

考技科目 COURSE	數理統計學	系級	統計系	日期 Date, Period	4月23日 第 / 節	試題編號 Course No.
----------------	-------	----	-----	-----------------------	----------------	--------------------

頁數 2-1

國立政治大學圖書館

1. (15%) Let X_1, X_2, \dots be independent random variables from $Uniform(0, 1)$. Define

$$N = \min\{n : \sum_{i=1}^n X_i > 1\}.$$

Find the distribution of N and compute $E(N)$. (Hint: You may find the density function of N via $P(N = n)$.)

2. (12%) Let the joint density function of X and Y be

$$f(x, y) = k(k-1)(y-x)^{k-2}, \text{ with } 0 < x \leq y < 1, k \in \{2, 3, 4, \dots\}.$$

Compute $g(x)$, $h(y)$, and $E(X|Y)$.

3. (a) (6%) Let $U \sim U(0, 1)$ and $V \sim U(-1, 1)$ are independent. Define

$$X = V/U, \text{ provided that } V^2 + U^2 \leq 1.$$

Find the distribution of X .

- (b) (9%) Let X_1, X_2 be independent r.v.'s distributed as $N(0, \sigma^2)$. Find the p.d.f. of $Y_1 = X_1^2 + X_2^2$ and $Y_2 = X_1/X_2$ and show that Y_1 and Y_2 are independent.

4. (12%) Suppose that X_1, \dots, X_n are a random sample from the distribution

$$F(x) = 1 - \left(\frac{\theta_1}{x}\right)^{\theta_2}, \theta_1 \leq x, \theta_1 > 0, \theta_2 > 0.$$

Find the M.L.E. (Maximum likelihood estimator) of θ_1 and θ_2 .

5. (16%) Let X_1, X_2, \dots are random variables. Write down the definition of the following three types of convergence: (X is also a random variable.)

- (a) $\{X_n\}$ converges to X in probability as $n \rightarrow \infty$.
- (b) $\{X_n\}$ converges to X in distribution as $n \rightarrow \infty$.
- (c) $\{X_n\}$ converges to X in quadratic mean as $n \rightarrow \infty$.

Also, show that (a) implies (b) and (c) implies (b), but (b) cannot imply (a) or (c).

考試科目 Course	數理統計學	系級	統計系	日期 Date, Period	第 4 月 23 日	頁數 Course No.
----------------	-------	----	-----	-----------------------	------------	------------------

2-2

6. (15%) Suppose X_1, \dots, X_n are a random sample from $U(-\theta, \theta)$, $\theta \in (0, \infty)$.

- (a) State the definition of "Sufficient statistic".
- (b) Find a sufficient statistic for θ .
- (c) Is the sufficient statistic in (b) minimal? If not, find a minimal sufficient statistic for θ .

7. (15%) Let X_1, \dots, X_n be a random sample from

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} I_{(0, \infty)}(x), \quad \theta \in \Theta = (0, \infty).$$

- (a) State the definition of UMP (Uniformly Most Powerful) test.
- (b) Find the UMP test for testing $H_0 : \theta \geq \theta_0$ vs. $H_1 : \theta < \theta_0$ at level of significance α .
- (c) Determine the minimum sample size n required to obtain power at least 0.95 against $\theta_1 = 500$ when $\theta_0 = 1000$ and $\alpha = 0.05$.

1. If you try to rent an apartment or buy a house, you will find that real estate representatives establish apartment rents and house prices on the basis of the square footage of the heated floor space. The data in the following table give the square footage (unit = 1,000 square feet) and sales prices (unit = \$1,000) on $n = 12$ houses randomly selected from those sold in a small city.

Square Feet, x	Price, y	Square Feet, x	Price, y
1.5	68.7	2.0	85.4
2.1	89.3	1.6	77.0
1.7	81.4	1.5	72.4
1.5	71.1	1.8	78.2
1.9	82.4	1.8	84.3
2.4	94.9	2.2	91.7

$\sum x_i = 22, \sum x_i^2 = 41.3, \sum y_i = 976.8, \sum y_i^2 = 80,265.66, \sum x_i y_i = 1,816.77.$

- a. Find the least squares regression line for these data. (15)
- b. Test to determine whether there is a linear relationship between the two variables. (Use $\alpha = 0.05$). (10)
- c. Compute the coefficient of determination, and interpret its value. (5)
- d. Estimate with 90% confidence that the mean sales price of a 2,000 square feet of heated floor space. (10)

2. A study was undertaken to determine the relative typing speeds that could be obtained when using three different brands of word processors. Each word processor was assigned to each of five secretaries, the order of assignment conducted in a random manner. The typing speed, in words per minute, for 10 minutes of typing was recorded for each secretary-word processor combination. The data obtained are given in the following.

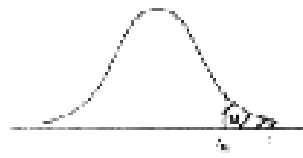
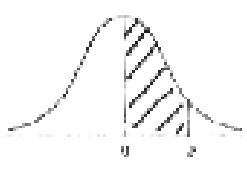
Word Processor Brand	Secretary					Total	Average
	1	2	3	4	5		
A	79	80	77	75	82	393	78.6
B	74	79	73	70	76	372	74.4
C	82	86	80	79	81	408	81.6
Total	235	245	230	224	239	1173	
Average	78.33	81.67	76.67	74.67	79.67		78.2

- a. Identify the design used for this experiment. (10)
 - b. Do the data provide sufficient evidence to indicate that the mean typing speed for the secretaries varies with the brand of word processor used? Test at $\alpha = 0.05$. (15)
3. Suppose that the time X necessary to process an application for a license plate for a newly purchased automobile is normally distributed with $\mu = 6$ min and $\sigma = 1.5$ min
- a. What is the probability that it takes at least 9 min to process a single application? (10)
 - b. If 10 such applications are process independently of one another, what is the probability that all take less than 9 min? (10)

4. The paper "Relative Controllability of Dissimilar Cars" reported results of an experiment to compare handling ability for two cars having quite different lengths, wheelbases, and turning radii. The observations are time in seconds required for subjects to park each car.

Subject:	1	2	3	4	5	6
Car A:	37.0	25.8	16.2	24.2	22.0	33.4
Car B:	17.8	20.2	16.8	41.4	21.4	38.4
Subject:	7	8	9	10	11	12
Car A:	58.2	33.6	24.4	23.4	21.2	36.2
Car B:	32.2	27.8	23.2	29.6	20.6	32.2

Does the data suggest that the average person will more easily handle one car than the other? Test the relevant hypothesis using $\alpha = 0.05$. (15)



NORMA. CURVE AREAS

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0949	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2421	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2825	.2855
0.8	.2885	.2915	.2945	.2975	.3005	.3035	.3065	.3095	.3125	.3155
0.9	.3185	.3215	.3245	.3275	.3305	.3335	.3365	.3395	.3425	.3455
1.0	.3485	.3515	.3545	.3575	.3605	.3635	.3665	.3695	.3725	.3755
1.1	.3785	.3815	.3845	.3875	.3905	.3935	.3965	.3995	.4025	.4055
1.2	.4085	.4115	.4145	.4175	.4205	.4235	.4265	.4295	.4325	.4355
1.3	.4385	.4415	.4445	.4475	.4505	.4535	.4565	.4595	.4625	.4655
1.4	.4685	.4715	.4745	.4775	.4805	.4835	.4865	.4895	.4925	.4955
1.5	.4985	.5015	.5045	.5075	.5105	.5135	.5165	.5195	.5225	.5255
1.6	.5285	.5315	.5345	.5375	.5405	.5435	.5465	.5495	.5525	.5555
1.7	.5585	.5615	.5645	.5675	.5705	.5735	.5765	.5795	.5825	.5855
1.8	.5885	.5915	.5945	.5975	.6005	.6035	.6065	.6095	.6125	.6155
1.9	.6185	.6215	.6245	.6275	.6305	.6335	.6365	.6395	.6425	.6455
2.0	.6485	.6515	.6545	.6575	.6605	.6635	.6665	.6695	.6725	.6755
2.1	.6785	.6815	.6845	.6875	.6905	.6935	.6965	.6995	.7025	.7055
2.2	.7085	.7115	.7145	.7175	.7205	.7235	.7265	.7295	.7325	.7355
2.3	.7385	.7415	.7445	.7475	.7505	.7535	.7565	.7595	.7625	.7655
2.4	.7685	.7715	.7745	.7775	.7805	.7835	.7865	.7895	.7925	.7955
2.5	.7985	.8015	.8045	.8075	.8105	.8135	.8165	.8195	.8225	.8255
2.6	.8285	.8315	.8345	.8375	.8405	.8435	.8465	.8495	.8525	.8555
2.7	.8585	.8615	.8645	.8675	.8705	.8735	.8765	.8795	.8825	.8855
2.8	.8885	.8915	.8945	.8975	.9005	.9035	.9065	.9095	.9125	.9155
2.9	.9185	.9215	.9245	.9275	.9305	.9335	.9365	.9395	.9425	.9455
3.0	.9485	.9515	.9545	.9575	.9605	.9635	.9665	.9695	.9725	.9755

CRITICAL VALUES OF T

DEGREES OF FREEDOM	CONFIDENCE LEVEL				
	1%	5%	10%	20%	50%
1	3.078	1.963	1.638	1.500	1.375
2	2.920	1.888	1.601	1.463	1.348
3	2.767	1.833	1.558	1.420	1.305
4	2.624	1.781	1.519	1.381	1.274
5	2.576	1.732	1.483	1.345	1.246
6	2.532	1.688	1.450	1.312	1.223
7	2.492	1.648	1.420	1.282	1.193
8	2.456	1.611	1.392	1.255	1.167
9	2.423	1.577	1.367	1.230	1.143
10	2.392	1.546	1.344	1.207	1.121
15	2.238	1.476	1.289	1.153	1.063
20	2.179	1.435	1.259	1.125	1.036
25	2.131	1.401	1.236	1.103	1.015
30	2.093	1.372	1.216	1.084	0.996
40	2.026	1.329	1.179	1.050	0.963
50	1.985	1.295	1.158	1.030	0.944
60	1.952	1.269	1.141	1.014	0.929
70	1.925	1.248	1.127	0.999	0.916
80	1.902	1.231	1.116	0.988	0.905
90	1.883	1.217	1.107	0.979	0.896
100	1.868	1.205	1.100	0.972	0.890
120	1.848	1.187	1.085	0.959	0.877
140	1.832	1.174	1.074	0.950	0.869
160	1.819	1.163	1.065	0.943	0.863
180	1.808	1.154	1.058	0.938	0.859
200	1.799	1.147	1.053	0.934	0.856
∞	1.782	1.145	1.050	0.932	0.855

國立政治大學圖書館

考試科目
COURSE

流計學

系級

流計所

日期
Date
Period

期
4月22日

第
節
Course No.

3-3

Table A.7 Critical Values From the F Distribution

df1	df2									
	1	2	3	4	5	6	7	8	9	10
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
2	19.00	18.51	18.16	17.84	17.54	17.27	17.02	16.79	16.58	16.39
3	10.13	9.55	9.25	8.98	8.74	8.52	8.31	8.11	7.92	7.74
4	7.71	7.01	6.75	6.52	6.31	6.11	5.92	5.74	5.57	5.41
5	6.61	5.79	5.56	5.35	5.16	4.98	4.81	4.64	4.48	4.33
6	5.99	5.14	4.92	4.72	4.54	4.37	4.21	4.05	3.90	3.75
7	5.59	4.74	4.52	4.32	4.15	3.98	3.82	3.67	3.52	3.37
8	5.32	4.48	4.26	4.06	3.89	3.73	3.57	3.42	3.27	3.12
9	5.12	4.28	4.06	3.86	3.69	3.53	3.37	3.22	3.07	2.92
10	4.98	4.14	3.92	3.72	3.55	3.39	3.23	3.08	2.93	2.78
11	4.84	4.00	3.78	3.58	3.41	3.25	3.09	2.94	2.79	2.64
12	4.73	3.89	3.67	3.47	3.30	3.14	2.98	2.83	2.68	2.53
13	4.63	3.80	3.58	3.38	3.21	3.05	2.89	2.74	2.59	2.44
14	4.56	3.74	3.52	3.32	3.15	2.99	2.83	2.68	2.53	2.38
15	4.51	3.69	3.47	3.27	3.10	2.94	2.78	2.63	2.48	2.33
16	4.46	3.65	3.43	3.23	3.06	2.90	2.74	2.59	2.44	2.29
17	4.43	3.62	3.40	3.20	3.03	2.87	2.71	2.56	2.41	2.26
18	4.41	3.60	3.38	3.18	3.01	2.85	2.69	2.54	2.39	2.24
19	4.39	3.58	3.36	3.16	2.99	2.83	2.67	2.52	2.37	2.22
20	4.38	3.57	3.35	3.15	2.98	2.82	2.66	2.51	2.36	2.21
21	4.37	3.56	3.34	3.14	2.97	2.81	2.65	2.50	2.35	2.20
22	4.36	3.55	3.33	3.13	2.96	2.80	2.64	2.49	2.34	2.19
23	4.36	3.54	3.32	3.12	2.95	2.79	2.63	2.48	2.33	2.18
24	4.35	3.53	3.31	3.11	2.94	2.78	2.62	2.47	2.32	2.17
25	4.35	3.53	3.31	3.11	2.94	2.78	2.62	2.47	2.32	2.17
26	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
27	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
28	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
29	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
30	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
31	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
32	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
33	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
34	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
35	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
36	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
37	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
38	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
39	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
40	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
45	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
50	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
60	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
70	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
80	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
90	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16
100	4.34	3.52	3.30	3.10	2.93	2.77	2.61	2.46	2.31	2.16

考級科目 Course	修級 Dept. & Class	日期 Date, Period	期 第	日 節	試題編號 Course No.
微積分 Calculus	統計 碩士生				

國立政治大學圖書館

1. (10 points)
 - (a) Find a point on the curve $y = x^2$ that is closest to the point (18,0).
 - (b) Give the tangent line to the curve at the point that you obtained in (a).

2. (10 points)
 - (a) Give the Taylor series of $\sin(x)$ about $x = 0$.
 - (b) Approximate $\sin(0.1)$ to the second decimal place.

3. (10 points)

Evaluate the sums.

 - (a) $1 \cdot 3 + 2 \cdot 4 + \dots + 98 \cdot 100$
 - (b) $1 - 1/2! + 1/3! - \dots + (-1)^{k+1} \cdot 1/k! + \dots$

4. (10 points)

Evaluate the integrals.

 - (a) $\int_{\frac{1}{2}}^{\frac{1}{3}} \frac{1}{2+x^2} dx$
 - (b) $\int_2^{10} x e^x dx$

5. (10 points)

Evaluate the limits.

 - (a) $\lim_{x \rightarrow 0^+} x^{2x}$
 - (b) $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x}$

6. (15%) Let $H = X(X^T X)^{-1} X^T$, where X is an $n \times p$ matrix of rank p and $X^T X$ is nonsingular. X^T is the transpose of X .
 - (1) Show that $I - H$ is a symmetric matrix, where I is the unit matrix of rank n . (4%)
 - (2) Show that H and $I - H$ are orthogonal. (4%)
 - (3) Also find that the traces of H and $I - H$. (7%)

7. (25%) Let $\Sigma = \begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix}$.
 - (1) Please verify whether Σ is a positive definite matrix. (4%)
 - (2) What are the eigenvalues of this matrix and its rank? (6%)
 - (3) What are the corresponding normalized eigenvalues of this matrix. Please also show that these eigenvectors are orthogonal. (7%)
 - (4) Produce a matrix that diagonalizes Σ based on the above results, and also compute Σ^{30} . (8%)

8. (10%) Please prove that $\text{rank}(AB) \leq \text{rank}(B)$ if A and B are any matrices and AB is defined.