

考試科目	基礎數學	所別	統計 4(4)	考試時間	3月16日 星期日 第一節
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1. Show that $1 - \frac{x^2}{2} < \cos x$ for all $x \in (0, \infty)$ 5 pts

2. Find $H'(2)$ given that $H(x) = \int_{2x}^{x^2-4} \frac{t}{1+\sqrt{t}} dt$ 5 pts

3. Determine whether the following improper integral converges or diverges, and find its value if it converges. 10 pts

$$\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}} dx$$

4. Find the following limits

(a) $\lim_{x \rightarrow 0} \frac{x - \sin^{-1} x}{\sin^3 x}$

(b) $\lim_{x \rightarrow \infty} (e^x + 1)^{\frac{1}{x}}$ 10 pts

5. Determine whether the following series converges or diverges

(a) $\sum \frac{(k!)^2}{(2k)!}$

(b) $\sum \frac{2^k k!}{k^k}$ 10 pts

6. $g(x, y) = \begin{cases} \frac{x^2 y^2}{x^4 + y^4} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$

(a) Show that $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$ both exist at $(0, 0)$ and their values at $(0, 0)$

(b) Show that $\lim_{(x, y) \rightarrow (0, 0)} g(x, y)$ does not exist. 15 pts

備 考 試 題 隨 卷 繳 交

命 題 委 員 :

(簽章) 97年 3月 7日

考試科目	基礎數學	所別	統計 A14	考試時間	3月16日 星期日	第一節
<p>7. For any idempotent and symmetric matrix A, prove that</p> <p>(a) A is non-negative definite 10pts</p> <p>(b) $\text{Trace } A = \text{rank } A$</p> <p>8. Let $L: V \rightarrow W$ be a linear transformation and $\dim V = \dim W$. Show that L is one-to-one iff L is onto. 10pts</p> <p>9. Let I_n be the identity matrix and J_n be the matrix of 1's, both of order n. Show that $aI_n + bJ_n = a^{n-1}(a+nb)$ 5pts</p> <p>10. Find an LU-factorization of the matrix $\begin{bmatrix} 4 & 2 & 3 \\ 2 & 0 & 5 \\ 1 & 2 & 1 \end{bmatrix}$. 10pts</p> <p>11. Let A be a symmetric matrix, prove that eigenvectors that belong to distinct eigenvalues of A are orthogonal. 10pts</p>						
備考	試題隨卷繳交					
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考試科目	數理統計	所別	統計 4141	考試時間	3月16日 星期日	第三節
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1. Let $Y_1 < Y_2 < Y_3 < \dots < Y_n$ be the order statistics of a random sample of size n from exponential distribution with p.d.f. $f(x) = e^{-x}$, $0 < x < \infty$, zero else where.

(a) Show that $Z_1 = nY_1$, $Z_2 = (n-1)(Y_2 - Y_1)$, ..., $Z_n = Y_n - Y_{n-1}$ are independent and each Z_i has the exponential distribution. (8%)

(b) Demonstrate that all linear function of Y_1, Y_2, \dots, Y_n , such that $\sum_{i=1}^n a_i Y_i$ can be expressed as linear function of independent random variables. (7%)

2. If we have a random sample X_1, X_2, \dots, X_n from a p.d.f.

$$f(x) = \frac{\theta^\kappa}{\Gamma(\kappa)} \exp(\kappa x - \theta e^x) \text{ for } -\infty < x < \infty.$$

(a) If $\kappa = 1$, find a complete and sufficient statistic, V , for θ . (3%)

(b) If $\kappa = 1$. Let $W_i = e^{X_i}$ for $i = 1, 2, \dots, n$, $\bar{W} = \frac{\sum_{i=1}^n W_i}{n}$ and $U = \prod_{i=1}^n \frac{W_i}{\bar{W}}$.

Show that U is independent of V . (5%)

(c) Suppose that Q is a test for testing $H_0: \kappa \leq 1$ versus $H_1: \kappa > 1$. What is an Unbiased test? (5%)

(d) If Q is an unbiased test, show that $E(Q|V) = \alpha$ almost everywhere when $\kappa = 1$. (5%)

(e) State how to construct a UMP unbiased test for $H_0: \kappa \leq 1$ versus $H_1: \kappa > 1$. (5%)

3. Let X_1, X_2, \dots, X_n be a random sample from $EXP(\theta, \eta)$, and let $Z_i = (X_i - \eta)/\theta$. Find the BLUEs (Best Linear Unbiased Estimator) for θ and η . (12%)

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考 試 科 目	數理統計學	所 別	統計 4141	考 試 時 間	3 月 16 日 星 期 日	第 三 節
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4. Suppose the joint pdf of X and Y is $f(x, y) = \begin{cases} e^{-y}, & y > x > 0 \\ 0, & \text{otherwise} \end{cases}$.

- a) Find the marginal pdf of X . (3%)
 b) Find $E(Y|x)$. (5%)
 c) Find the correlation coefficient of X and Y . (8%)

5. Consider a random sample of size n form a population with pdf: $f(x) = (1-p)^{x-1} p$, $0 < p < 1$,
 $x = 1, 2, 3, \dots$

- a) Show that $1/\bar{X}_n$ converges in probability to $P(X=1)$. (5%)
 b) Show that $(\bar{X}_n - 1)/\bar{X}_n^2$ converges in probability to $P(X=2)$. (5%)
 c) Find the asymptotic distribution of $(\bar{X}_n - 1)/\bar{X}_n^2$. (10%)

6. Let X be the lifetime of a machine. Suppose that the pdf of X is

$$f(x) = \frac{x^{\kappa-1}}{\Gamma(\kappa)\beta^\kappa} e^{-x/\beta}, \quad x > 0 \text{ where } \beta, \kappa > 0.$$

- a) Find the mode of X . (6%)
 b) Let $h(x) = a \exp(-cx)$ be the cost function of operating this machine to the lifetime $X = x$, where $a > 0$. In order that the expected cost of operation of this machine exists, what is the range for the constant c ? What is the expected cost? (8%)

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考試科目	統計方法	所別	統計系 ⁴¹⁴	考試時間	3月16日 星期日	第 四 節
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1. Below is information on the traffic flow (1000's of cars per 24 hours) and lead content of bark on trees near the highway ($\mu\text{g/g}$ dry wt) for a sample of 11 days.

Day	Traffic Flow, x	Lead Content, y	Day	Traffic Flow, x	Lead Content, y
1	8.3	227	7	17.0	728
2	8.3	312	8	24.3	945
3	12.1	362	9	24.3	738
4	12.1	521	10	24.3	759
5	17.0	640	11	33.6	1263
6	17.0	539			

$$\begin{aligned} \sum x_i &= 198.3 \\ \sum x_i^2 &= 4,198.03 \\ \sum y_i &= 7,034 \\ \sum y_i^2 &= 5,390,382 \\ \sum x_i y_i &= 149,354.4 \end{aligned}$$

- Develop the regression equation for lead content based on traffic flow. Comment on the slope value. (10 pts)
 - Determine the coefficient of determination. Interpret its value. (10 pts)
 - Determine the 95% prediction interval for the lead content of a particular bark when the traffic flow is 25,000 cars a day. (10 pts)
2. A study of the relationship between facility conditions at coffee shops and aggressiveness in the pricing of coffee reported the accompanying data based on a sample of 441 shops. At the 0.01 significance level can we conclude that the facility conditions and pricing policy are related? (10 pts)

Condition	Substandard	Pricing Policy		
		Aggressive	Neutral	Nonaggressive
Standard	24	15	17	
Modern	52	73	80	
	58	86	36	

3. The objective of a community research project is to determine whether married men are more community conscious before fatherhood or after five years of fatherhood. A test designed to measure community consciousness was administered to a sample of men before fatherhood, and the same test was given to them five years after fatherhood. The test scores are given in the followings.

Name	Before		Name	After	
	Fatherhood	Fatherhood		Fatherhood	Fatherhood
Tim	110	114	Jeff	186	196
Andy	157	159	Doug	116	116
Bob	121	120	Alan	160	140
Mike	96	103	Ed	149	142
John	130	139			

Test at the 0.05 level. H_0 : There is no difference in community consciousness before and after fatherhood.
 H_1 : There is a difference. (10 pts)

Note: We are not sure that the scores are normally distributed.

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考試科目	統計方法	所別	統計 4141	考試時間	3月16日 星期四 第四節
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4. (5 points) A particular type of electronic item is produced by only two companies, Alamo Electronic Company and South Jersey Products. Suppose Alamo produced 60% of the items and that 92% of the items produced by Alamo are non-defective. Suppose that the probability of defectiveness (of all items produced by the two companies) is 0.088. Given that a newly produced item is defective. What is the probability that it is produced by South Jersey Products?
5. (30 points) In a study on smoking habits in a large community, a scientist wishes to test the null hypothesis $H_0 : p = 0.3$ against the alternative hypothesis $H_1 : p > 0.3$, where p is the unknown proportion of smokers in the entire community. A simple random sample size 36 is drawn from the community and the scientist decides to reject H_0 if and only if the sample proportion of smokers is 0.5 or more. Let \hat{p} denote the sample proportion.
- (a) (5 points) Under the null hypothesis, what does the Central Limit Theorem say about the sampling distribution of \hat{p} ?
- (b) (5 points) What will be the approximate significance level?
- (c) (5 points) If the actual sample proportion of smokers equals to 0.37, then what will be the approximate p -value of the test?
- (d) (5 points) Construct the 95% confidence interval for p .
- (e) (5 points) What is the probability that the true p lies in the interval that you constructed in (d)?
- (f) (5 points) If the desired error of margin is 0.1, how many observations are needed in order to a 95% confidence interval for p ?
6. (15 points) A consumer organization wants to know whether there is a difference in the price of a particular type of cereal at three different types of stores. The price of the cereal was checked in a sample of five discount stores, five drugstores, and five department stores. The results are shown below.

Discount	12	13	14	12	15
Drugstore	15	17	14	18	17
Department	19	17	16	20	19

Set the significance level at 0.05.

- (a) (4 points) Write down H_0 and H_1 .
- (b) (4 points) Calculate the observed F -statistic.
- (c) (4 points) Find the critical value.
- (d) (3 points) Would you reject H_0 ?

備 考 試 題 隨 卷 繳 交

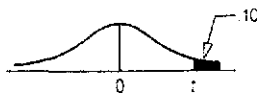
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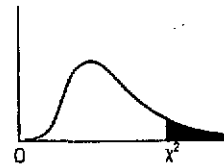
考試科目	統計方法	所別	統計系 4141	考試時間	3月16日 星期日	第 四 節
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Student's *t* Distribution



Example: With $df=9$ and .10 area in the upper tail, $t=1.383$

Critical Values of Chi-Square



Example: With 17 df and a .02 area in the upper tail, $\chi^2=30.995$

df	Confidence Intervals					
	80%	90%	95%	98%	99%	99.9%
	Level of Significance for One-Tailed Test					
	0.100	0.050	0.025	0.010	0.005	0.0005
df	Level of Significance for Two-Tailed Test					
	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646

Degrees of Freedom, df	Right-Tail Area			
	0.10	0.05	0.02	0.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.668	13.277
5	9.236	11.070	13.388	15.086
6	10.645	12.592	15.033	16.812
7	12.017	14.067	16.622	18.475
8	13.362	15.507	18.169	20.090
9	14.684	16.919	19.679	21.666
10	15.987	18.307	21.161	23.209
11	17.275	19.675	22.618	24.725
12	18.549	21.026	24.054	26.217
13	19.812	22.362	25.472	27.688
14	21.064	23.685	26.873	29.141
15	22.307	24.996	28.259	30.578
16	23.542	26.296	29.633	32.000
17	24.769	27.587	30.995	33.409
18	25.989	28.869	32.346	34.805
19	27.204	30.144	33.687	36.191
20	28.412	31.410	35.020	37.566
21	29.615	32.671	36.343	38.932
22	30.813	33.924	37.659	40.289
23	32.007	35.172	38.968	41.638
24	33.196	36.415	40.270	42.980
25	34.382	37.652	41.566	44.314
26	35.563	38.885	42.856	45.642
27	36.741	40.113	44.140	46.963
28	37.916	41.337	45.419	48.278
29	39.087	42.557	46.693	49.588
30	40.256	43.773	47.962	50.892

n	2α						
	.15	.10	.05	.04	.03	.02	.01
	α						
	.075	.050	.025	.020	.015	.010	.005
4	0						
5	1	0					
6	2	2	0	0			
7	4	3	2	1	0	0	
8	7	5	3	3	2	1	0
9	9	8	5	5	4	3	1
10	12	10	8	7	6	5	3
11	16	13	10	9	8	7	5
12	19	17	13	12	11	9	7
13	24	21	17	16	14	12	9
14	28	25	21	19	18	15	12
15	33	30	25	23	21	19	15
16	39	35	29	28	26	23	18
17	45	41	34	33	30	27	23
18	51	47	40	38	35	32	27
19	58	53	46	43	41	37	32
20	65	60	52	50	47	43	37
21	73	67	58	56	53	49	42
22	81	75	65	63	59	55	48
23	89	83	73	70	66	62	54
24	98	91	81	78	74	69	61
25	108	100	89	86	82	78	68
26	118	110	98	94	90	84	75
27	128	119	107	103	99	92	83
28	138	130	116	112	108	101	91
29	150	140	126	122	117	110	100
30	161	151	137	132	127	120	109
31	173	163	147	143	137	130	118
32	186	175	159	154	148	140	128
33	199	187	170	165	159	151	138

Wilcoxon *T* Values

備 考試題隨卷繳交

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