1. Let X be a nonnegative continuous random variable with cdf F and pdf f. Show that

(15%) $$EX = \int_0^\infty (1-F(x))\,dx$$

in the sense that, if either side exists, so does the other end and the two are equal.

2. Let $X_1, X_2, \cdots, X_n$ be i.i.d. random variables with common density function f. Also, let

$$M_n = \max(X_1, \cdots, X_n), \quad N_n = \min(X_1, X_2, \cdots, X_n)$$

(10%) Find the distribution of $R_n = M_n - N_n$ if f is given as

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

3. Let $X \sim \text{Poisson}(\lambda)$, find the limiting distribution

(10%) of $Y = \dfrac{X-\lambda}{\sqrt{\lambda}}$ as $\lambda \to \infty$.

4. Prove that: if T is an UMVUE of $\tau(\theta)$, then

(10%) T is unique.

5. Let $X \sim N(0,1)$ under H_0 and $X \sim \text{Cauchy}(1,0)$ under H_1. Find a most powerful test of size α

(5%) for testing H_0 against H_1 and find its power.

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from $U(0,\theta)$. and let $M_n = \max(X_1, \cdots, X_n)$

(15%) (a) Find an UMVUE of θ

(5%) (b) Is the estimator in (a) consistent? why?

(5%) (c) Find an UMP size α test of $H_0: \theta \le \theta_0$ versus

線性模式	系級	統計	日期 Date, Period	六月十日 第 2 節	試題編號 CourseNo.

1. In a simple linear regression model, $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, i = 1,\ldots,n$, where $\varepsilon_i \sim i.i.d.\, N(0,\sigma^2)$.

   (1) Let $\mu_{y|x^*}$ denote the mean of y when $x = x^*$. Derive the test statistic for $H_0: \mu_{y|x^*} = c$ versus $H_a: \mu_{y|x^*} \neq c$ and find its [redacted] distribution.

   (20 points)

   (2) Suppose that x_i can be selected anywhere in the interval [A,B], and if the sample size n is an even integer, suggest how to select $x_i, i = 1,\ldots,n$, so that $\mathrm{Var}(\hat{\beta}_1)$ is minimized. (10 points)

2. Suppose that $Y = \mu + \varepsilon$, where $\varepsilon \sim N(0,\sigma^2 I_n)$. Under the full model, $\mu = X\beta + Z\phi$, where X is $n \times p$, Z is $n \times k$, and rank$([X \quad Z]) = p+k$. But under the reduced model, $\mu = X\beta$. If the true model is the full model.

   (1) First find the bias in estimating β caused by fitting the reduced model. Then find condition for it to be unbiased. (12 points).

   (2) First find the bias in estimating σ^2 caused by fitting the reduced model. Then find condition for it to be unbiased. (12 points)

3. Consider problem 2 again. Suppose that $X'Z = 0$. Derive the test statistic to determine if the full model can be reduced and find its [redacted] distribution.

   (20 points)

4. Suppose $Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$, $i, j = 1,2$, where $\varepsilon_{ij} \sim i.i.d.\, N(0,\sigma^2)$.

   (1) Write down the above model in matrix form. (6 points)

   (2) Prove whether each of the followings is estimable or not.

   a. $\mu + \alpha_1$. (10 points)

   b. $\alpha_1 - \alpha_2 + \beta_1 - \beta_2$. (10 points)