The Theoretical Superiority of the Interest Rate's Liberalization over Its Instrument

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摘 要

本文對利率自由化在理論上的相對優越性，提供了一個極有參考性與完整的補充。無論吾人是採用凱因斯的，或是古典的，或是隨機性理性預期的總體模型來作分析，皆可得到相當一致的明確結論，即利率官訂將導致經濟動態調整的不穩定與價格難以收斂等嚴重後果。尊重利率價格調整機能確實有其理論上的根據與優點。

ABSTRACT

This paper renders a piece of useful information and sound supplement regarding the theoretical comparative advantages of the interest rate's liberalization over its instrument.

Adopting any one of three approaches, i.e., the Keynesian-Hicksian macromodel, the Classical macromodel, and the stochastic rational expectations macromodel, we obtain a quite consistent and unique conclusion that the interest rate officially pegged policy will result in serious problems such as local unstability, destabilization, and indeterminancy. These findings enhance our belief that the interest rate's liberalization does exist the theoretical superiority. This implies that the adherence to the interest rate price mechanism by the monetary authority is valid in its nature.

I. Introduction

This paper is intended to show the theoretical relative advantages of the interest rate's liberalization from the following three approaches. The first is from the viewpoint of adjustment dynamics of a simple Keynesian macromodel. We employ the familiar IS-LM framework for illustrative purpose, and show that, in accordance with the stability conditions, the interest rate's liberalization increases the likelihood of a locally stable economic system.

The second approach is that we utilize a version of the conventional Classical macromodel to show that pegging the interest rate at a given level as alleged by the real bills doctrine will result in serious problems of both indeterminate solution and destabilization of the relevant endogenous variables.

The third turns to the viewpoint of the choice of instruments under a stochastic rational expectations macromodel. We put the Lucas (1973) supply function into the IS-LM framework in a way of stochastic rational expectations form and find that the price level can't converge if the monetary authority pegs the interest rate under this stochastic economics regime.

Despite we adopt any one of these three approaches, i.e., Keynesian, Classical, and stochastic rational expectations, we obtain a considerably consistent and unique conclusion that the interest rate's liberalization does have the theoretical advantage of dynamically stabilizing and rationalizing the economic system, in addition to its other beneficial effects such as those mentioned in Shea (1979), and Tsiang (1979). Thus, this paper enriches a sound theoretical argument for the interest rate's liberalization.

II. The Dynamic Adjustment Advantage for the Interest Rate's Liberalization from Keynesian IS-LM Model

A conventional macromodel considered here is

$$I(i,y) - S(y) = 0 \quad \text{———— IS equilibrium locus} \tag{1}$$

$$L(i,y) - M = 0 \quad \text{———— LM equilibrium locus} \tag{2}$$

where I=investment, S=saving, L=demand for money, M=fixed money supply, i= interest rate, y=real income.

The dynamic adjustment equations for this model are specified as usual:

$$dy/dt = k_1 [I(i,y) - S(y)], \qquad 0 \leqslant k_1 \leqslant \infty, \tag{3}$$

$$di/dt = k_2 [L(i,y) - M], \qquad 0 \leqslant k_2 \leqslant \infty. \tag{4}$$

We take Taylor's linear approximation for equations (3) and (4) around the initial equilibrium point (y^*, i^*), and let the general solution be $y=y^* + \alpha e^{\lambda t}$, and $i=i^* + \beta e^{\lambda t}$, where α and β are nonzero constants, the above system then becomes:

$$\begin{bmatrix} k_1(I_y - S_y) - \lambda & k_1 I_i \\ k_2 L_y & k_2 L_i - \lambda \end{bmatrix} \begin{bmatrix} \alpha e^{\lambda t} \\ \beta e^{\lambda t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

To find values for λ such that equation (5) yields nonzero α and β, we derive the following auxiliary equation:

$$\lambda^2 - [k_1(I_y - S_y) + k_2 L_i]\lambda + k_1 k_2 [L_i(I_y - S_y) - I_i L_y] = 0 \tag{6}$$

with the roots λ_1 and λ_2 satisfying

$$\lambda_1 + \lambda_2 = k_1(I_y - S_y) + k_2 L_i \tag{7.1}$$

$$\lambda_1 \cdot \lambda_2 = k_1 k_2 [L_i(I_y - S_y) - I_i L_y] \tag{7.2}$$

According to Wang (1985), as $I_y > S_y$ (IS slopes upward), the sufficient conditions for this model to be dynamically stable are such that the relative speed of dynamic adjustment, k_2/k_1, be no less than some critical value $(I_y - S_y)/-L_i$, regardless of whether $\Delta > 0$ or $\Delta < 0$, where

$$\Delta = [k_1(I_y - S_y) + k_2 L_i]^2 - 4k_1 k_2 [L_i(I_y - S_y) - I_i L_y].$$

In the case of $\Delta > 0$, node and saddlepoint solutions will occur. In the another case of $\Delta < 0$, focus and center types will appear. If k_2/k_1 is less than $(I_y - S_y)/-L_i$, the aforementioned types of critical point of node, saddlepoint, focus, and center will be trapped into the unstable regime.

One measure of the interest rate liberalization is the sensitivity of the interest rate to excess demand in money, which is to be reflected by the magnitude of k_2. As the interest rate adjustment mechanism is completely liberalized, k_2 will approach $+\infty$. In this situation, the relative speed of adjustment, k_2/k_1, will achieve its upper limit for a given k_1, and enlarges its own possibility of being no less than the critical value $(I_y - S_y)/-L_i$.

This can lead the economy to stable node and saddlepoint if $\Delta > 0$, or stable focus and center if $\Delta < 0$.

On the contrary, if the interest rate is officially pegged, k_2 approaches 0. Now the relative speed tends toward its lower limit, and is more likely to be less than $(I_y - S_y)/-L_i$, which contributes to the destabilizing forces. In the extreme case of $k_2 = 0$, LM is horizontal at a certain interest rate, and the system generates an unstable saddlepoint. Any exogenous shock can only be absorbed by the income adjustment, making the real economic variables very sensitive to outside disturbances.

From this examination, we may conclude that the interest rate liberalization does have the advantage of dynamically stabilizing the economic system under a simple Keynesian IS-LM framework. In fact, this conclusion is very consistent with the argument proposed by the macrodisequilibrium economists such as Clower (1965), Leijonhufvud (1967), Solow and Stiglitz (1968), Barro and Grossman (1971), . . .

III. The Indeterminancy and Destabilization Disadvantage for the Interest Rate Pegged Policy from the Classical Model

A version of the Classical model discussed here is adapted from Sargent (1979, Chap. I):

$$y = F(K,N) \tag{8}$$

$$W/P - F_N(K,N) = 0 \tag{9}$$

$$N = N^S(W/P) \tag{10}$$

$$C = C(y - \overline{T} - \overline{\delta}\overline{K} - [(M + \overline{B})/P]\overline{\pi}, \overline{i} - \overline{\pi}) \tag{11}$$

$$I = I(q(\overline{K}, N, \overline{i} - \overline{\pi}) - 1) \tag{12}$$

$$y - (C + I + \overline{G} + \overline{\delta}\overline{K}) = 0 \tag{13}$$

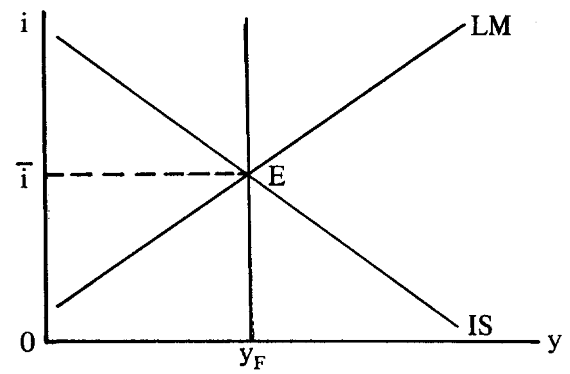
$$L(\overline{i}, y) - M/P = 0 \tag{14}$$

Where y=real income, N=employment, P=price level, W=wage rate, C=consumption, I=investment, M=money supply, G=government spending, T=taxes, K=capital stock, π=anticipated rate of inflation, B=government bond, i=nominal interest rate, δ=depreciation rate.

This model contains seven equations. The endogenous variables are y, N, P, W, C, I, and M, while the exogenous variables are $\overline{G}$, $\overline{T}$, $\overline{B}$, $\overline{K}$, $\overline{\pi}$, $\overline{\delta}$, and $\overline{i}$.

The real bill doctrine was alleged that the monetary authority attempted to peg the interest rate i by permitting the money supply to be whatever it must be to meet the demand for loans intended to finance "real" investment at an interest rate set with a view of accommodating commerce and business. The effect of the rule is to make the interest rate an exogenous variable, and the money supply endogenous. The monetary authority simply stands ready to buy or sell whatever quantities of government bonds are offered at the regulated interest rate.

The Classical model is block recursive, because equations (8)-(10) determine y, N, and W/P. Equations (11)-(14) are then four equations that, given the previously determined values of y, N, and W/P, can determine the four endogenous variables C, I, P, and M. The workings of the model can be illustrated with Figure 1. The equations (8)-(10) determine output at its full employment level, y_F. The equations (11)-(14) simultaneously determine a level of aggregate demand which equals y_F at the officially pegged interest rate $\bar{i}$. Since the interest rate mechanism can not adjust to make aggregate demand equal the full employment output supply, the entire burden falls on the price level and the money supply, which influence consumption through their effect on perceived real disposable income. The price level and the endogenous money supply issued by the government jointly adjust to make the IS curve and the portfolio balance LM curve pass through the intersection of $\bar{i}$ and y_F.



Let us suppose that there is an increase in government expenditures to illustrate how the model works. Since the interest rate and output are fixed, the price level and the money supply must adjust to reduce perceived disposable income and then consumption by just enough to offset the increase in G. In particular, the change in P and the change is G must obey [1]

$$dG = -c_1([M + \bar{B})/P^2]\,\bar{\pi})\,dP \tag{15}$$

Where c_1 is the marginal propensity to consume out of perceived disposable income. If $M + \bar{B}$ and π are both positive, prices must fall in order to increase the real value of the government's debt to the public and the anticipated real capital losses on that debt, thereby decreasing disposable income and consumption.

A fall in M is implied by the fall in P through the portfolio balance equation (14). However, notice that the dynamic response of this Classical model to an increase in G in the situation of positive $[(M+\bar{B})/P^2]\bar{\pi}$ will lead to the destabilization of the system. For G rise will create an excess demand for goods at the initial i, P, and y_F, which usually leads to rising prices. But if $[(M+\bar{B})/P^2]\bar{\pi}$ is positive, a fall in P is required to restore equilibrium, so that the system is driven away from equilibrium. On the other hand of negative $[(M+\bar{B})/P^2]\bar{\pi}$, the system will be stable due to rising prices. But this premise scarcely happens in our economy world.

If $\pi(M+B) = 0$, then P no longer appears in the consumption function, and the equations (11)-(13) of the system are three equations containing only two variables, C and I. The consumption function determines C, while the investment function determines I. But in general the values of C and I so determined will not be satisfied to the national income equation (13), where y is full employment output y_F having been determined by equations (8)-(10). That leads to the "overdeterminancy" problem for the two variables C and I. Although the model overdetermines C and I, P and M are underdetermined by the seventh equation, which is not capable of determining P and M.

Even if i is pegged at the "correct" level, i.e., at the intersection of the y_F line and the IS curve, the price level and the money supply are still indeterminate. It is true that if i is pegged at the correct level, the solution for y_F equals the aggregate demand from the IS curve. But it remains true that the portfolio balance LM curve determines only the ratio M/P and can not determine the level of either M or P.[2]

Wicksell (1965) argued that pegging the interest rate too low would set off increases in the price level and money supply of indefinitely large magnitudes. The model itself possesses no equilibrium when the monetary authorities peg the interest rate at such a level. Wicksellian analysis criticizes the "real bills" doctrine. Basing on the above analysis, we conclude that any attempt to peg the interest rate in the Classical model is destabilizing and indeterminate.[3]

IV. The Indeterminancy Disadvantage for the Interest Rate Pegged Policy from the Stochastic Rational Expectations Model.

In a stochastic rational expectations (SRE) model, should the Central Bank peg the interest rate as its instrument? We will answer this question by considering the following simple SRE macromodel:

$$y_t = r(P_t - {}_tP^*_{t-1}) + \lambda y_{t-1} + u_t, \quad r > 0 \tag{16}$$

$$m_t - P_t = -y_t + bi_t + \varepsilon_{1t}, \quad b < 0 \tag{17}$$

$$y_t = c(i_t - ({}_{t+1}P^*_{t-1} - P_t)) + \varepsilon_{2t}, \quad c < 0$$

Where all the variables except i_t are in the natural-log form; i_t is the interest rate, y_t is the real output, m_t is the money supply, P_t is the price level, u_t, ε_{1t} and ε_{2t} are each serially independent stationary random processes with means of zero and finite constant variances.

We supplement (16)-(18) with

$$_{t}P^{*}_{t-1} = E[P_t | I_{t-1}], \tag{19}$$

$$_{t+1}P^{*}_{t-1} = E[P_{t+1} | I_{t-1}], \tag{20}$$

Where I_{t-1} includes all lagged endogenous and exogenous variables in the information set of the authority. Using (19) and taking conditional expectations in (16)-(18), we have

$$Ey_t | I_{t-1} = \lambda y_{t-1} \tag{21}$$

$$Em_t | I_{t-1} - EP_t | I_{t-1} = Ey_t | I_{t-1} + bEi_t | I_{t-1} \tag{22}$$

$$Ey_t | I_{t-1} = c(Ei_t | I_{t-1} - E(P_{t+1} - P_t) | I_{t-1}) \tag{23}$$

The monetary authority has the option of using a feedback rule on previous information for setting i_t and letting m_t be whatever it must be to achieve portfolio balance at the desired pegged i_t; or alternatively of setting m_t via a feedback rule on previous information and letting i_t be whatever it must to equilibrate the system. Under a money supply rule, $Em_t | I_{t-1}$ is given and the distribution of prices is determined by solving (22) and (23):

$$EP_t | I_{t-1} = \frac{1}{1-b} Em_t | I_{t-1} - \frac{1+bc^{-1}}{1-b} Ey_t | I_{t-1} - \frac{1}{1-b} EP_{t+1} | I_{t-1} \tag{24}$$

Since $b < 0$, thus $0 < -b/(1-b) < 1$.

We can solve the above difference equation in $EP_t | I_{t-1}$ in the forward direction to get

$$EP_t | I_{t-1} = \frac{1}{1-b} \sum_{j=0}^{\infty} (\frac{-b}{1-b})^j Em_{t+j} | I_{t-1} - \frac{1+bc^{-1}}{1-b} \sum_{j=0}^{\infty} (\frac{-b}{1-b})^j Ey_{t+j} | I_{t-1} \tag{25}$$

where we are imposing the terminal condition

$$\lim_{n \to +\infty} (\frac{-b}{1-b})^n EP_{t+n} | I_{t-1} = 0 \tag{26}$$

provided that the rule is such that

$$\sum_{j=0}^{\infty} (\frac{-b}{1-b})^j Em_{t+j} | I_{t-1}$$

converges, equation (25) determines a finite expected price level $EP_t | I_{t-1}$ this has the effect of asserting that agents will not expect accelerating inflation or deflation in the absence of money supply changes. Thus, there exists a money supply rule that delivers a finite conditionally expected price level and can be tailored to set $EP_t | I_{t-1}$ at some desired level.

On the other hand, consider an interest rate pegged policy, which determines $Ei_t | I_{t-1}$. Since both $Ei_t | I_{t-1}$ and $Ey_t | I_{t-1}$ are already determined by the pegged policy and eq. (16), we have that eq. (17) determines $Em_t | I_{t-1} - EP_t | I_{t-1}$ and eq. (18) must determine $EP_t | I_{t-1}$. From the conditional expectation of (18), we have

$$EP_t | I_{t-1} = EP_{t+1} | I_{t-1} + c^{-1} Ey_t | I_{t-1} - Ei_t | I_{t-1} \tag{27}$$

The solution of the above difference equation is

$$EP_t | I_{t-1} = \frac{1}{c} \sum_{j=0}^{n} Ey_{t+j} | I_{t-1} - \sum_{j=0}^{n} Ei_{t+j} | I_{t-1} + EP_{t+n+1} | I_{t-1} \tag{28}$$

which requires a terminal condition in the form of an exogenously given expected price level $EP_{t+n+1} | I_{t-1}$. An increase in this terminal condition value results in a one-for-one increase in $EP_t | I_{t-1}$. Thus, the expected price $EP_t | I_{t-1}$ is underdetermined by the model itself, being dependent on our viewpoint for the terminal condition value that in effect determines the price level. That is, the model itself is incapable of restricting the price level.

The economics reasons behind the underdetermined expected price level is that the public correctly expects that the authority will accommodate whatever quantity of money is demanded at the pegged interest rate under the interest rate rule. The public will expect that any increase in P_t will be met by an increase in m_t. There is no mechanism to anchor the expected price level. The terminal condition that we have to impose to determine the $EP_t | I_{t-1}$ is very much stricter under an interest rate rule than what we had to impose under the money supply rule. This is not simply a matter of choosing the wrong level or rule for the interest rate. The most serious issue is that there is no interest rate rule that is associated with a determinate price level in a SRE macroeconomics model.

V. Conclusion

Recently, the monetary authority in Taiwan has been devoting to the financial reform of the interest rate's liberalization. This paper renders a piece of useful information and sound supplement regarding the theoretical comparative advantages of the interest rate's liberalization over its instrument.

We demonstrate this argument from the following three approaches: the Keynesian-Hicksian macromodel, the Classical macromodel, and the stochastic rational expectations macromodel. Adopting any one of these three methods, we obtain a quite consistent and unique conclusion that the interest rate officially pegged policy will result in serious problems such as local unstability, destabilization, and indeterminancy.

These findings enhance our belief that the interest rate's liberalization does exist the theoretical superiority. Thus, our demonstration shows that the adherence to the interest rate adjustment mechanism by the monetary authority is rather valid in its nature.

Footnote

1. $dG = -c_1 [\frac{-(dM+d\bar{B})\bar{\pi}}{P} + \frac{(M+\bar{B})}{P^2}\bar{\pi}\, dP]$

   assume the open market operation obey the constraint, dM=-dB, thus

   $dG = -c_1 [\frac{M+\bar{B}}{P^2}\bar{\pi}] dP$

2. This model also illustrates the danger of naive "equation counting" as a technique supposedly capable of determining whether a model possesses a unique equilibrium. The equality between the number of equations and number of variables does not suffice to guarantee that the equations have a solution. The problem is that the system decomposes in an unfortunate way, the last three equations forming an independent subset involve only I and C, while the last is an independent equation involving both P and M.
3. The criticism of the real bills doctrine based on the classical analysis described here is correct only in a system that contains neither a Pigou effect nor nonzero perceived real capital gains on the government's debt to the public. This is consistent with the argument of Metzler's point (1951).

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