答案請依題目順序依序填寫(由上而下)於答案簿上 未依要求填寫者不以計分

1. The quality engineer suspects that the average weights of all products would be smaller than 220mg. The weights of all products are normal distribution. She takes a sample of size 16 from the products and finds the sample mean is 190mg, standard deviation is 12mg. (5%x4)

1-1. Estimate the 99% confidence interval for average weight of all products.

1-2. Test quality engineer's suspicion. α =0.05.

1-3. If the quality engineer claims that the average weight of all products would be smaller than 200mg. How do clients test her claims.

1-4. Estimate the 95% confidence interval for weight standard deviation of all products.

2. The quality engineer suspects that the average weights of all products would be smaller than 220mg. She takes a sample of size 36 from the products and finds the sample mean is 190mg, standard deviation is 20mg.

2-1. To have 95% confidence to control the sampling error within 5 mg, how large the sample size should be taken. (If the sample size is unknown) (5%)

2-2. Test quality engineer's suspicion using P value. α =0.05. (7%)

2-3. In fact, the average weight of all products is 200mg. Calculate type II error probability. (7%)

2-4. Draw the OCC for (2). (7%)

3. Manufacturing engineer proposes that factors A and B may be the causes of product lifetime variation. Experimental data is collected as follows, and 12 experiments are performed randomly.

A	B	observations	sum of observations
-	-	460, 440, 420	1320
-	+	110, 105, 108	323
+	-	320, 350, 380	1050
+	+	938, 1002, 1060	3000

Total sum=5693 Total sum of square(SST)=1292283

3-1. What is the experimental design? (for example RBD, factorial design, confounding design, or…) (3%)

3-2. Estimate the effects of A, B, and interaction AxB. (6%)

3-3. Give an ANOVA table and test the significance of A, B, and AxB, α =0.05.. (10%)

3-4. What are the data assumptions for (3). (6%)

3-5.Write down the fitted regression model based on the test results of ANOVA table. (8%)

4. Multiple choice: (the answer is (1), (2), (3), or.....) (3%x7)

4-1. To investigate which factors are important for effect. Which tool can be used?

a. Pareto diagram b. ANOVA c. t distribution d. Regression analysis

(1) b+d (2) a+b (3) b+d (4) a+c (5) a+b+d (6) none of above

4-2. A single replicate of 2^2 factorial design has 4 treatments. Since the time limitation, the 4 treatments should be performed by two operators. Which experimental design is appropriate?

(1) RBD (2) factorial design (3) fractional factorial design (4) confounding design

4-3. To test if two independent population means are same, which statistic can be used?

a. Mann-Whitney statistic b. paired t statistic c. Z statistic d. Wilcoxon signed rank

(1) a (2) b (3) c (4) a+c (5) c+d (6) b+c (7) none of above

4-4. To test if a coin is fair, which statistic can be used?

a. Z statistic b. x^2 statistic c. t statistic d. F statistic

(1) a (2) b (3) c (4) d (5) a+b (6) a+c 7. b+d

4-5. To investigate the effect of a new drug, which statistic can be used?

a. Z statistic b. paired t statistic c. F statistic in ANOVA(CRD) d. F statistic in ANOVA(RBD)

(1) a (2) b (3) c (4) d (5) a+d (6) b+d (7) a+c (8) b+c

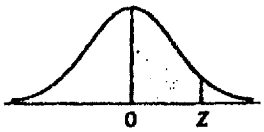
4-6. About the F statistic, which one is incorrect?

(1) used in ANOVA (2) inference for $(\frac{\sigma_2}{\sigma_1})^2$ (3) $t^2(v) = F(v,1)$ (4) $F_{1-\alpha}(v1,v2) = \frac{1}{F_\alpha(v2,v1)}$

4-7. A uniform distribution with pdf $f(x) = \frac{1}{b-a}, a < x < b$. The expectation E(X)=?

(1) (b-a)/2 (2) 1/2 (3) $\frac{b^2-a^2}{12}$ (4) (a+b)/2

TABLE A.5 AREAS OF THE STANDARD NORMAL DISTRIBUTION



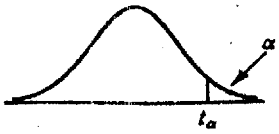
The entries in this table are the probabilities that a standard normal random variable is between 0 and Z (the shaded area).

	SECOND DECIMAL PLACE IN Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998									
4.0	.49997									
4.5	.499997									
5.0	.4999997									

考試科目	統計學	所別	統計系	考試時間	4月18日(上下)午第一節 星期日

TABLE A.6 CRITICAL VALUES OF t

CRITICAL VAL



DEGREES OF FREEDOM	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.808
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

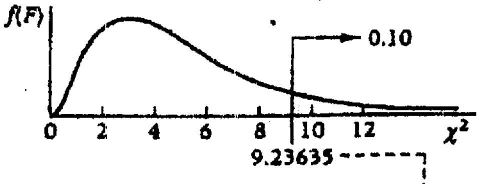
From M. Merrington, "Table of Percentage Points of the t-Distribution," *Biometrika*, 1941, 32, 300. Reproduced by permission of the *Biometrika* trustees.

備考	試題隨卷繳交

試科目	統計學	所別	統計系	考試時間	4月18日 星期日 上午第一節

TABLE A.8 THE CHI-SQUARE TABLE

VALUES OF χ^2 FOR SELECTED PROBABILITIES



Example: df (Number of degrees of freedom) = 5, the tail above χ^2 = 9.23635 represents 0.10 or 10% of the area under the curve.

AREA IN UPPER TAIL

Degrees of Freedom	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	$392,704 \times 10^{-10}$	$157,088 \times 10^{-9}$	$982,069 \times 10^{-9}$	$393,214 \times 10^{-8}$	.0157908	2.70554	3.84146	5.02389	6.63490	7.87944
2	.0100251	.0201007	.0506356	.102587	.210720	4.60517	5.99147	7.37776	9.21034	10.5966
3	.0717212	.114832	.215795	.351846	.584375	6.25139	7.81473	9.34840	11.3449	12.8381
4	.206990	.297110	.484419	.710721	1.063623	7.77944	9.48773	11.1433	13.2767	14.8602
5	.411740	.554300	.831211	1.145476	1.61031	9.23635	11.0705	12.8325	15.0863	16.7496
6	.675727	.872085	1.237347	1.63539	2.20413	10.6446	12.5916	14.4494	16.8119	18.5476
7	.989265	1.239043	1.68987	2.16735	2.83311	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.344419	1.646482	2.17973	2.73264	3.48954	13.3616	15.5073	17.5346	20.0902	21.9550
9	1.734926	2.087912	2.70039	3.32511	4.16816	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55821	3.24697	3.94030	4.86518	15.9871	18.3070	20.4831	23.2093	25.1882
11	2.60321	3.05347	3.81575	4.57481	5.57779	17.2750	19.6751	21.9200	24.7250	26.7569
12	3.07382	3.57056	4.40379	5.22603	6.30380	18.5494	21.0261	23.3367	26.2170	28.2995
13	3.56503	4.10691	5.00874	5.89186	7.04150	19.8119	22.3621	24.7356	27.6883	29.8194
14	4.07468	4.66043	5.62872	6.57063	7.78953	21.0642	23.6848	26.1190	29.1413	31.3193
15	4.60094	5.22935	6.26214	7.26094	8.54675	22.3072	24.9958	27.4884	30.5779	32.8013
16	5.14224	5.81221	6.90766	7.96164	9.31223	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.69724	6.40776	7.56418	8.67176	10.0852	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.26481	7.01491	8.23075	9.39046	10.8649	25.9894	28.8693	31.5264	34.8053	37.1564
19	6.84398	7.63273	8.90655	10.1170	11.6509	27.2036	30.1435	32.8523	36.1908	38.5822
20	7.43386	8.26040	9.59083	10.8508	12.4426	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.03366	8.89720	10.28293	11.5913	13.2396	29.6151	32.6705	35.4789	38.9321	41.4010
22	8.64272	9.54249	10.9823	12.3380	14.0415	30.8133	33.9244	36.7807	40.2894	42.7958
23	9.26042	10.19567	11.6885	13.0905	14.8479	32.0069	35.1725	38.0757	41.6384	44.1813
24	9.88623	10.8564	12.4011	13.8484	15.6587	33.1963	36.4151	39.3641	42.9798	45.5585
25	10.5197	11.5240	13.1197	14.6114	16.4734	34.3816	37.6525	40.6465	44.3141	46.9278
26	11.1603	12.1981	13.8439	15.3791	17.2919	35.5631	38.8852	41.9232	45.6417	48.2899
27	11.8076	12.8786	14.5733	16.1513	18.1138	36.7412	40.1133	43.1944	46.9630	49.6449
28	12.4613	13.5648	15.3079	16.9279	18.9392	37.9159	41.3372	44.4607	48.2782	50.9933
29	13.1211	14.2565	16.0471	17.7083	19.7677	39.0875	42.5569	45.7222	49.5879	52.3356
30	13.7867	14.9535	16.7908	18.4926	20.5992	40.2560	43.7729	46.9792	50.8922	53.6720
40	20.7065	22.1643	24.4331	26.5093	29.0505	51.8050	55.7585	59.3417	63.6907	66.7659
50	27.9907	29.7067	32.3574	34.7642	37.6886	63.1671	67.5048	71.4202	76.1539	79.4900
60	35.5346	37.4848	40.4817	43.1879	46.4589	74.3970	79.0819	83.2976	88.3794	91.9517
70	43.2752	45.4418	48.7576	51.7393	55.3290	85.5271	90.5312	95.0231	100.425	104.215
80	51.1720	53.5400	57.1532	60.3915	64.2778	96.5782	101.879	106.629	112.329	116.321
90	59.1963	61.7541	65.6466	69.1260	73.2912	107.567	113.145	118.136	124.116	128.299
100	67.3276	70.0648	74.2219	77.9295	82.3581	118.498	124.342	129.561	135.807	140.169

1. Suppose the random sample $Y_1, Y_2, \ldots, Y_n$ comes from a distribution with probability density function (p.d.f.)

$$f(y) = \begin{cases} \frac{1}{\theta_2} e^{-(y-\theta_1)/\theta_2} & y > \theta_1 \\ 0 & \text{otherwise}, \end{cases}$$

where $\theta_2 > 0$.

(a) Find the method of moments estimators for θ_1 and θ_2. (7%)

(b) Find the maximum likelihood estimators for θ_1 and θ_2. (8%)

(c) Suppose a random sample of size 4 yields

1 4 2 5

Calculate both the method of moments and the maximum likelihood point estimates of θ_1 and θ_2. (4%)

2. Suppose the random variables $Y_1, Y_2, \cdots, Y_n$ are such that $\sum_{i=1}^{n} \ln Y_i$ is distributed as normal with mean 0 and variance $n\beta^2$. Find a $(1-\alpha)100\%$ confidence interval for β. (15%)

3. Let $Y_1, Y_2 \sim^{i.i.d.} N(0,1)$. Find the distribution of the following random variables:

(a) $\frac{(Y_1 - Y_2)}{\sqrt{2}}$ (5%)

(b) $(Y_1 + Y_2)^2 / (Y_1 - Y_2)^2$ (5%)

(c) $(Y_1 + Y_2) / \sqrt{(Y_1 - Y_2)^2}$ (5%)

(d) Y_1^2 / Y_2^2 (5%)

4. For each of the n independent individuals, there are two measures of interest: x_i which is some known value, and Y_i which is a random variable distributed as $Normal(\alpha + \beta x_i, \sigma^2)$, for known σ^2. In other words,

$Y_i \sim Normal(\alpha + \beta x_i, \sigma^2)$ $i = 1, \ldots, n$

$Y_1, \ldots, Y_n$ mutually independent

$x_1, \ldots, x_n$ known

σ^2 known

We want test $H_0 : \alpha = 2, \beta = 0$ versus H_1 : not H_0.

(a) Outline the steps for deriving a likelihood ratio test (LRT). (5%)

(b) Find the maximum likelihood estimators for α and β. (8%)

(c) Actually find the LRT, and show that the rejection region (RR) is the form

$$\left\{\sum_{i=1}^{n}(Y_i-2)^2-\sum(Y_i-\hat{\alpha}_{mle}-\hat{\beta}_{mle}x_i)^2>k\right\}$$, where k is a constant which

is specified ,

$\hat{\alpha}_{mle}$: the maximum likelihood estimator of α

$\hat{\beta}_{mle}$: maximum likelihood estimator of β (13%)

5. Let $Y_1, Y_2, \ldots, Y_n \sim^{i.i.d.} Poisson(\lambda)$. Define $\theta=(1+\lambda)e^{-\lambda}$.

   (a) Find a sufficient statistic for θ. (6%)

   (b) Find an unbiased estimator of θ. (6%)

   (c) Find the UMVUE (Uniformly Minimum Variance Unbiased Estimator) of θ. (8%)

備考 試題隨卷繳交