國立政治大學九十八學年度研究所博士班入學考試命題紙

考試科目	數理統計	所別	統計	考試時間	5月16日 星期六	第1節

1. (20 pts, 10 pts for each part) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables having the uniform distribution $U[0, \theta]$.

   (a) Show that when $\theta \in \Theta = (0, \infty)$, the UMVUE of θ is $\frac{n+1}{n} X_{(n)}$.

   (b) Suppose that $\Theta = (1, \infty)$, show that the UMVUE for θ is

   $$I_{[0,1]}(x_{(n)}) + \frac{n+1}{n} x_{(n)} I_{(1,\infty)}(x_{(n)}).$$

2. (32 pts, 8 pts for each part) Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. $N(\mu, \sigma^2)$, where $\mu \in R$ and $\sigma^2 \in R^+$.
   We are interested in testing the null hypothesis $H_0 : not \ H_1$, against the alternative hypothesis $H_1 : \mu > 0, \sigma^2 < 1$.

   (a) Find the Likelihood ratio test for the above null and alternative hypotheses.

   (b) Can the above hypotheses use intersection-union method?

   (c) Find the intersection-union test.

   (d) What is the difference between part (a) and (c)?

3. (48 pts, 12 pts for each part) Suppose that we have a random sample ($X_1, X_2, \ldots, X_n$) form a distribution with a Lebesgue probability density function f, which is given by $f(x) = \frac{\beta^\lambda}{\Gamma(\lambda)} \exp(\lambda x - \beta e^x)$ for all $x \in (-\infty, \infty)$, where β and λ are positive parameters and $\Gamma(\cdot)$ is the gamma function. Suppose that α is a constant in $(0,1)$.

   (a) Suppose that $\lambda = 1$. Find a statistic V that is complete and sufficient for β.

   (b) Suppose that $\lambda = 1$. Let $Y_i = e^{X_i}$ for $i = 1, \ldots, n$. $\bar{Y} = n^{-1} \sum_{i=1}^{n} Y_i$ and

   $W = \prod_{i=1}^{n} (Y_i / \bar{Y})$. Show that W is independent of the statistic V in part (a).

   (c) Suppose that φ is a test for testing $H_0 : \lambda \le 1$ vs. $H_1 : \lambda > 1$, such that $\beta_\varphi \le \alpha$ under H_0 and $\beta_\varphi \ge \alpha$ under H_1, where β_φ is the power function of φ. Show that $E(\varphi | V) = \alpha$ almost everywhere when $\lambda = 1$, where V is the statistic in part (a).

   (d) Find a UMPU level α test for the $H_0 : \lambda \le 1$ vs. $H_1 : \lambda > 1$. Also, express the rejection region of the UMPU test using the statistic W in part (b), if possible.

備考	試題隨卷繳交