考試科目	數理統計	所別	統計學系	考試時間	5月22日(六)第1節

1. (15pts). Let (X,Y) be a bivariate normal random vector with parameters μ_1, μ_2, σ_1^2, σ_2^2, and ρ. Find a necessary and sufficient condition for $S = X+Y$ and $T = X-Y$ to be independent.

2. A method of generating random variables from a bounded pdf is the following. Let $f(x)$ be a bounded pdf on $[a,b]$ and $c = \max_{a\le x\le b} f(x)$. Let X and Y be independent, with $X \sim$ uniform$[a,b]$ and $Y \sim$ uniform$[0,c]$. Let d be a number greater than b and define a new random variable $W = \begin{cases} X & \text{if } Y \le f(X) \\ d & \text{if } Y > f(X) \end{cases}$.

   (a) (4pts). Please find $P(W = d)$.

   (b) (4pts). Please find $P(W \le w)$ for $a \le w \le b$.

   (c) (12pts). Using (a) and (b), explain how how a random variable with pdf $f(x)$ can be generated from uniform random variables.

3. In a large population of married men (none of whom have been widowed), a fraction, p, have been divorced at least once. The following procedure is followed: A box contains 100 envelopes. $100x$ of these contain the questions: " Have you been divorced?" The other $100(1-x)$ contain the question: "Is this your first marriage?" Assume that x is known and $0 < x < 1$. Note that if a person would answer "yes" ("no") to the first question he would necessarily answer "no" ("yes") to the second. We assume that everyone answers truthfully. N married men were selected at random from the population. Each of them was asked to pick an envelope from the box, read its contents, return it to the box, and then answered the question that only the subject knows.

   (a) (4pts). Please compute the probability, denoted by $r(p,x)$, that a subject would answer "yes".

   (b) (4pts). Let Y denote the total number of men who answered "yes". Find the distribution of Y.

   (c) (16pts). Find the MLE of p. Note, you may want to plot $r(p,x)$ as a function of p for different values of x and observe the features of $r(p,x)$.

備註	試題隨卷繳交

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4. Let X_1, X_2 be iid uniform$[\theta, \theta+1]$. For testing $H_0 : \theta = 0$ versus $H_1 \theta > 0$, we have two competing tests: $\phi_1(X_1, X_2)$: Reject H_0 iff $\max(X_1, X_2) = X_{(2)} > \sqrt{0.995} (\approx 0.997497)$ and $\phi_2(X_1, X_2)$: Reject H_0 iff $X_1 + X_2 > k$.

   (a) (5pts). Please find size of ϕ_1. Show your work.

   (b) (5pts). Please find the value of k such that ϕ_2 has the same size as ϕ_1. Show your work.

   (c) (10pts). It has been computed that the power function of ϕ_1 is $p_1(\theta) = \begin{cases} 1 - (\sqrt{0.995} - \theta)^2 & \text{if } 0 < \theta \le \sqrt{0.995} \\ 1 & \text{if } \sqrt{0.995} < \theta \end{cases}$. Please calculate the power function of ϕ_2. Show your wok.

   (d) (5pts). Draw a well-labeled graph of each power function in (c). Prove or disprove: ϕ_2 is a more powerful test than ϕ_1.

5. (16pts). Consider the LINEX loss given by $L(\theta, a) = e^{c(a-\theta)} - c(a-\theta) - 1$, where c is a positive constant. As the constant c varies, the loss function varies from very asymmetric to almost symmetric. Suppose $X|\theta \sim f(x|\theta)$. Please show that the Bayes estimator of θ, using a prior π, is given by $\delta_\pi(X) = \frac{-1}{c} \log E(e^{-c\theta}|X)$.

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