

考試科目	基礎數學 41412	所別	統計學系	考試時間	之月27日(六)第一節
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- (10%) Find the rank of $A = \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$, where a, b , and c are real numbers.
- (10%) Let A, B , and C are matrices and \tilde{x}, \tilde{y} , and \tilde{z} are vectors. If $C^{-1} = A^{-1} + B^{-1}$ and $C^{-1}\tilde{z} = A^{-1}\tilde{x} + B^{-1}\tilde{y}$, then show that $\tilde{z} = \tilde{x} + (I + AB^{-1})^{-1}(\tilde{y} - \tilde{x})$.
- (10%) Find the inverse of the following matrix $\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$.
- (10%) Find all 2×2 matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $AB = BA$, where $B = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$.
- (10%) Find the eigenvalues of matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$.
- (10%) Discuss $\lim_{x \rightarrow 0^+} \frac{\sin(x)}{|x|}$, $\lim_{x \rightarrow 0^-} \frac{\sin(x)}{|x|}$, and $\lim_{x \rightarrow 0} \frac{\sin(x)}{|x|}$.
- (7%) If $s_n = \frac{1}{2 \times 1} + \frac{1}{3 \times 2} + \dots + \frac{1}{(n+1) \times n}$, for $n = 1, 2, 3, \dots$, then $\lim_{n \rightarrow \infty} s_n = ?$
(7%) Prove that $\lim_{n \rightarrow \infty} n^{1/n} = 1$. (Hint: Binomial expansion on $(1 + a_n)^n$, where $a_n = n^{1/n} - 1$.)
- Compute the following integrals:
(5%) (a) $\int \frac{x^2}{\sqrt{1-x^3}} dx$
(5%) (b) $\int \cos(\pi t) \cos(\sin(\pi t)) dt$
- Compute the following derivatives:
(5%) (a) $\frac{d}{dx} \left(\int_0^x x^2 t^2 dt \right)$
(5%) (b) $\frac{d}{dx} \left(\int_1^x \cos(x+t) dt \right)$
- (6%) Find the Taylor (or polynomial) expansion for $x \cos(x)$.

備

註

- 作答於試題上者，不予計分。
- 試題請隨卷繳交。

考試科目	數理統計學 1413	所別	統計學系	考試時間	2月27日(六)第三節
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1. A recent study reports that the average time spent online is 7 hours per day.
- (a) What is the probability that time spent online tomorrow will exceed 12 hours? (4%)
- (b) If the variance of time spent on line per day is 16, what is the probability that time spent online tomorrow will be between 2 to 12 hours? (6%)

2. The joint distribution of pollutant standards index (PSI) collected under without using an air filter (X_1) and with using air filter (X_2) is

$$f(x_1, x_2) = \begin{cases} 1/4, & 0 \leq x_1 \leq 4, 0 \leq x_2 \leq 2, 2x_2 \leq x_1 \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Find $E(X_2 | x_1)$ and $V(X_2 | x_1)$. (10%)
- (b) Let $W = X_1 - X_2$ be the reduction in PSI due to using an air filter. Find the probability density function of W , and $E(W)$. (10%)

3. Let X_1, \dots, X_n be a random sample from $f(x) = \frac{(\ln \beta)^x}{x! \beta}$, $x = 0, 1, \dots$, $\beta > 1$.

- (a) Find the complete sufficient statistic for β , and the MLE of β . (10%)
- (b) Find the UMVUEs of $\ln \beta$ and $(\ln \beta)^2$, respectively. (6%, 8%)

4. Let X_1, \dots, X_n be a random sample from a continuous distribution function $F(x)$, and $F'(x) = f(x)$. Let $X_{(k)}$ denote the k^{th} order statistic.

(a) For some α , where $0 < \alpha < 1$, let x_α satisfy $F(x_\alpha) = \alpha$ and $f(x_\alpha) > 0$. Moreover, $m/n \rightarrow \alpha$.

- (a1) Show that $F(X_{(m)})$ converges in probability to a constant and find that constant. (6%)
- (a2) Find the limiting distribution of $\sqrt{n}(F(X_{(m)}) - \alpha)$. (6%)

(Hint: The limiting distribution of $\sqrt{n}(X_{(m)} - x_\alpha)$ is $N(0, \alpha(1-\alpha)/[f(x_\alpha)]^2)$.)

- (b) Find the limiting distribution of $Y_n = n[1 - F(X_{(n)})]$. (10%)

5. Let X_1, \dots, X_n be a random sample from $f(x) = \frac{\beta x^{\beta-1}}{\lambda} \exp(-\frac{x^\beta}{\lambda})$, $x > 0$, $\lambda > 0$, and β is known.

- (a) Find the likelihood ratio size α test of $H_0: \lambda = \lambda_0$ vs. $H_1: \lambda \neq \lambda_0$. (10%)
- (b) Find an equal tailed $100(1-\alpha)\%$ confidence interval for λ . (6%)

(c) Let the prior of λ be $\pi(\lambda) = \frac{b^a}{\Gamma(a)\lambda^{a+1}} \exp(-\frac{b}{\lambda})$, $\lambda > 0$, $a, b > 0$.

Find the posterior distribution and the posterior mean of λ . (8%)

備註 一、作答於試題上者，不予計分。
二、試題請隨卷繳交。

考試科目	統計方法 41414	所別	統計學系	考試時間	2月27日(六)第4節
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1. The following data resulted from an article reported for both a sample of 40 right handed males and a sample of 87 right-handed females the number of individuals whose feet were the same size, had a bigger left than right foot (a difference of half a shoe size or more), or had a bigger right than left foot.

	L = R	L > R	L < R	Sample size
Males	10	2	28	40
Females	18	55	14	87

Does the data indicate that sex has a strong effect on the development of foot asymmetry? Use the 0.05 significance level.

- State the appropriate null and alternative hypotheses. (5 pts)
- Perform the test and draw your conclusion. (15 pts)
- Place a bound or bounds on the p -value. (5 pts)

2. A recent investigation of the relationship between traffic flow (1000's of cars per 24 hours) and PM2.5 content of bark on trees near the highway yield the following data.

Tree	Traffic flow	PM2.5 content
1	8.3	227
2	8.3	312
3	12.1	362
4	12.1	521
5	17.0	640
6	17.0	539
7	17.0	728
8	24.3	945
9	24.3	738
10	24.3	759
11	33.6	1263
$\Sigma(\text{data})$	198.3	7034
$\Sigma(\text{data})^2$	4,198.03	5,390,382

- Develop the regression equation that can be used to describe how the PM2.5 content depends on the traffic flow. (10 pts)
- Determine and interpret the coefficient of determination, r^2 . (10 pts)
- Can we conclude that there is a positive correlation between traffic flow and PM2.5 content? Use the 0.05 significance level. (10 pts)

3. A modification has been made to reduce the process time of certain gadget. Because the modification increases the cost, it will be incorporated only if sample data strongly indicates that the modification has decreased true average processing time.

Original process	8.6	5.1	4.5	5.4	6.3	6.6	5.7	8.5
Modified process	5.5	4.0	3.8	6.0	5.8	4.9	7.0	5.7

Assume that both processing time distributions are skewed. Can we conclude, at the 0.01 significant level, that the modified process reduces process time?

- State the appropriate null and alternative hypotheses. (5 pts)

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b. Perform the test and draw your conclusion. (15 pts)

4. Three types of cereal grain grown in a certain region were analyzed to determine thiamin content, resulting in the following data.

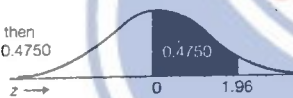
Type A	8	11	10		
Type B	3	2	1	3	2
Type C	3	4	5	4	

Does the data suggest that at least two of the grains differ with respect to true average thiamin content? Use the 0.05 significance level.

- State the appropriate null and alternative hypotheses. (5 pts)
- Perform the test and draw your conclusion. (15 pts)
- What assumptions do you need to make in order to perform b. (5 pts)

Areas under the Normal Curve

Example:
If $z = 1.96$, then
 $P(0 \text{ to } z) = 0.4750$



II. Percentage Points of the t Distribution*

α	.40	.25	.10	.05	.025	.01	.005	.0025
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773
6	.265	.727	1.440	1.943	2.447	3.143	3.707	4.317
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.019
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

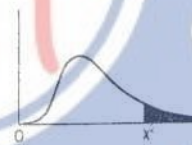
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IV. Percentage Points of the F Distribution (continued)

$F_{1-\alpha, n_1, n_2}$	Degrees of Freedom for the Numerator (n_1)																			
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞	
n_2	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.33	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	4.05	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

Critical Values of Chi-Square

This table contains the values of χ^2 that correspond to a specific right-tail area and specific number of degrees of freedom.



Example: With 17 df and a .02 area in the upper tail, $\chi^2 = 30.995$

Degrees of Freedom, df	Right-Tail Area			
	0.10	0.05	0.02	0.01
1	2.706	3.841	5.412	6.635
2	4.605	5.991	7.824	9.210
3	6.251	7.815	9.837	11.345
4	7.779	9.488	11.866	13.277
5	9.236	11.070	13.388	15.086
6	10.645	12.592	15.033	16.812
7	12.017	14.067	16.622	18.475
8	13.362	15.507	18.168	20.090
9	14.684	16.919	19.679	21.666
10	15.987	18.307	21.161	23.209
11	17.275	19.675	22.618	24.725
12	18.549	21.026	24.054	26.217
13	19.812	22.362	25.472	27.688
14	21.064	23.685	26.873	29.141
15	22.307	24.996	28.259	30.578
16	23.542	26.296	29.633	32.000
17	24.769	27.587	30.995	33.409
18	25.989	28.869	32.346	34.805
19	27.204	30.144	33.687	36.191
20	28.412	31.410	35.020	37.566
21	29.615	32.671	36.343	38.932
22	30.813	33.924	37.659	40.289
23	32.007	35.172	38.968	41.638
24	33.196	36.415	40.270	42.980
25	34.382	37.652	41.566	44.314
26	35.563	38.885	42.856	45.642
27	36.741	40.113	44.140	46.963
28	37.916	41.337	45.419	48.278
29	39.087	42.557	46.693	49.588
30	40.256	43.773	47.962	50.892

備

註

- 一、作答於試題上者，
- 二、試題請隨卷繳交。