考試科目	基礎數學	所別	統計學系	考試時間	2 月 27 日(日) 第一節

1. (15%) Let A, B, and C be $n \times n$ matrices. Which of the following statements are correct? (multiple choices)

   (a) $AB = BA$.

   (b) If $AB = I_n$, then $BA = I_n$, where I_n is the identity matrix.

   (c) A is nonsingular if and only if $Ax = 0$ has only the trivial solution $x = 0$.

   (d) A is singular if and only rank$(A) < n$.

   (e) A is singular if and only zero is not an eigenvalue of A.

2. (9%) Consider mappings from R^2 to R^2 (i.e. from xy-plane to xy-plane). Find the matrix that

   (a) reflects vectors with respect to the line $x = y$.

   (b) represents the counterclockwise rotation by 90^o.

   (c) projects vectors onto the line $x = y$.

3. (6%) Let $A = \begin{pmatrix} 3 & 1 & 2 & -1 \\ 2 & 0 & 3 & -7 \\ 1 & 3 & 4 & -5 \\ 0 & -1 & 1 & -5 \end{pmatrix}$.

   (a) What is the dimension of the null space of A?

   (b) Determine the rank of A.

4. (20%) Let

$$A = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}.$$

   (a) (8%) Which of above matrices are positive definite?

   (b) (8%) Which of above matrices are orthogonally diagonalizable?

   (c) (4%) Orthogonally diagonalize the above matrix if it is orthogonally diagonalizable.

請注意：背面還有試題。

備註	試 題 隨 卷 繳 交

考試科目	基礎數學	所別	統計學系	考試時間	2月27日(日) 第一節

5. Define a real-valued function f by $f(\alpha)=\int_0^\infty x^{\alpha-1}e^{-x}dx$.

(a) Show that for any $\alpha>1$, $f(\alpha)=(\alpha-1)f(\alpha-1)$. **(6%)**

(b) Calculate $f(n)$ for any positive integer $n>1$. **(6%)**

(c) The Stirling's formula states that for $n\in N$, $\lim_{n\to\infty}\frac{n!}{n^{n+(1/2)}e^{-n}}=\sqrt{2\pi}$.

Consider a sequence $A_n=\binom{2n}{n}p^n(1-p)^n$, $0<p<1$.

Show that $A_n\approx\frac{[4p(1-p)]^n}{\sqrt{n\pi}}$ as $n\to\infty$. **(7%)**

(d) What is the condition of p such that $\sum_{n=1}^{\infty}A_n$ will converge ? **(7%)**

Note: $\binom{m}{n}$ denotes possible number of combinations that we choose n items from m items.

For some real value p, $0<p<1$, define the function f by

$$f(x)=\begin{cases}(1-p)^{x-1}p, & x=1,2,3,\ldots\\ 0 \quad , & \text{otherwise}\end{cases}$$

6. (a) Calculate $F(y)=\sum_{x=1}^{[y]}f(x)$, where $[y]$ denotes the largest integer $\le y$, and $y>0$. **(6%)**

(b) Which of the following are true ? (multiple choice)

(i) $F(y)$ is differentiable (ii) $F(y)$ is left-continuous

(iii) $F(y)$ is right-continuous (iv) $F(y)$ is a step function

(v) $F(y)$ is increasing (vi) $\lim_{y\to\infty}F(y)=1$ **(6%)**

7. A function $f:(a,b)\mapsto R$ is *convex* on (a, b) if

$$f(rx+(1-r)y)\le rf(x)+(1-r)f(y) \quad \text{for all } a<x<y<b \text{ and } 0\le\lambda\le1.$$

(a) Which of the following are convex on $(0,\infty)$? (multiple choice)

(i) $1/x$ (ii) $\log x$ (iii) $-\log x$ (iv) e^{-x} (v) $e^{-(x-1)^2/2}$ (vi) $\tan^{-1}x$ **(6%)**

(b) An alternative definition for a convex function is that for $\mu\in(a,b)$,

$$f'(\mu)(x-\mu)+f(\mu)\le f(x) \quad \text{for all } x\in(a,b)$$

Show that if $f'(x)$ exists and $f''(x)\ge0$ for all $x\in(a,b)$, then f is convex on (a, b). **(6%)**

備註	試題隨卷繳交

考試科目	數理統計學	所別	統計學系	考試時間	2 月 27 日(日) 第三節

1. (a) Suppose A and B are two disjoint events. Find the condition(s) such that A^c and B^c are also disjoint. (4pts)

   (b) Suppose $P(A_i)=1/(4+i)$ for $i=1,2,3$. What is the lower bound for $P(A_1^c \cap A_2^c \cap A_3^c)$? (5pts)

2. Let X_i's be independent Poisson random variables with mean μ_i, $i=1,\cdots,n$.

   (a) Find the distribution of $\Sigma_{i=1}^n X_i$. (5pts)

   (b) Derive the conditional distribution of $(X_1,\cdots,X_n)$ given $\Sigma_{i=1}^n X_i = k$. (5pts)

   (c) Find $Cov(X_j, X_m \mid \Sigma_{i=1}^n X_i = k)$. (8pts)

   (d) Let n=1 and focus on X_1 now. Suppose μ_1 is random with pdf as $\lambda \exp(-\lambda\mu_1)$, for $\mu_1>0$, and 0 otherwise. Find the unconditional mean and variance for X_1. (6pts)

3. Let $X_1,\cdots,X_n$ be a random sample from a continuous distribution with the pdf: $f(x)=2x$, for $0<x<1$, and zero otherwise.

   (a) Find the approximate distributions of $\overline{X}_n$ and $\exp(-\overline{X}_n)$, respectively. (8pts)

   (b) Let n =2, $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1+X_2)$. Derive the joint pdf of Y_1 and Y_2. Explain whether Y_1 and Y_2 being independent or not. (9pts)

備註	試題隨卷繳交

請注意：背面還有試題。

考試科目	數理統計學	所別	統計學系	考試時間	2 月 27 日(日) 第三節

4. Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of length-of-life measurements of components whose length of life follows an exponential distribution with mean θ, and

$$f(y) = \frac{1}{\theta} e^{-y/\theta}, \ 0 < y < \infty.$$

a. Find the MLE (maximum likelihood estimator) of $e^{-t/\theta}$, where $e^{-t/\theta} = P(Y > t)$, and t is a constant.. (5 pts)

b. Let

$$U = \begin{cases} 1, & Y_1 > t, \\ 0, & otherwise. \end{cases}$$

Show that U is an unbiased estimator of $e^{-t/\theta}$. (5 pts)

c. Show that $W = \sum_{i=1}^{n} Y_i$ is the minimal sufficient statistic for θ. (5 pts)

d. Show that the conditional p.d.f. for Y_1, given $W = w$, is

$$f(y_1 \mid w) = \begin{cases} \left(\frac{n-1}{w^{n-1}}\right)(w - y_1)^{n-2}, & 0 < y_1 < w, \\ 0, & otherwise. \end{cases}$$ (5 pts)

e. Find the MVUE (minimum variance unbiased estimator) of $e^{-t/\theta}$. (5 pts)

f. Find a uniformly most powerful critical region of size α for testing H_0: $\theta = \theta_0$ against H_1: $\theta > \theta_0$. (10 pts)

5. Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the following probability density function

$$f(y) = \frac{1}{\theta} e^{-(y-\tau)/\theta}, \ \tau < y < \infty.$$

Find the likelihood ratio test for testing H_0: $\tau = \tau_0$ against H_1: $\tau > \tau_0$, with θ unknown. (15 pts)

備註	試題隨卷繳交

考試科目	統計方法	所別	統計學系	考試時間	2 月 27 日(日) 第四節

1. True or False. **Explain** if your answer is false. (注意：僅回答「False」，但未加註文字說明者，將不予計分。)
   (1) (3%) A friend of yours plans to toss a fair coin 200 times. You watch the first 40 tosses, noticing that she got only 16 heads. But then you get bored and leave. Since the coin is fair, the number of heads you expect her to have when she has finished the 200 tosses is 100.
   (2) (3%) Some marathons allow two runners to "split" the marathon by each running a half marathon. Alice and Sharon plan to "split" a marathon. Alice's half marathon times average 92 minutes with a standard deviation of 4 minutes, and Sharon's half-marathon times average 96 minutes with a standard deviation of 2 minutes. Assume that the women's half marathon times are independent. The expected time for Alice and Sharon to complete a full marathon is 92 + 96 = 188 minutes with a standard deviation of 6 minutes.
   (3) (3%) Disjoint (mutually exclusive) sets are always independent.
   (4) (3%) Bias is usually reduced when sample size is larger.
   (5) (3%) The five-number summary of credit hours for 24 students in an introductory statistics class is:

Min	Q1	Median	Q3	Max
13.0	15.0	16.5	18.0	22.0

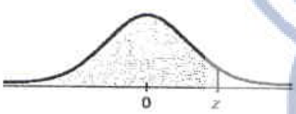
   There are no outliers in the data.
   (6) (3%) You should never analyze data with outliers.
   (7) (3%) Environmental researchers have collected data on rain acidity for years. Suppose that a Normal model describes the acidity (pH) of rainwater, and that water tested after last week's storm had a z-score of 1.8. This means that the acidity of that rain had a pH 1.8 higher than average rainfall.
   (8) (3%) Suppose a Normal model describes the number of pages printer ink cartridges last for. If you keep track of printed page counts for the 50 printers in your company's office, the histogram for those page counts will be symmetric.
   (9) (3%) It could be true that the correlation between height and weight is 0.568 inches per pound.
   (10) (3%) A correlation of zero between two quantitative variables means that there is no association between the two variables.

備註	試題隨卷繳交	請注意：背面還有試題。

考試科目	統計方法	所別	統計學系	考試時間	2月27日(日)第四節

2. Police estimate that 80% of drivers now wear their seatbelts. They set up a safety roadblock, stopping cars to check for seatbelt use.
   (1) (3%) How many cars do they expect to stop before finding a driver not wearing a seatbelt?
   (2) (3%) What is the probability that the first unbelted driver is in the 6^{th} car stopped?
   (3) (3%) What is the probability that the first 10 drivers are all wearing their seatbelts?
   (4) (3%) If they stop 30 cars during the first hour, find the mean and standard deviation of the number of drivers expected to be wearing seatbelts.
   (5) (8%) If they stop 120 cars during this safety check, would you be surprised if they find at least 35 drivers not wearing their seatbelts? Explain. Find the corresponding probability.

Table Z (cont.)
Areas under the standard Normal curve



z	Second decimal place in z 0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

備註	試題隨卷繳交

考 試 科 目	統計方法	所 別	統計學系	考 試 時 間	2月27 日(日) 第四節

3. 政大商學院民意與市場調查中心發佈一項中正紀念堂民眾滿意度調查結果，這是一項對台灣地區年滿 20 歲的民眾所做的電話訪問，共計成功訪問 1,039 位有效樣本，在 95%的信心水準下，最大抽樣誤差為 3.04%，而其中 73%的民眾對中正紀念堂的整體表現表示滿意或非常滿意。
   (a) 請問最大抽樣誤差為 3.04%是如何計算出來的？(5%)
   (b) 請問在上一題的計算當中，需要用到哪些統計定理？(5%)

4. 政大統研所如果未來欲採用申請方式入學，錄取標準之一可能是採用該學生大學在校成績。為了準確地找出能在政大統研所表現傑出學生之預測變數，政大統研所於是蒐集其系所學生大學在校成績及其在政大統研所之成績，探討是否可採取學生大學在校成績當做錄取標準之可能性。請針對下列蒐集資料時所遇到之不同資料尺度狀況，寫出你認為可行之統計分析方法，並列出虛無假設(Null Hypothesis)、對立假設(Alternative Hypothesis)、統計方法假設條件、檢定統計量及其分佈。
   (a) 如果所蒐集到的學生資料尺度狀況為：在政大統研所之成績為 0~100 分，在大學在校成績也為 0~100 分。(10%)
   (b) 如果所蒐集到的學生資料尺度狀況為：在政大統研所之成績為 0~100 分，在大學在校成績有五個等級【優、甲、乙、丙、丁】。(10%)
   (c) 如果所蒐集到的學生資料尺度狀況為：在政大統研所之成績有三個等級【A、B、C】，在大學在校成績是五個等級【優、甲、乙、丙、丁】。(5%)

5. 政大商學院資料採礦研究中心接受某一跨國 B to B 企業委託，對其顧客做一深入探討分析。
   (1) 請針對下列(a)、(b)小題所蒐集資料時所遇到之不同狀況，寫出你認為可行之統計分析方法，並列出其虛無假設及檢定統計量。
      (a) 此跨國企業欲探討其金字塔端頂級兩類(A、B)顧客之購買金額之分佈是否有顯著不同，隨機抽出 A 類公司顧客 8 家，B 類公司顧客 9 家，又發現 A、B 兩類公司其個別購買金額不是常態分佈。(5%)
      (b) 欲探討兩類(甲、乙)不同產業顧客之平均購買金額是否有顯著不同，隨機抽出甲類公司顧客 80 家，乙類公司顧客 90 家。(5%)
   (2) 此跨國企業約有 100 萬家顧客，如欲針對這 100 萬家顧客之所屬產業、類型、地區、規模…等等之屬性，探討其購買模式，可應用哪些資料採礦方法來分析。(5%)

備 註	試 題 隨 卷 繳 交