考試科目	數理統計	所別	統計學系	考試時間	5月12日(六)第 1 節

You need to show your work (that is, how you get the answer) to get points.

1. (10 points) Let Y be any random variable. Find c such that $E(|Y-c|)$ is minimized.

2. (20 points) Let f be a density function and F its corresponding cdf. Consider the following sampling procedure.
   step 1. Sample Y from f
   step 2. If $Y \in (a,b)$, set $X = Y$. Else, go to step 1.
   Find the cdf and pdf of X.

3. (30 points) Let X and Y be two r.v.'s.
   (a) Find $m(Y)$ that minimizes $E(X - m(Y))^2$.
   (b) Show that $\text{Var}(E(X|Y)) \leq \text{Var}X$.
   (c) When does the equality in (b) hold?

4. (20 points) Suppose that $X_t = \rho X_{t-1} + Z_t$, where $|\rho| < 1$ and $\{Z_t\}$ are i.i.d $N(0, \sigma^2)$.
   (a) Find the distribution of X_t.
   (b) Find $\text{Cov}(X_t, X_{t-h})$.

5. (20 points) Consider a linear model:

$$y_i = x_i\beta + e_i, \quad i = 1, ..., n,$$

where e_i are iid $N(0, \sigma^2)$ variates, and β and σ^2 are unknown parameters. Consider the following prior on β and σ^2:

$$p(\beta, \sigma^2) \propto 1/\sigma^2.$$

The posterior density of (β, σ^2) is proportional to the prior times the likelihood. That is, $p(\beta, \sigma^2|y) \propto p(\beta, \sigma^2)L(\beta, \sigma^2)$.
(a) Obtain the joint posterior density $p(\beta, \sigma^2|y)$.
(b) Obtain the marginal posterior density $p(\sigma^2|y)$.

備註	試題隨卷繳交