科目 urse	數學	開課系級 Dept, & Class	統計系	日 期 Date, Period	4 月 27 日 第 2 節	試題編號 Course No.	225

5、(a) (5%) Show that if $a \neq b$, then

$$a^n + a^{n-1}b + a^{n-2}b^2 + \cdots + ab^{n-1} + b^n = \frac{a^{n+1} - b^{n+1}}{a - b}$$

(b) (5%) Use the result in part (a) to find A^n if

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 1 & 0 & c \end{bmatrix}$$

6. (10%) Find a 3×3 matrix A that has eigenvalues $\lambda = 0, 1$, and -1 with corresponding eigenvectors

$$\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

respectively. (Hint: Consider the product of eigenvalues matrix and eigenvectors matrix.)

7. Find the rank and nullity of the matrix; then verify that *rank(A) + nullity(A) = the number of columns in A*. (Each part counts for 5%)

$$(a) A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix} \quad (b) A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

8. (10%) Find A^n if n is a positive integer and

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

9. (10%) Find the inverse matrix A^{-1} of A, and check your result by showing that $AA^{-1} = I$.

$$\begin{bmatrix} 1 & 1+i & 0 \\ 0 & 1 & i \\ -i & 1-2i & 2 \end{bmatrix}$$

注意: 附常態分配表,卡方分配表,t分配表,F分配表於最後.

(15%) 1. The length of time, X, to complete a certain key task in house construction is an exponentially distributed random variable with a mean of 10 hours. The cost, C, of completing this task is related to the square of the time to completion by the formula $C = 100 + 30X + 4X^2$.
Find the expected value of C.

(15%) 2. Let $X_1, X_2, \cdots, X_n$ denote a random sample from a normal distribution $N(\mu, 16)$. To test $H_0: \mu = 25$ against $H_1: \mu = 23$, define the critical region to be $C = \{(x_1, x_2, \cdots, x_n);\ \bar{x} \le c\}$. Find the sample size n and the constant c such that $\alpha = \beta = 0.10$.

(15%) 3. In 1980, 28% of the CEOs (chief executive officers) of major U.S. corporations had a degree beyond the bachelor's, 59% possessed a bachelor's degree, and 13% had a high school diploma. A recently taken sample of 500 CEOs showed that 208 of them have a degree beyond the bachelor's, 252 have a bachelor's degree, and 40 possess a high school diploma. Using the 1% significance level, can you reject the null hypothesis that the current percentage distribution by degree of all CEOs is the same as for 1980?

(20%) 4. The following table gives the time taken (in minutes) to assemble one unit of a certain product by six randomly selected workers on two types of machines.

Worker	1	2	3	4	5	6
Machine I	23	26	19	24	26	22
Machine II	21	24	23	25	24	25

(a) Test at the 5% significance level if the mean time taken to assemble a unit of the product is different for the two types of machines.

(b) If you planned to report the results of the statistical test in part (a), what P-value would you report?

(c) Explain what conditions must hold true in order to perform the test in part (a).

(10%) 5. (a) What property does the least squares line (i.e. $y = a + bx$) have when compared with any other line, say, $y = a' + b'x$ that might be used to describe the set of data: $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$?

(b) Give the definition of the measure of usefulness of the regression line and its computational formula without proof.

(25%) 6. A firm has 3 branches located in different regions but doing rather similar work. An efficiency study was carried out to see if workers of a particular classification differed in effectiveness among the branches. A sample of 5 random time intervals was taken from each branch and certain relevant observations of activities going on during these periods were recorded with the following results.

	Branch		
	A	B	C
Sample means, $\bar{x}_i$	49	54	47
Sample variance, s_i^2	270	200	220

(a) In a one-way ANOVA for k independent samples with equal sample size c, prove that (1) $\frac{SS(B)}{k-1} = cs_{\bar{x}}^2$, and (2) $\frac{SS(W)}{k(c-1)} = \frac{\sum_{i=1}^{k} s_i^2}{k}$, where $s_{\bar{x}}^2 = \frac{\sum_{i=1}^{k}(\bar{x}_i - \bar{x})^2}{k-1}$, $SS(B)$ is the between-samples sum of squares, and $SS(W)$ is the within-samples sum of squares.

(b) Using the results in part (a) and the F test, determine if there is a significant difference in workers' effectiveness among the three branches at $\alpha = 0.05$.

考試科目 Course	數學	開課系級 Dept, & Class	統計學系	日期 Date, Period	月 日 第 節	試題編號 CourseNo.	

1. Find the Maclaurin series for the following functions : (15%)

(1) $f(x)=\frac{1}{1-x}$ (2) $g(x)=\frac{1}{1+x^2}$ (3) $h(x)=\tan^{-1}x$

Also find the interval of convergence and the associated radius of convergence.

2. Consider the production function $f(x,y)=600x^{\frac{2}{3}}y^{\frac{1}{3}}$ and the cost constraint $400x+200y=30{,}000$ dollars. How many units of labor (x) and how many units of capital (y) should be used in order to maximize production? (10%)

3. Use polar coordinates to evaluate (10%)

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx+\int_{1}^{\sqrt{2}}\int_{0}^{x} xy\,dy\,dx+\int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\,dy\,dx$$

4. Find the approximate value of (15%)

$$\int_0^3 \sqrt{4+x^3}\,dx$$

by (1) the trapezoidal rule (2) Simpson's rule. Take $n=6$.