributed to unobservable constructs that are usually measured in empirical studies by a variety of observable indicators or proxies. These observable indicators or proxies can then be viewed as measures of latent variables with measurement errors. Maddala and Nimalendran (1996) show that the use of these indicators as theoretical explanatory variables may cause EIV problems. Bentler (1983) also emphasizes the estimated results of the traditional simultaneous equation model has no meaning when variables have measurement errors. Therefore, the latent variable covariance structure model is provided and applied in corporate finance. Titman and Wessels (1988), Chang et al. (2009) and Yang et al. (2009), mitigate the measurement problems of proxy variables, and apply structure equation models (e.g. LISREL model and MIMIC model) to determine capital structure decision. Maddala and Nimalendran (1996) use the structure equation model to examine the effect of earnings surprises on stock prices, trading volumes, and bid-ask spreads.

Goldberger (1972) and Jöreskog and Goldberger (1975) developed a structure equation model with multiple indicators and multiple causes of a single latent variable, MIMIC mode, and obtained maximum likelihood estimates of parameters. Fig. 1 shows the path diagram that depicts a simplified MIMIC model in which variables in a rectangular box denote observable variables, while variables in an oval box are latent constructs. In this diagram, observable variables X_1, X_2, and X_3 are causes of the latent variable η, while Y_1, Y_2, and Y_3 are indicators of η. In our study, X's are determinants of capital structure (η), which are then measured by Y's.

Jöreskog and Sörbom (1989) show that the full structural equation (LIEREL model) can be restricted to a MIMIC model. We here discuss the structural model and show how structural model can be restricted to a MIMIC model.

3.6.1. Structural model (LISEREL model)

A structural equation model is composed of two sub-models – structural sub-model and measurement sub-model. The structural model can be defined as

$$\eta = \Gamma X + \zeta, \tag{37}$$

[3] Please see Lee (1973) for the solution of Eq. (33).

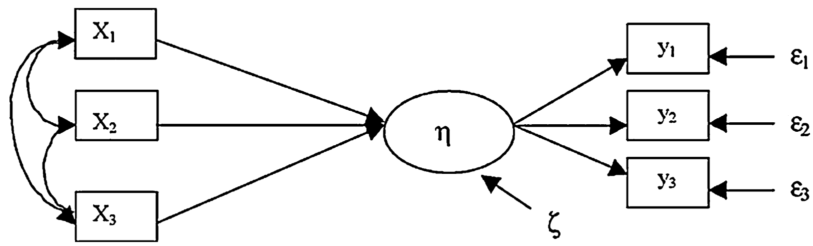

Fig. 1. Path diagram of a simplified MIMIC model.

$$Y = \Lambda_y \eta + \varepsilon, \tag{38}$$

where Y is a vector of indicators of the latent variable η, and X is a vector of causes of η.

The latent variable η is linearly determined by a set of observable exogenous causes, $X = (x_1, x_2, \ldots, x_q)'$, and a disturbance ζ. The latent variable η, in turn, linearly determines a set of observable endogenous indicators, $Y = (y_1, y_2, \ldots, y_p)'$ and a corresponding set of disturbance, $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)'$.[4]

3.6.2. MIMIC model

Substituting Eq. (48) into Eq. (49), we obtain a reduced form:

$$Y = \Lambda_y \eta + \varepsilon = \Lambda_y(\gamma' X + \zeta) + \varepsilon = \Pi' X + z \tag{39}$$

In structural equation modeling, the total effect of a cause variable on an indicator can be measured as the sum of the direct effect and the indirect effect. Since a MIMIC model is a reduced form of a structural equation model, the total effect of MIMIC model, denoted as Π' in Eq. (39), comes merely from the indirect effect.

Since the scale of the latent variable is unknown, the factor indeterminacy is a common problem in the MIMIC model, as in other structure equation models. We can obtain infinite parameter estimates from the reduced form by arbitrarily changing the scale of the latent variables. However, by fixing the scales of latent variables, one can solve the indeterminacy problem. Two methods are usually adopted to fix the scale of latent variables. One method is normalization in which a unit variance is assigned to each latent variable, while another method is to fix a non-zero coefficient at unity for each latent variable.

In terms of estimation of the parameters, Jöreskog and Goldberger (1975) adopt the normalization method to deal with the factor indeterminacy problem and use maximum likelihood estimation method in structural equation modeling to estimate parameters. The maximum likelihood estimates for the parameters of the model are obtained at the minimization of the fit function as follows:

$$F = \log||\Sigma|| + \mathrm{tr}(S\Sigma^{-1}) - \log||S|| - (p - q), \tag{40}$$

where Σ is the population covariance matrix; S is the model-implied covariance matrix; p is the number of exogenous observable variables; and q is the number of endogenous observable variables. Minimization of the fit function can be done by the LISREL program provided by Jöreskog and Sörbom (1981).

3.7. Bayesian approach

Zellner (1970) uses the Bayesian approach to deal with measurement problems in the estimation of regression relationships containing unobservable independent variables. Zellner (1970) shows that the Bayesian approach can obtain optimal estimates under a finite sample. Several studies use Bayesian approaches to examine cost of capital (e.g. Lee & Wu, 1989) and asset pricing models (e.g. Ang & Chen, 2007; Davis, 2010; Geweke & Zhou, 1996; McCulloch & Rossi, 1991).

Davis (2010) develops a Bayesian approach and uses U.S. firm level data to reexamine the capital asset pricing model. The Bayesian approach can estimate all parameters simultaneously in one step and effectively avoid the errors-in-variables problem on the estimators induced from two-pass capital asset pricing test.

Davis (2010) uses a Bayesian approach to simultaneously estimate coefficients of the following three equation system.

$$r_{i,t,y} = \alpha_{i,y} + \gamma_{i,y} r_{m,t,y} + \delta_{i,y} r_{m,t-1,y} + \varepsilon_{i,t,y},$$
$$\text{where } \varepsilon_{i,t,y} \sim N(0, \sigma^2_{r_{i,y}}); \tag{41}$$

$$\overline{r_{i,y} - r_{f,y}} = c_{0,y} + c_{m,y}\beta_{i,y} + \eta_{i,y}, \quad \text{where } \eta_{i,y} \sim N(0, \sigma^2_y) \text{ and}$$
$$\beta_{i,y} \sim \gamma_{i,y} + \delta_{i,y}; \tag{42}$$

$$\mathbf{c}_y = \begin{bmatrix} c_{0,y} \\ c_{m,y} \end{bmatrix} \sim N(\mathbf{c}, \mathbf{V}_c), \quad \text{where } \mathbf{c} = \begin{bmatrix} c_0 \\ c_m \end{bmatrix} \text{ and}$$
$$\mathbf{V}_c = \begin{bmatrix} \sigma_0^2 & \sigma_{0m}^2 \\ \sigma_{0m}^2 & \sigma_m^2 \end{bmatrix}; \tag{43}$$

where $r_{i,t,y}$ is firm i's return in month t during the time period y, and $\overline{r_{i,y} - r_{f,y}}$ denotes the average monthly excess return for firm i during the time period y. The model allows firm-level βs to vary over each time period, y. The model also assumes that the joint normality of stock returns and market returns, contemporaneously estimated βs, and average excess returns are statistically independent.

In addition to deal with errors-in-variables problem, there are several advantages using the Bayesian approach to test the capital asset pricing model. First, the Bayesian approach allows βs to vary over time periods and firms and controls the inherent uncertainty associated with firm-level βs. Second, the Bayesian approach can modify the distribution assumptions in stock returns and market returns. Third, the Bayesian inference is free from the use of asymptotic approximations and therefore can be used under finite sample. Fourth, the Bayesian approach takes parameter uncertainty associated with all the model parameters into account.

4. Applications of errors-in-variables models in finance research

For the last four and a half decades, EIV models have been used to correct estimation bias associated with empirical results in various finance-related research issues. We here review four kinds of research, cost of capital, asset pricing models, capital structure, and investment equation, and discuss how EIV models can remedy measurement error problems induced from finance-related

[4] Stapleton (1978) further develops MIMIC with more latent variables.

research. It is therefore useful to understand the statistical properties of these EIV models in situations resembling real research question.

The main focus of this section is to discuss the measurement error problems on various empirical studies related to finance research and investigate how EIV models can remedy such problems. However, in empirical studies, it is impossible to observe variables without measurement error. We cannot evaluate EIV models and suggest a best EIV model for a certain circumstances. Instead, we here provide Table 1 to summarize the application of EIV models in finance-related research. Research topics, EIV models, specialties of EIV model, and results for each study are included in Table 1.

4.1. Cost of capital

Miller and Modigliani (1966) developed a theoretical expression for the value of a firm from which the firm's cost of capital could be derived. They assume a perpetual stream of earnings from real assets, and a constant capitalization rate (ρ), at which the market discounts the uncertain pure (unlevered) equity stream of earnings for some risk classes and perfect markets. It is thus possible to estimate the market capitalization rate (and thus the cost of capital) of a group of firms by performing a cross-sectional regression of the market value of the firm's equity on the expected average earnings of the firm, the market value of debt, and the growth resulting from the above-average investment opportunities. The above analysis suggests a cross-sectional regression:

$$(V-\tau_c D)=a_0+a_1\overline{X}(1-\tau_c)+a_2(\text{growth potential})+\varepsilon, \tag{44}$$

where V is sum of the market value of all securities issued by the firm, τ_c is the corporate tax rate, D is the market value of a firm's debt, and $\bar{X}$ is the expected level of average annual earnings generated by current assets.

To avoid heteroscedasticity of regression residuals, the equation must be adjusted to compensate for the dominance of the large companies. Miller and Modigliani (1966) use weighted least square to adjust the standard deviation of the error term to firm size (deflating each variable by the book value of total assets). Therefore Eq. (51) can be adjusted to:

$$\frac{(V-\tau_c D)}{A}=\frac{a_0}{A}+a_1\bar{X}\frac{(1-\tau_c)}{A}+a_2\frac{\Delta\bar{A}}{A}+u, \tag{45}$$

where $u=\varepsilon/\text{A}$. With this reformulation, the regression equation is expected to be homogeneous, that is, to have no constant term, and the term A, total assets, is used to avoid heteroscedasticity.

An additional problem beyond that of heteroscedasticity is the possible error of measurement associated with the earnings term. Since anticipated average earnings are essentially unobservable, accounting-statement estimates of earnings must be used instead. Therefore, the true relation between value and anticipated earnings, when replaced by the observable estimates, implies a simultaneous system of relationships:

$$V_i^*=\alpha X_i^*+\sum_j \beta_j Z_{ij}+u_i, \tag{46}$$

$$X_i=X_i^*+v_i, \tag{47}$$

$$X_i^*=\sum_j \delta_j Z_{ij}+w_i, \tag{48}$$

where $V_i^*=(V_i-\tau_c D_i)/A_i$, $X_i^*=(\bar{X}(1-\tau_c))/A_i$ (the true anticipated earnings); v_i = measurement errors associated with current earnings; X_i = observable estimate of earnings derived from the accounting statements; and Z_{ij} = other relevant variables determining earnings. Equations (46)–(48) are related to anticipated earnings and a set of explanatory variables which may also be correlated with the firm's anticipated earnings.

In addition, the earnings variable used in the regression only approximates the true value of anticipated earnings, varying by the error of measurement, v_i. The system represents the simultaneous determination of two endogenous variables, V^* and X, by the Z_j exogenous variables. In regressing:

$$V_i^*=\alpha X_i+\sum \beta_j Z_{ij}+U' \tag{49}$$

the coefficients will be biased. The coefficient for earnings, α, will have a downward bias.

In an attempt to remedy the simultaneous-equation bias, Miller and Modigliani (1966) use an instrumental-variable approach. In this approach, the endogenous variable X is first regressed against all the instrumental variables, Z_j, to obtain estimates of the various coefficients. These estimates are then used to develop a new variable, X, which is

$$\hat{X}_i^*=\sum \hat{\delta}_j Z_{ij} \tag{50}$$

Depending on the choice of Z_j, the new estimate of earnings, $\hat{X}_i^*$, should be relatively free of the error measurement. It can then be used in the second-stage regression as the earnings variable. The resulting estimates of α and β can be shown to be consistent.

Miller and Modigliani (1966) hypothesized that the constant term was really zero. The reduction of bias on the estimates through the use of the two-stage process also seems to support the hypothesis that the constant term is zero. Miller and Modigliani (1966) state that the reason the constant term was significantly different from zero for the direct least-squares cases was that the error of measurement for earnings was large. This error is reduced by the two-stage process.

Higgins (1974) derives and tests a finite-growth model for the estimation of the cost of capital and share price of electric utility industry between 1960 and 1968. He suggests that the market value of equity is related to the trend of earnings and the trend of population in utility's service area. Assuming that observations of a variable consist of a true component and a random element, if such random elements have zero mean and are serially uncorrelated, the smoothing procedure can reduce potential errors in measurement. Empirical results show that the extrapolation of historical population trends is superior to the conventional use of change of capital, and share prices are not a positive function of dividends as often suggested.

Zellner (1970) proposes a least squares regression method to deal with potential errors-in-variables problems. He shows that his methods utilize more information than traditional instrumental variables methods do in dealing with an errors-in-variables problem. Lee and Wu (1989) further apply Zellner's method to reexamine Miller and Modigliani's (1966) cost of capital estimation for utility industry and obtain better cost of capital estimates than OLS methods and instrumental variable method.

More recently, Pastor, Sinha, and Swaminathan (2008) propose an implied cost of capital which is calculated by earnings forecasts and argue that the implied cost of capital can capture time variation in expected stock returns. Ortiz-Molina and Phillips (2014) adopt Pastor et al. (2008) method to investigate the relationship between real asset liquidity and the cost of capital, and find the implied cost of capital can mitigate measurement error problem on determine the cost of capital. Guay, Kothari, and Shu (2011) further propose the implied cost of capital corrected sluggish analyst forecast to

Table 1
Applications of errors-in-variables models in finance research.

Study	Issue	Method	Specialties/conditions	Results
Miller and Modigliani (1966)	Determinants of cost of capital	Instrumental variable method	Have to find exogenous variables	– Measurement error problem matters
Black et al. (1972)	CAPM test	Grouping (10 groups)	Panel data; 2-pass estimation	– Reject both the CAPM and the zero-beta CAPM
Blume and Friend (1973)	CAPM test	Grouping (12 groups)	Panel data; 2-pass estimation	– Linear model is better than quadratic model in explaining expected return – Reject both the CAPM and the zero-beta CAPM
Fama and MacBeth (1973)	CAPM test	Grouping (20 groups), period by period	Panel data; 2-pass estimation	– Find a linear relationship between the expected return and beta risk, beta is the only risk measure in explaining expected return, and risk premium is greater than zero – CAMP and efficient capital market hold
Higgins (1974)	Determinants of cost of capital	Smoothing procedure	Assume that measurement error has zero mean and serially uncorrelated	– The extrapolation of historical population trends is superior to the conventional use of change of capital, and share prices are not a positive function of dividends as often suggested
Lee (1977)	CAPM test	Wald's Grouping/Instrumental Variable	Adjust for measurement error of market return in first-step	– Estimated risk premium is larger than realized risk premium – Reject CAPM
Litzenberger and Ramaswamy (1979)	CAPM test	MLE, OLS, GLS	For individual stocks	– Before-tax expected rates of return are linearly related to systematic risk and dividend yield – MLE can obtain consistent estimators without losing efficiency – CAPM is rejected because of non-zero $\hat{\gamma}_0$
Cheng and Grauer (1980)	CAPM test	Grouping (20 groups)	Price-level testing (Invariance Law)	– Neither framework of Invariance Law or security market line can accommodate the possibility that the CAPM may hold for each period – Reject CAPM
Gibbons (1982)	CAPM test	One-step Gauss-Norman Procedure (40 groups)	One-step estimation	– Gauss-Norman procedure can increase the precision of estimated risk premium – Reject CAPM
Titman and Wessels (1988)	Determinants of capital structure	LISREL model	Deal with EIV problem due to the imperfect representation of proxy variables for interested attributes	– Do not support for four of eight propositions on the determinants of capital structure – A firm's capital structure is not significantly related to its non-debt tax shields, volatility of earnings, collateral value of assets, and future growth
Lee and Wu (1989)	Determinants of cost of capital	Zellner's EV method	Use sample information to estimate the ratio of error variances and construct an operational estimator	– Obtain better cost of capital estimates
MacKinlay and Richardson (1991)	CAPM test	GMM	Release the assumption of the normality of asset returns	– Conclusions of mean-variance efficiency vary by settings
Shanken (1992)	CAPM test	MLE	For individual stocks; deal with small-sample bias in the second-step cross-sectional regression estimates	– The adjustment does not have much effect on Fama and MacBeth's (1973) conclusion – Support CAPM
Fama and French (1992)	CAPM test	2-way grouping (10 × 10 groups)	Take size and book-to-market ratio into account	– The market capitalization and the book-to-market ratio can replace beta altogether – Reject CAPM
Jagannathan and Wang (1996)	CAPM test	Multifactor Asset Pricing Model	Test conditional CAPM	– Including human capital and business cycle can increase explanatory power of expected return – Support CAPM

Table 1 (*Continued*)

Study	Issue	Method	Specialties/conditions	Results
Kim (1995, 2010)	CAPM test	MLE	Closed form adjustment; for individual stocks or groups	– MLE method can effectively adjust the errors-in-variables bias and CAPM holds – Support CAPM
Kim (1997)	CAPM test	MLE	Closed form adjustment; for individual stocks or groups; for multifactor estimation	– Linear relationship between beta and expected return – Book-to-market ratio has significant explanatory power for expected return, but size has not
Lewbel (1997)	Elasticity of patent applications to R&D expenses	Instrumental variable method	Applicable if no outside data available for use as instruments	– TSLS estimator can mitigate the effects of measurement error – The estimated elasticity of patent applications with respect to R&D expenditures yields very close to one
Erickson and Whited (2000)	Test *q* theory	GMM	For balanced panel data	– Cash flow does not affect firms' financial decision, even for financially constrained firms – Support the *q* theory if measurement error is taken into account
Pastor et al. (2008)	Determinants of cost of capital	Implied cost of capital	Use earnings forecasts to compute implied cost of capital	– The implied cost of capital can capture time variation in expected stock returns
Chang et al. (2009)	Determinants of capital structure	MIMIC model	Allow several observable variables as indicators without multicollinearity problem	– Seven constructs, growth, profitability, collateral value, volatility, non-debt tax shields, uniqueness, and industry, as determinants of capital structure have significant effects on capital structure decision
Yang et al. (2009)	Determinants of capital structure	LISREL model	Jointly determine capital structure and return	– Stock returns, expected growth, uniqueness, asset structure, profitability, and industry classification are main determinants of capital structure – Leverage, expected growth, profitability, firm value, and liquidity can explain stock returns – The capital structure and stock return, in addition, are mutually determined by each other
Almeida et al. (2010)	Test *q* theory	GMM and instrumental variables method	Simple instrumental variable – lagged variable	– Estimators from GMM are unstable across different specifications and not economically meaningful – Estimators from a simple instrumental method are robust and conform to *q* theory
Davis (2010)	CAPM test	Bayesian approach	One-step estimation	– Positive relationship between excess return and market risk – Support CAPM
Jagannathan et al. (2010)	CAPM tests	Three-stage cross-sectional regression	Adjust for rolling window betas	– Dealing with measurement error in rolling window betas – Support CAPM
Guay et al. (2011)	Determinants of cost of capital	Corrected implied cost of capital	Sluggish analyst forecasts may result measurement error on implied cost of capital	– The corrected implied cost of capital can improve the ability to explain cross-sectional variation in future stock returns
Da et al. (2012)	CAPM tests	Three-stage cross-sectional regression	Adjust for rolling window betas	– Dealing with measurement error in rolling window betas – Support CAPM
Erickson and Whited (2012)	Test *q* theory	GMM	For unbalanced panel data	– Instrumental variables, dynamic panel estimators, and high-order moment estimators can perform well under correct specification – Developing a minimum distance technique allowing high-order moment estimators be used in unbalanced panel data

Table 1 (*Continued*)

Study	Issue	Method	Specialties/conditions	Results
Ortiz-Molina and Phillips (2014)	Determinants of cost of capital	Implied cost of capital	Use earnings forecasts to compute implied cost of capital	– The implied cost of capital can mitigate measurement error problem on determine the cost of capital
Lee and Tai (2014)	Determinants of capital structure	SEM with CFA approach	Jointly determine capital structure and return	– SEM with CFA approach outperforms MIMIC model and 2SLS method in terms of the joint determinants of capital structure and stock return

improve the ability to explain cross-sectional variation in future stock returns.

4.2. Capital asset pricing model

The capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) implies that the expected returns on securities and their market risks (β) are positively and linearly correlated and that market risks have sufficient power to explain expected returns of securities. Black et al. (1972), Fama and MacBeth's (1973), and others use the two-step method to test CAPM. In the first step, estimated betas are obtained by time-series market model for each security. In the second step, the estimated betas are used in testing the linear relationship between betas and expected returns on securities. Because estimated betas are subjected to a measurement error (estimation error) problem, there exists an EIV problem in the second step. The EIV problem will result in estimating the explanatory power of beta and the estimated rate of return on beta risk. More specifically, the EIV problem leads to an underestimation of the coefficient associated with beta risk. Although the EIV problem exists in the two-step method of the asset pricing test, most researchers do not carefully use sophisticated econometric and statistical methods to deal with this kind of problem. Roll (1969, 1977) shows that the testing asset pricing model suffers an EIV problem, concluding that (i) no correct and unambiguous test of the theory has appeared in the literature, and (ii) practically no possibility exists that such a test can be accomplished in the future. Roll and Ross (1994) show that the measurement error problem of market rate of return can bias the empirical test of CAPM.

Several studies focus on beta estimation in the first step to solve the EIV problem in testing CAPM. Brennan (1970), Lee and Jen (1978) and Roll (1977) show that the possible measurement error in market beta risk is the unobserved market rate of return and risk-free rate of return.[5] To improve the beta estimator, Fabozzi and Francis (1978) and Lee and Chen (1979) use the random coefficient procedure to estimate random coefficient betas. Brennan and Schwartz (1977), Brennan (1979) and Brown and Warner (1980) also provide different types of market models which can produce different results in predicting rates of return, testing efficient market hypotheses, and measuring security price performance.

To reduce the impact of the measurement error problem in the second step, Black et al. (1972) use the grouping method in the capital asset pricing model test. Although the results support the linear relationship between the systematic risk and expected return, CAPM cannot hold because of the non-zero intercept term and the lower market premium in the cross-sectional regression. Blume and Friend (1973) and Fama and MacBeth (1973) also use the grouping method in testing CAPM and show that the CAPM is valid. However, Jagannathan, Skoulakis, and Wang (2009) show that the time series average of the cross-sectional estimators converges in probability to the true value of the estimator. Although their results support CAPM indicating a linear relationship between the systematic risk and the expected return, the lower value of time-series average of the market premium shows that the measurement error problem still exists after Fama and MacBeth's (1973) grouping method.

Considering the measurement errors of the market rate of return and risk-free rate of return, Lee (1977) uses two EIV estimation methods, Wald's two-group grouping method and Durbin's instrumental variable method, to adjust the estimated beta risk in the first step of the capital asset pricing test. Although correcting the measurement errors induced from the unobservable market rate of return, Lee (1977) finds that the predictive ability of the capital asset pricing model is still poor.

Litzenberger and Ramaswamy (1979) derive an after-tax version of CAPM and show that, in the equilibrium, the before-tax expected return on a security is linearly related to its systematic risk and its dividend yield. Litzenberger and Ramaswamy (1979) further empirically test both the before tax and the after-tax versions of CAPM. Instead of grouping method, Litzenberger and Ramaswamy (1979) use maximum likelihood estimation in the second-step regression to test the before-tax and the after-tax versions of capital asset pricing model. Although maximum likelihood estimators are consistent, the average risk premium is small and not significantly different from zero.

Given the EIV bias in the two-step CAPM test, Gibbons (1982) introduces a one-step Gauss-Newton procedure and uses the maximum likelihood method to obtain the estimated price of systematic risk. Because the one-step Gauss-Newton procedure does not use estimated beta as an explanatory variable in a regression model, the measurement errors problem of estimated beta can be avoided. Shanken (1992) shows that using generalized least square (GLS) on the second step of CAPM test can yield an estimator identical to the Gauss-Newton estimator obtained by Gibbons' (1982) maximum likelihood method. Gibbons (1982) shows that the Gauss-Newton procedure increases the precision of estimated risk premium, but rejects the mean-variance efficiency of the market portfolio.

Shanken (1992), who provides a modified version of the two-step estimator by using maximum likelihood estimation, finds that Fama and MacBeth's two-step procedure overstates the precision of the estimator in the second-step and therefore provides an adjusted standard error for the estimator in the second-step regression. Jagannathan and Wang (1998a, 1998b) further

[5] Roll (1969), Roll (1977) and Lee and Jen (1978) show that the observed market rate returns in terms of stock market index are measured with errors since the stock market index does not include all assets which investors can invest. Lee and Jen (1978) have theoretically shown how beta estimate and Jensen performance measures can be affected by both constant and random measurement errors of R_m and R_f. Diacogiannis and Feldman (2011), Green (1986), Roll and Ross (1994) and Gibbons and Ferson (1985) have argued that market portfolio measure with errors is an inefficient portfolio and show how the inefficient benchmark can affect theoretical CAPM derivation. Diacogiannis and Feldman (2011) provide a pricing model that uses inefficient benchmarks, a two beta model, one induced by the benchmark, and one adjusting for its inefficiency.

release Shanken's assumption that asset returns are conditional homoscedasticity to derive a more general standard error for the second-step estimator by generalized least squares.

Fama and French (1992) use a two-way sort grouping method to control for size effect, and find a weak relationship between beta risk and expected return. Before reaching the conclusion that the capital asset pricing has not been valid in the recent years, one possible reason that may be considered is that the measurement error problem cannot be fully eliminated by the grouping method, and results of CAPM test may vary depending on the portfolio formation technique (e.g. Ahn et al., 2009).

To deal with the problem of EIV in testing CAPM, Kim (1995) provides a maximum likelihood method, extracting information associated with the relationship between the measurement error variance and idiosyncratic error variance and incorporating such information into the maximum likelihood estimation in the second step of the capital asset pricing model test. Given the assumption that the disturbance term of the market model is homoscedasticity, the corrected factors for the traditional least squares estimators of the cross-sectional regression coefficients can be obtained. Although Kim's (1995) maximum likelihood method can only deal with the EIV problem of the estimated beta in the first pass, the maximum likelihood method can test, besides the capital asset pricing model, the multifactor asset pricing models. Kim (1995) uses maxima likelihood method to reexamine CAPM and multi-factor asset pricing model and finds more support for the role of market beta risk and less support for the role of firm size. His results show that the prominent risk factors (e.g. size, book-to-market ratio, and momentum factors) might result a different explaining power for cross-sectional stock returns after correcting the EIV problem.

MacKinlay and Richardson (1991) use generalized method of moments (GMM) to test the mean–variance efficiency. They theoretically show that the estimator from GMM and the estimator from maximum likelihood method are equivalent when stock returns are conditionally homoscedasticity, but GMM can avoid the EIV problem by estimating coefficients in one step. Empirical and simulation results show that the conclusion mean–variance efficiency of market indexes is sensitive to the model settings.

Chen (2011) offers an empirical examination of various EIV estimation methods in the testing of CAPM, including the grouping method, the instrumental variable method, and the maximum likelihood method. Both potential measurement error problems of market return in the first pass and estimated beta in the second pass are corrected by either the grouping method or the instrumental variable method. Chen (2011) shows that empirical results support the role of market beta in the capital asset pricing model after correcting the EIV problem.

To deal with the measurement error problem associated with testing both CAPM and APT, Lee and Wei (1984) and Wei (1984) use the MIMC model to test whether APT outperformed CAPM. Betas are obtained from simultaneous equation system, and a cross-sectional regression of the security return against its β will be used to test the CAPM. They conclude that the beta estimated from the MIMC model by allowing measurement error on the market portfolio does not significantly improve the OLS beta estimate, and MLE estimator does a better job than the OLS and GLS estimators in the cross-sectional regressions because the MLE estimator takes care of the measurement error in beta.

More recently, Jagannathan, Kim, and Skoulakis (2010) and Da, Guo, and Jagannathan (2012) use three-stage cross-sectional regression to correct the errors-in-variables problem from the rolling-window betas. Their empirical findings support CAPM in explaining option-adjusted stock returns at the individual stock level.

4.3. Capital structure

Titman and Wessels (1988), Chang et al. (2009) and Yang et al. (2009) use structure equation models (e.g. LISREL model and MIMIC model) to mitigate the measurement problems of proxy variables when working on capital structure theory. Titman and Wessels (1988) use LISREL method to investigate determinants of capital structure. In the structure equation model, they use 15 indicators associated with eight latent variables and set 105 restrictions on the coefficient matrix. Empirical results, however, do not support four of eight propositions on the determinants of capital structure. Specifically, their results show that a firm's capital structure is not significantly related to its non-debt tax shields, volatility of earnings, collateral value of assets, and future growth. One possible reason for the poor results is that the indicators used in the empirical study do not adequately reflect the nature of the attributes suggested by financial theory.

Chang et al. (2009) apply a Multiple Indicators and Multiple Causes model (MIMIC) with refined indicators to reexamine Titman and Wessels (1988) work on determinants of capital structure. Chang et al. (2009) examine the seven indicator factors as follows: growth, profitability, collateral value, volatility, non-debt tax shields, uniqueness, and industry. Their empirical results show that the growth is the most influential determinant on capital structure, followed by profitability, and then collateral value. Under a simultaneous cause–effect framework, their seven constructs as determinants of capital structure have significant effects on capital structure decision.

Yang et al. (2009) apply a LISREL model to find determinants of capital structure and stock returns, and estimate the impact of unobservable attributes on capital structure decisions and stock returns. Using leverage ratios and stock returns as two endogenous variables and 11 latent factors as exogenous variables, Yang et al. (2009) find that stock returns, expected growth, uniqueness, asset structure, profitability, and industry classification are main determinants of capital structure, while leverage, expected growth, profitability, firm value, and liquidity can explain stock returns. In addition, the capital structure and stock return are mutually determined by each other.

Lee and Tai (2014) develop a simultaneous determination model to identify the joint determinants of capital structure and stock returns. The structural equation model with confirmatory factor analysis shows that stock return, asset structure, growth rate, industry classification, uniqueness, volatility and financial rating, profitability, government financial policy, and managerial entrenchment are key factors in determining a firm's capital structure. Such results are robust in the MIMIC and two-stage least square methods.

4.4. Measurement error in investment equation

Modern q theory, developed by Lucas and Prescott (1971) and Mussa (1977), shows that the shadow value of capital, marginal q, is the firm manager's expectation of the marginal contribution of new capital goods to future profits. Marginal q, therefore, should summarize the effects of all factors relevant to the investment decision. Lucas and Prescott (1971) and Hayashi (1982) show that the equality of marginal q with average q is under the assumptions of constant returns to scale and perfect competition. Because the marginal q is unobservable in the real world, most of empirical studies adopt Lucas and Prescott (1971) and Hayashi's (1982) assumption and use average q instead of marginal q to test q theory. In addition, if financial markets' valuation of the capital will be equal to the manager's valuation, average q, should equal an observable value, Tobin's q, defined as the ratio of the market value to the replacement value. Most empirical studies use Tobin's q as a proxy for marginal q to test the q theory of investment. However, their empirical results are inconsistent to the q theory (e.g. Blundell, Bond, Devereux, and Schiantarelli, 1992; Fazzari, Hubbard, and Petersen, 1988; Gilchrist & Himmelberg, 1995; Schaller, 1990).[6]

The model introduced by Fazzari et al. (1988) is

$$\frac{I_{it}}{K_{it}} = \eta_i + \beta q^*_{it} + \alpha \frac{CF_{it}}{K_{it}} + u_{it}, \tag{51}$$

where I_{it} represents the investments of firm i at time t, K_{it} is capital stock of firm i at time t, q^*_{it} is the marginal q_{it}, CF_{it} is cash flow of firm i at time t, η_i is the firm-specific effect, and u_{it} is the innovation term.

Almeida et al. (2010) show that OLS estimated coefficient of independent variable with measurement error, q^*_{it}, will be biased downward, and OLS estimated coefficient of the independent variable without measurement error, CF_{it}/K_{it}, will be biased upward. Following Eq. (51), if one of independent variables has measurement error and the other independent variables has no measurement error; the asymptotic biases of estimated coefficients can be defined as:

$$\text{p}\lim \hat{\beta} - \beta = \frac{-\beta\sigma^2_{\varepsilon}}{\sigma^2_{CF/K} - b_{CF/K,q^*} + \sigma^2_{\varepsilon}}, \tag{52}$$

and

$$\text{p}\lim \hat{\alpha} - \alpha = \beta b_{CF/K,q^*}\left(\frac{\sigma^2_{\varepsilon}}{\sigma^2_{\varepsilon} + \sigma^2_{CF/K}(1 - R^2_{CF/K,q^*})}\right), \tag{53}$$

in which $\sigma^2_{CF/K}$ is the variance of CF/K, σ^2_{ε} is the variance of error term between unobserved marginal q and observable average q^*, $b_{CF/K,q^*}$ is the auxiliary regression coefficient of a regressing q^* on CF/K, and $R^2_{CF/K,q^*}$ is the correlation coefficient between q^* and CF/K. We know that $\sigma^2_{CF/K} - b_{CF/K,q^*} + \sigma^2_{\varepsilon}$ is generally positive, so the estimated coefficient of q^* is downward biased. In addition, the direction of bias of estimated coefficient of CF/K will depend on the signs of β and $b_{CF/K,q^*}$. Given that q and cash flow are positively correlated, we can get the conclusion of Almeida et al. (2010) that $\hat{\beta}$ is downward biased and $\hat{\alpha}$ is upward biased.

Erickson and Whited (2000) argue that the measurement error of marginal q can result in different implications in empirical q models. They incorporate an EIV model to reexamine the empirical work done by Fazzari et al. (1988). By using generalized method of moments (GMM), Erickson and Whited (2000) obtain consistent estimators that the information contained in the third- and higher-order moments of the joint distribution of the observed regression variables. The estimator precision and consistency can be increased by exploit the information afforded by an excess of moment equations over parameters. Results show that cash flow does not affect a firms' financial decision, even for financially constrained firms, and the q theory is held if measurement error is taken into account.

Almeida et al. (2010) use Monte Carlo simulations and real data to compare the performance of generalized method of moments and instrumental variables approach dealing with measurement error problems in investment equations. In Monte Carlo simulations, they find estimators of GMM proposed by Erickson and Whited (2000) are biased for both mismeasured and well-measured regressors when the data have individual-fixed effects, heteroscedasticity, or no high degree of skewness. In contrast, the instrumental variable method results fairly unbiased estimators under those same conditions. Almeida et al. (2010) further empirically examine the investment equation introduced by Fazzari et al. (1988) by using GMM and instrumental variable method. Almeida et al. (2010) adopt Biorn's (2000) method using the lags of the variable as instruments in testing the investment equation. Empirical results show that estimators from generalized method of moments are unstable across different specifications and not economically meaningful, while estimators from a simple instrumental method are robust and conform to q theory. Almeida et al. (2010) conclude that instrumental method yields more consistent estimators and support the q theory in the investment equation.

In contrast, Erickson and Whited (2012) compare the ability of three errors-in-variables models, instrumental variables, dynamic panel estimators, and the high-order moment estimators in investment equation. They conclude that all of three models can perform well under correct specification, while the high-order moment estimators often outperform the instrumental variables and dynamic panel estimators in terms of bias and dispersion. Erickson and Whited (2012) also demonstrate a minimum distance technique to extend the high-order moment estimators used on unbalanced panel data.

5. Conclusion

In this paper, we investigate theoretical issues related to errors-in-variables (EIV) problem, and review how existing EIV estimation methods deal with measurement error problem. We first show how EIV problems affect the coefficients of independent variables in the regression model. We then discuss how classical method, mathematical programming method, grouping method, instrumental variable method, maximum likelihood method, and LISREL method deal with EIV problems. We further investigate how alternative EIV models have been used in empirical finance research. We find that the empirical research of cost of capital, asset pricing, capital structure, and investment equation have used alternative EIV methods to improve the empirical results. Not only can the reader of this paper understand the important research topics in finance, but also can the reader realize how measurement error problems affect the results of empirical work in such research topics. Finally, we suggest that future empirical studies on finance related issues should pay more efforts to deal with EIV problems and obtain more robust empirical results.

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