考試科目	數理統計	所別	統計系博士班	考試時間	5月26日 星期六 第｜節

1. The following statements are sometimes true and sometimes false. Give two examples for each statement – one for which the statement is true and one for which the statement is false. (24%)
   (a) An unbiased estimator is consistent.
   (b) A consistent estimator is unbiased.
   (c) If $T_1(X_1,\cdots,X_n)$ is an unbiased estimator of θ_1 and $T_2(X_1,\cdots,X_n)$ is an unbiased estimator of θ_2, then T_1T_2 is an unbiased estimator of $\theta_1\theta_2$.

2. Let X and Y be discrete random variables whose joint distribution is defined by this table.

		x	
		1	2
y	1	$\frac{1}{4}-\theta$	$\frac{1}{4}+\theta$
	2	$\frac{1}{4}+\theta$	$\frac{1}{4}-\theta$

Here θ is a parameter that satisfies $-\frac{1}{4}\le\theta\le\frac{1}{4}$. (20%)

(a) Show that X and Y are each ancillary statistics.
(b) Show that $X+Y$ is not an ancillary statistic.
(c) Find a sufficient statistic for θ.
(d) Are X and Y independent?
(e) Prove the following general result. Consider a model with parameter θ. Suppose X and Y are independent and X and Y are each ancillary statistics under this model. Then, for any function $T=g(X,Y)$ is ancillary.

備考	試題隨卷繳交

命題委員：106 （簽章） 96年5月11日

考試科目	數理統計	所別	統計系博士班	考試時間	5月26日 星期六 第1節

3. Let $X_1,\ldots,X_n$ be a random sample from logistic distribution with cdf $F(x)=1/(1+e^{-x})$. Let $V_n=\max(X_1,\ldots,X_n)$. (16%)
   (a) Find $\lim_{n\to\infty} P\{V_n-\log n\le 0\}$
   (b) Find $\lim_{n\to\infty} P\{|V_n-\log n|\le 1\}$

4. Let $X_1,\ldots,X_n$ be a random sample from a population having probability density function $f(x)=\begin{cases}\frac{1}{\sigma}\exp\{-\frac{x-\mu}{\sigma}\} & x>\mu,\\ 0 & \text{otherwise,}\end{cases}$ where $\mu>0$ and $\sigma>0$. (20%)
   (a) Find the population mean.
   (b) Find a minimal sufficient statistic for $\theta=(\mu,\sigma)$.
   (c) Find the maximum likelihood estimator of $\mu+\sigma$.
   (d) We wish to test the hypothesis $H_0:\mu\le\mu_0$ vs. $H_1:\mu>\mu_0$. Find the likelihood ratio test for the above hypotheses.

5. Suppose that $X_1,\ldots,X_n$ is a random sample from the distribution $P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!}$, for $x=0,1,2,\ldots$, and $0<\lambda<+\infty$. Let Y be the number of X_i's that the value 0. (20%)
   (a) Argue that Y follows the binomial distribution with parameters n and p. Find an expression for p as a function of λ.
   (b) Using the method of moments, find an estimator of λ $(\tilde{\lambda})$ based on Y only.
   (c) Find the large sample approximation of the expectation and variance for $\tilde{\lambda}$.
   (d) Using the approximation obtained in part (c), determine the asymptotic efficiency of $\tilde{\lambda}$ relative to the standard estimator, $\bar{X}$. Evaluate $ARE(\tilde{\lambda},\bar{X}\,|\,\lambda)$ at $\lambda=0.1$, 0.5 and 1.0 and comment briefly.

備考	試題隨卷繳交

命題委員： 107 (簽章) 96 年 5 月 11 日