考試科目	數理統計	所別	統計學系	考試時間	5 月 21 日(六) 第一節

1. (15 pts) Suppose that we observe independent and identically distributed random variables $X_1, \ldots, X_n$, and X_1 has a probability density function f, where

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(\log(x) - \log(1-x) - \mu)^2}{2}\right)\left(\frac{1}{x} + \frac{1}{1-x}\right)$$

for $0 < x < 1$. Here μ is an unknown parameter and log is the natural logarithm.

   (a) Find the maximum likelihood estimator for μ.

   (b) Find the likelihood ratio test of size α for testing

$$H_0 : \mu = 0 \text{ versus } H_1 : \mu \neq 0.$$

   Express the rejection region in terms of a test statistic and give the distribution of the test statistic under H_0.

2. (35 pts) Suppose that $(X_1, \ldots, X_n)$ is a random sample from the uniform distribution on $[0, \theta]$, where $\theta > 0$ is an unknown parameter. Let

$$X_{(n)} = \max_{1 \le i \le n} X_i$$

and

$$X_{(1)} = \min_{1 \le i \le n} X_i.$$

   (a) Show that $X_{(n)}$ is a consistent estimator for θ.

   (b) Find an UMVUE (uniformly minimum variance unbiased estimator) for θ.

   (c) Can we conclude that $X_{(n)}$ is a better estimator for θ than the UMVUE in Part (b) based on their mean squared errors?

   (d) Determine whether $X_{(n)}$ and $X_{(n)}/X_{(1)}$ are independent.

   (e) Construct a 95% confidence interval for θ based on $X_{(n)}$.

備註	試題隨卷繳交

考 試 科 目	數理統計	所 別	統計學系	考 試 時 間	5 月 21 日(六) 第一節

3. (15 pts) Suppose that $(X_1, \ldots, X_n)$ is a random sample from $N(\mu, 1)$, where μ is an unknown parameter. Consider the problem of estimating μ. Suppose that the loss of estimation when μ is estimated by a and the true value for μ is μ_0 is given by

$$\ell(a, \mu_0) = \begin{cases} (a-\mu_0)^2, & \text{if } |a-\mu_0| > 1; \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the Bayes rule for this estimation problem when the prior for μ is $N(0, 1)$.

   (b) Find the Bayes rule for this estimation problem when the prior for μ puts probabilities 0.6 and 0.4 at -1 and 1 respectively.

4. (35 pts) Suppose that we observe independent and identically distributed pairs $(X_1, Y_1), \ldots, (X_n, Y_n)$. Suppose that

$$P(X_1 = 1) = p = 1 - P(X_1 = 0),$$

$$P(Y_1 = y | X_1 = 1) = p^y (1-p)^{1-y} \text{ for } y \in \{0, 1\},$$

and

$$P(Y_1 = y | X_1 = 0) = (1-p)^y p^{1-y} \text{ for } y \in \{0, 1\},$$

where p is an unknown parameter and $0 < p < 1$.

   (a) Find all value(s) of p such that X_1 and Y_1 are independent.

   (b) Find a sufficient statistic for p. The dimension of the sufficient statistic should be no more than 4.

   (c) Express the maximum likelihood estimate of p in terms of the sufficient statistic found in Part (b).

   (d) Suppose that $n = 3$ and the observed (X_1, Y_1), (X_2, Y_2), (X_3, Y_3) are $(1, 0)$, $(0, 1)$ and $(0, 0)$ respectively. Find the maximum likelihood estimate of p.

   (e) Suppose that $n = 2$ but X_1 and X_2 are missing. The observed values for Y_1 and Y_2 are 0 and 1 respectively. Find the maximum likelihood estimate of p.

備 註	試 題 隨 卷 繳 交