

考試科目	數理統計學	所別	統計所	考試時間	4月21日 星期六 下午第2節
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1. (12%) Let X and Y are bivariate normally distributed with density function

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{ \frac{1}{1-\rho^2} \left[\left(\frac{X-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{X-\mu_x}{\sigma_x}\right)\left(\frac{Y-\mu_y}{\sigma_y}\right) + \left(\frac{Y-\mu_y}{\sigma_y}\right)^2 \right] \right\}$$

Derive the conditional density function of $f(X|Y)$, $E(X|Y=y)$, and $Var(X|Y=y)$.

2. (20%) Let X_1, X_2, \dots, X_n be a set of random sample from $Uniform(\alpha, \beta)$. Also, let $Y_1 = \min\{X_1, X_2, \dots, X_n\}$, $Y_n = \max\{X_1, X_2, \dots, X_n\}$, and $R = Y_n - Y_1$.
- (a) Compute the values of $E(Y_n)$ and $Var(Y_1)$.
- (b) Find the density function of R .
- (c) Find the MLE and the moment estimators of (α, β) .

3. (16%) Let X be a random variable having the geometric distribution, i.e. $f(x) = \theta(1-\theta)^x$, $x = 0, 1, \dots$ with $0 < \theta < 1$. Let $U(X)$ be defined as follows:

$$U(X) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases}$$

Show that $U(X)$ is a UMVU estimator of θ , and also state (by your own words) why $U(X)$ is not a reasonable estimator to use. Find a better UMVUE than $U(X)$.

4. (12%) Suppose X_1, X_2, \dots, X_n are a set of random sample from $B(p)$. Then use the moment generating function to show that $\frac{\sum_{i=1}^n X_i - np}{\sqrt{np(1-p)}} \xrightarrow{n \rightarrow \infty} N(0, 1)$.

Note that you cannot use the Central Limit Theorem to show the result.

5. (15%) Let X_1, X_2, \dots, X_n be independent random variables with density p.d.f.

$$f(x) = \frac{\lambda^x}{x!} e^{-\lambda}, \quad x \in \{0, 1, 2, \dots\} \quad \text{and} \quad \theta > 0.$$

- (a) Define the uniformly most powerful critical region of size α .
- (b) Using the statistic $\sum_{i=1}^n X_i$ to construct a uniformly most powerful critical region for testing $H_0: \lambda = \lambda_0$ against $H_1: \lambda > \lambda_0$. Let $\lambda_0 = 4$, $n = 25$, then find a uniformly most powerful critical region of size $\alpha = 0.05$ (using normal approximation).

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

(簽 章) 91 年 3 月 29 日

考試科目	數理統計學	所別	統計所	考試時間	4月21日 星期日	下午第 2 節
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6. (13%) Let X follow a binomial distribution, i.e. $B(n, p)$.
- (a) Show that the likelihood ratio statistic for testing $H_0: p=0.5$ against $H_1: p \neq 0.5$ is equivalent to $|2X - n|$.
- (b) Using the normal approximation with continuity correction, find the critical value corresponding to a level 0.05 test, if $n = 100$.
7. (12%) Taiwan Lottery (電腦彩券) is to randomly pick up 6 balls from a box of 42 balls, numbered 1, 2, ..., 42.
- (a) What kind of distribution is it for picking up 6 out of 42 balls?
- (b) If the lottery is a fair game, then the numbers picked up must be random. Define the "randomness" in this case and state how to check if the numbers picked up are random.

Note: For standard normal distribution,

$$P(Z \leq 1.284) = 0.9, \quad P(Z \leq 1.645) = 0.95, \quad P(Z \leq 1.96) = 0.975, \quad P(Z \leq 2.326) = 0.99.$$

備 考 試 題 隨 卷 繳 交

命 題 委 員:

(簽章) 91年3月29日

考試科目	統計學	所別	統計系	考試時間	4月2日 上午第 / 節 星期 日 不
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國立政治大學圖書館

- Each customer of store 1 is asked how much money he or she spent on merchandise. Let (μ_1, σ_1) be the population mean and standard deviation of spent money for store 1. A survey of 100 customers from store 1 is taken, and asked how much money they spent. The results are as follows. Sample mean $(\bar{X}_1) = \$15$, sample standard deviation $(S_1) = \$3$. (unit: thousand)
 - What is the distribution of the sample mean? (4%)
 - Estimate the 95% confidence interval for σ_1 . (5%)
 - Suppose that the sample mean \bar{X}_1 is unknown. What is the confidence level (probability) to control the sample mean \bar{X}_1 to be within \$0.5 of the population mean μ_1 ? (5%)
 - Suppose that the sample size n is unknown. How large a sample is required if \bar{X}_1 is to be within \$0.5 of the population mean μ_1 with 95% confidence. (5%)
 - Test $H_0: \mu_1 = 18$ vs $H_1: \mu_1 < 18$, $\alpha = 0.05$. What is the conclusion? (5%)
 - What is the P value in problem (5), and give conclusion. (5%)
 - For problem (5), what is the critical value (\bar{X}_L) on the rejection region. (5%)
 - Under problem (5) and (7), calculate type II error probability and power. (in fact, $\mu_1 = 16$). (6%+3%)
 - Draw the OCC for problem (5). (5%)
- A supervising inspector of incoming quality again wants to know whether there is any difference among the average lifetimes under the four temperatures. The following data are obtained.

Temp. (°C)	lifetime (unit: hour)						Total
15	18	15	25	18	22	25	123
20	10	15	18	20	18	20	101
25	10	5	1	3	5	6	30
30	10	6	15	12	5	6	54
							308

Note: $\sum \sum y_{ij}^2 = (18^2 + 15^2 + \dots + 6^2) = 5123.167$

- If the data is collected by complete randomized design (CRD) then describes the procedure of the experimental design. (5%)

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考試科目	統計學	所別	統計學	考試時間	4月21日(土) 星期日 下午第一節
<p>(2) Do ANOVA table and test if X (temp.) is a significant factor for lifetime? $\alpha = 0.05$ (10%)</p> <p>(3) What are the assumptions for the ANOVA? (4%)</p> <p>(4) Estimate the 95% confidence interval for population variance σ^2, under the assumptions in (3). (4%)</p> <p>(5) Estimate the 95% confidence interval for average lifetime (μ_1) when temp. is $15^\circ C$. (4%)</p> <p>(6) Estimate the 95% confidence interval for ($\mu_1 - \mu_2$). (μ_2 is average lifetime when temp. is $20^\circ C$). (5%)</p> <p>(7) Write down a regression model (or form), which express the relationship between lifetime and temperature, based on the result in problem (2). (5%)</p> <p>(8) If the 24 experiments are run in 6 days. Each temperature is assigned to each day, the order of assignment is in a random manner. Identify the design used for the experiment. (5%)</p>					
3. Multiple choices (1% x 10)					
<p>(1) When sample size is large, t distribution would be approximated to A. X^2 distribution B. non-standard normal distribution C. standard normal distribution D. n large means $n < 30$.</p> <p>(2) X^2 statistic is A. very sensitive to normality assumption B. Very robust (insensitive to normality assumption) C. Far way from standardized normal distribution when n is large. D. with degree of freedom n.</p> <p>(3) To have a smaller type I and type II error probabilities is to A. reduce sample size B. increase sample size C. all of above D. none of above</p> <p>(4) We take a sample of size 50 from an exponential distribution. The distribution of a sample mean is a A. uniform distribution B. Binomial distribution C. Poisson distribution D. Normal distribution</p> <p>(5) Contingency tables should not be used with expected cell frequencies A. Less than the number of rows. B. less than the number of columns. C. less than 5 D. Less than 30.</p> <p>(6) A chi-square goodness of fit test is to be performed to see if data fit the Poisson distribution. There are 10 categories, and lambda must be estimated. How many degrees of freedom should be used?</p>					

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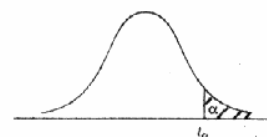
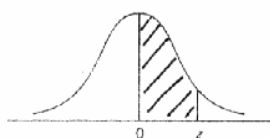
A. 8 B. 7 C. 6 D. 5

(7) A chi-square goodness of fit test is
A. two-tailed test B. right-tail test C. left-tail test D. none of above

(8) The sum of all residuals, in regression analysis, is
A. 1 B. 2 C. 0 D. 0.5

(9) If the slope of a simple linear regression model is positive, then the correlation coefficient is
A. negative B. positive C. zero D. do not know

(10) The relationship among errors, in the regression model, is
A. dependent B. not correlated C. linear D. nonlinear



NORMAL CURVE AREAS

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3486	.3508	.3531	.3554	.3577	.3600	.3623
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

CRITICAL VALUES OF t

DEGREES OF FREEDOM	t _{.100}	t _{.050}	t _{.025}	t _{.010}	t _{.005}
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.177	2.675	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.705
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.369	2.632
100	1.290	1.660	1.984	2.364	2.626
120	1.289	1.658	1.980	2.358	2.617
140	1.288	1.656	1.977	2.353	2.611
160	1.287	1.654	1.975	2.350	2.607
180	1.286	1.653	1.973	2.347	2.603
200	1.286	1.653	1.972	2.345	2.601
∞	1.282	1.645	1.960	2.326	2.576

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

(簽 章)

年

月

日

考試科目	統計學	所別	統計學	考試時間	4月21日(上) 星期日 下午第一節
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國立政治大學圖書館

DEGREES OF FREEDOM	$\chi^2_{.99}$	$\chi^2_{.90}$	$\chi^2_{.80}$	$\chi^2_{.70}$	$\chi^2_{.60}$	$\chi^2_{.50}$	$\chi^2_{.40}$	$\chi^2_{.30}$	$\chi^2_{.20}$	$\chi^2_{.10}$	$\chi^2_{.05}$
1	0.0000393	0.0001571	0.0009821	0.0039321	0.0157908	2.70554	3.84146	5.02389	6.63490	7.87944	
2	0.0100251	0.0201007	0.0506356	0.102587	0.210720	4.60517	5.99147	7.37776	9.21034	10.5966	
3	0.0717212	0.114832	0.215795	0.351846	0.584375	6.25139	7.81473	9.34840	11.3449	12.8381	
4	0.206990	0.297110	0.484419	0.710721	1.063623	7.77944	9.48773	11.1433	13.2767	14.8602	
5	0.411740	0.554300	0.831211	1.145476	1.61031	9.23635	11.0705	12.8325	15.0863	16.7496	
6	0.675727	0.872085	1.237347	1.63539	2.20413	10.6446	12.5916	14.4494	16.8119	18.5476	
7	0.989265	1.239043	1.68987	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777	
8	1.34419	1.646482	2.17973	2.73264	3.48954	13.3616	15.5073	17.5346	20.0902	21.9550	
9	1.734926	2.087912	2.70039	3.32511	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893	
10	2.15385	2.55821	3.24697	3.94030	4.86518	15.9871	18.3070	20.4831	23.2093	25.1882	
11	2.60321	3.05347	3.81575	4.57481	5.57779	17.2750	19.6751	21.9200	24.7250	26.7569	
12	3.07382	3.57056	4.40379	5.22603	6.30380	18.5494	21.0261	23.3367	26.2170	28.2995	
13	3.56503	4.10691	5.00874	5.89186	7.04150	19.8119	22.3621	24.7356	27.6883	29.8194	
14	4.07468	4.66043	5.62872	6.57063	7.78953	21.0642	23.6848	26.1190	29.1413	31.3193	
15	4.60094	5.22935	6.26214	7.26094	8.54675	22.3072	24.9958	27.4884	30.5779	32.8013	
16	5.14224	5.81221	6.90766	7.96164	9.31223	23.5418	26.2962	28.8454	31.9999	34.2672	
17	5.69724	6.40776	7.56418	8.67176	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185	
18	6.26481	7.01491	8.23075	9.39046	10.8649	25.9894	28.8693	31.5264	34.8053	37.1564	
19	6.84398	7.63273	8.90655	10.1170	11.6509	27.2036	30.1435	32.8523	36.1908	38.5822	
20	7.43386	8.26040	9.59083	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968	
21	8.03366	8.89720	10.28293	11.5913	13.2396	29.6151	32.6705	35.4789	38.9321	41.4010	
22	8.64272	9.54249	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7956	
23	9.26042	10.19567	11.6885	13.0905	14.8479	32.0069	35.1725	38.0757	41.6384	44.1813	
24	9.88623	10.8564	12.4011	13.8484	15.6587	33.1963	36.4151	39.3641	42.9798	45.5585	
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9278	
26	11.1603	12.1981	13.8439	15.3791	17.2919	35.5631	38.8852	41.9232	45.6417	48.2899	
27	11.8076	12.8786	14.5733	16.1513	18.1138	36.7412	40.1133	43.1944	46.9630	49.6449	
28	12.4613	13.5648	15.3079	16.9279	18.9392	37.9159	41.3372	44.4607	48.2782	50.9933	
29	13.1211	14.2565	16.0471	17.7083	19.7677	39.0875	42.5569	45.7222	49.5879	52.3356	
30	13.7867	14.9535	16.7908	18.4926	20.5992	40.2560	43.7729	46.9792	50.8922	53.6720	
40	20.7065	22.1643	24.4331	26.5093	29.0505	51.8050	55.7585	59.3417	63.6907	66.7659	
50	27.9907	29.7067	32.3574	34.7642	37.6886	63.1671	67.5048	71.4202	76.1539	79.4900	
60	35.5346	37.4848	40.4817	43.1879	46.4589	74.3970	79.0819	83.2976	88.3794	91.9517	
70	43.2752	45.4418	48.7576	51.7393	55.3290	85.5271	90.5312	95.0231	100.425	104.215	
80	51.1720	53.5400	57.1532	60.3915	64.2778	96.5782	101.879	106.629	112.329	116.321	
90	59.1963	61.7541	65.6466	69.1260	73.2912	107.565	113.145	118.136	124.116	128.299	
100	67.3276	70.0648	74.2219	77.9295	82.3581	118.498	124.342	129.561	135.807	140.169	

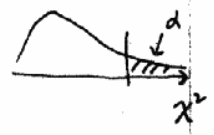


Table A.7 Critical Values $F_{\alpha; v_1, v_2}$ for the F Distribution

v_2	$\alpha = .05$																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
50	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

