避險期貨在管理階層誘因契約中之角色

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摘要

本文目的在於設計含有「避險期貨」之誘因契約，以解決代理成本與誘因風險之問題。經由理論模型之推導，得證出股東若能運用「避險期貨」設計誘因契約，可有效解決代理成本之問題，並可減少誘因風險。結果說明在管理者行動可被有效監督之下，完全避險之誘因契約可達到最適風險分攤原則；而在管理者行動無法被完全監督之下，股東需權衡提供誘因契約所伴隨而來之風險成本，其中研究結果說明若股東欲誘使管理者選擇最適之努力程度提升，則在其他條件不變下，其所設計之誘因契約之風險權數亦需相對提高。並且研究結果符合 Holmstrom (1979)之分析，說明若股東未將持有股份全數售予管理者，則其最適行動必劣於 first-best action。另外，由比較靜態分析發現，當管理者努力之邊際報酬提升，股東會提供更多的誘因以誘使管理者更加努力，而其最適努力程度亦會提升；當管理者愈趨避風險或面對更多不確定性時，股東應降低其薪酬契約中之風險成本，且管理者之最適努力程度亦會下降。

關鍵詞：避險期貨、誘因風險、誘因契約、代理成本
The Role of Hedged Futures in Managerial Incentive Contracts

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Abstract

Constructing from Holmstrom’s (1979) framework and following linear-exponential-normal (LEN) formulation, we design an incentive contract involving “hedged futures” to solve the agency cost problem. The main results show that when the manager’s action can be perfectly inferred, the stockholder can simply pay a fixed payment or elaborately design an achievable risk-free incentive contract, and both contracts can effectively achieve the optimal risk sharing. On the other hand, when the manager’s action cannot be perfectly inferred, the optimal contract trades off benefits from providing incentives with the cost of imposing risk on a risk-averse manager. Further, the higher the optimal level of manager’s effort to motivate, the fewer the optimal hedge ratio required to induce the manager to work harder, ceteris paribus. Besides, consistent with the analysis of Holmstrom (1979), the second-best action is strictly lesser than the first-best action if the stockholder does not “sell” the whole share to the manager. Also, the results of comparative analysis indicate that when the marginal rate of return increases, the stockholder will provide more incentive to induce the manager to work harder and the optimal level of manager’s effort will rise as well. When the manager is more risk-averse or faces more uncertainty, the stockholder needs to reduce the risk in the compensation contract, such as increasing the hedge ratio, and the optimal level of manager’s effort will decrease as well.

Keywords: Hedged futures, Incentive risk, Incentive contract, Agency cost.

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1. INTRODUCTION

In the agency theory (Ross 1973; Jensen and Meckling 1976; Myers and Majluf 1984), when the manager (or the owner) owns 100 percent of the ownership, any behavior that damages the firm’s value would be fairly evaluated by outside investors, so that the manager requires entirely bearing the whole agency cost; that is, the capital market can discipline the manager through effectively rational expectation. However, while the organization is expanding, it is inevitable to experience the separation of ownership and control. At the moment, the manager only owns few or none shares of the firm, who is called the “professional manager”; then, the capital market can no longer discipline the manager through fair evaluation, so that the manager isn’t required to completely bear the agency cost raised by his deviant behavior.

When the professional manager’s welfare (or utility) is less related with the prices of the company’s stocks, capital market could not discipline the manager for his deviant behavior. Whereas if the stockholder (i.e., the principal) has an impact on the manager’s welfare through designing mechanism (including the manager’s compensation, promotion opportunity, and so on), the manager would be required to bear the cost raised by damaging the company’s value. Therefore, the aim of the present paper is to design a managerial compensation contract, in light of the stockholder, in order to force the professional manager to think about the whole company’s performance when he pursues his self-utility maximization, so as to achieve the convergence of both sides’ objectives.

With regard to the compensation contract, viewing current environment filled with global and intense competition, many industries in the high technology and communication sectors need to keep their excellent technicians. To attract the technicians and make their objectives in line with the stockholders’ interests, corporations usually establish a priority for employees to purchase their companies’ stocks at the employee’s price (i.e., lower than the market price). Furthermore, corporations will grant the stocks or options as performance award, but usually set a requirement on prohibiting employees from trading the granted stocks (or options) within a certain period, for the anticipation that employees would regard the future development of the company as individual objectives. After observing this situation, we desire to describe the current situation with a form of derivative financial commodity.

So far, much research has been done on exploring solutions for agency cost. Haugen and Senbet (1981) suggested that call or put option could solve the problems associated with excessive nonpecuniary consumption. However, as
described by Lambert, Larcker and Verrecchia (1991), when the manager was a risk averter, the manager’s valuation on an incentive contract would be lower than the stockholder’s. Lambert, Larcker and Verrecchia (1991) noted that the lower manager’s valuation was owing to the incentive contracts being un-hedged. Thus, Hemmer (1993) recommended that a compensation package involving stock options could be designed by using other components of the compensation package as a hedging tool only if the agent chooses the action desired by the principal, and therefore would shield the manager from risk.

Hemmer’s (1993) model assumed that the level of agent’s effort \( \{x^h, x^l\} \) was dichotomy and the natural state’s \( (\varepsilon) \) distribution was symmetric and binomial with the result that incentive contracts involving “hedged” options could solve the problem of agency cost and achieve the optimal risk-sharing. Therefore, extending from Hemmer’s (1993) model, we would like to design incentive contracts involving other “hedged” derivative financial commodities to explore the ability of solving the problems associated with the agency cost and incentive-related risk. Specifically, constructing from Holmstrom’s (1979) framework, we design an incentive contract involving “hedged futures”, putting emphasis on the hedge for the manager pursuing self-utility maximization, to investigate first best solution and second best solution. This emphasis is resulting from a rational manager would prefer “utility-maximization hedge”, not just “risk-minimum hedge”.

In this paper, the compensation package involving “hedged futures” designed, similar to the method of Hemmer (1993), consists of stock futures and a cash component in which the cash component provided as an insurance for the futures is, therefore, negatively correlated with the value of the futures. Further, following linear-exponential-normal (LEN) formulation of Holmstrom and Milgrom’s (1987) framework, we obtain the relationship between the manager’s optimal level of effort, optimal hedge ratio, and the optimal fixed payment under the incentive contract involving “hedged futures”.

Much research has been done on investigating the effect of the stock option as an incentive mechanism since lots of corporations have been granting stock options to compensate managers. However, currently, more and more investors are complaining about the opportunistic behavior of the manager given stock options offered as an incentive mechanism. In response to investors’ complaint, more and more corporations announce to adjust compensation policies, such as replacing stock options granted with the payment of restricted stocks or cash. Thus, it appears that seeking other alternative type of contracts is prevailing. Since few studies have done on the effect of other derivative financial commodities (e.g. stock futures) on
solving agency problems, we endeavor to explore the effect of an incentive contract involving stock futures on solving agency problems.

Further, in view of the requirement about prohibiting employees from trading the granted stocks within a certain period, we suggest that an incentive contract involving “hedged futures”, alike the granted stocks prohibited from trading in a specified period, would be able to describe this situation and effectively disclose the trading timing and quantities. Thus, although we provide stock futures as the structure of the contract for analysis in this paper, it generalizes to introduce other claims on the company’s output. For example, the restricted stock with a specific employee’s price can be used as well. Therefore, the purpose of the paper we focus on is to investigate the effect of an incentive contract involving “hedged futures” on solving the problems associated with the agency cost and incentive-related risk. With meaningful implications, this research may provide an alternative to the problem of the agency cost and incentive-related risk.

The organization of the paper is as follows. Section 1 introduces the motivation of investigating the effect of an incentive contract involving “hedged futures”. Section 2 reviews the related literature of applying derivative financial commodities to managerial incentive contracts for solving agency problems. In sections 3 and 4, we shall separately investigate first best solution and second best solution for exploring the effect of an incentive contract involving “hedged futures” on solving the problems associated with the incentive-related risk. Section 5 summaries the main results and discusses the implications of the future research.

2. LITERATURE REVIEW

Recently, more and more derivative financial commodities have been invented, so some research has been done on involving derivative financial commodities in managerial incentive contracts so as to induce the agent to make more alignment with the principal. Haugen and Senbet (1981) suggested that call option or put option could solve problems associated with excessive non-pecuniary consumption. Haugen and Senbet showed that when the manager held the call option of the managed corporation, he made non-pecuniary consumption one more unit, and then the firm’s value decreased one unit; although the value of his own stock decreased less one unit, the call option’s value would also reduce at that time. Therefore, under the well-designed mechanism, call option could eliminate the executive’s incentive to increase perk consumption. Otherwise, Haugen and Senbet reported that call and put option could utilize simultaneously to solve the problem associated with risk transferring.
Nevertheless, Lambert, Larcker and Verrecchia (1991) showed that the manager’s valuation on the incentive contracts was not equivalent to the shareholder’s. They found that when the manager was risk-averse and his other wealth related to the stock price of the managed firm, his valuation on the incentive contracts was lower than the shareholder’s. Besides, the shareholder measured the option-involved remuneration by the Black-Scholes’ (1973) model and the basic assumption of the Black-Scholes’ (1973) model was a risk-neutral manager, these resulted in an actual risk-averse manager unable to receive the risk premium as payoff, and thus, reduced his valuation on the incentive contracts.

Therefore, Hemmer (1993) recommended involving “hedged options” in incentive contracts, and showed that these contracts not only solved the problem associated with agency cost but also achieved the optimal risk-sharing. Consequently, Hemmer suggested that under asymmetric information, the principal could design a contract using a hedged combination so as to induce the agent to achieve the objective the principal desired and the agent could also bear no incentive-related risk. What a hedged combination was to make the components in the incentive plan of the manager’s remuneration become negative relationship, which could reduce the incentive risk of the remuneration.

On the assumption that the degree of the agent’s effort was dichotomy and the natural state’s distribution was symmetrically binomial, Hemmer (1993) proved that when the agent chose the action that the principal desired, the incentive contract was risk-free; otherwise, the compensation package involved risk. Hirshleifer and Suh (1992) also showed that “hedging” could mitigate agency problems. Therefore, starting from Hemmer (1993), we shall explore the effect of another “hedged” derivative financial commodity (e.g. “hedged futures”) on solving the problem associated with the agency cost and incentive-related risk.

Since a great many of corporations grant stock options to compensate managers, much research has been done on the benefit and deficiency of the stock option as an incentive mechanism\(^1\). Some researchers claimed that options would be an inefficient remuneration mechanism (Jenter 2001; Meulbroek 2001; Hall and Murphy 2002). Others argued that the empirical relationship of the stock options granted to managers displayed inconsistent with the incentive purpose (Yermack 1995; Bertrand and Mullainathan 2001; Tian 2004). Another researchers presented that managers manipulated the pay-setting process of granted options for their own profit (Yermack 1997; Bens, Nagar and Wong 2002; Baker, Collins and Reitenga

\(^1\) See surveys on compensation (Murphy 1999; Core, Guay and Larcker 2003); also see Hanlon, Rajgopal and Shevlin (2003).
Hall and Murphy (2003) also explored the trouble with stock options and identified several problems with options granted to managers.

Tian (2001) investigated the optimal contracting problem between the corporation and CEO under asymmetric information. He found that the optimal contract (i.e., the second-best contract) depended on the risk- and effort-averse degree of the executive and his personal investment opportunities unrelated to the managed corporation. Incentive compensation, including stocks and options, which was an essential part of the optimal contract accounted for 14% to 100% of total pay. However, for a risk-averse CEO, options might not be a compatible incentive because of the risk- and effort-averse degree of the CEO. When CEO’s degree of risk- and effort-aversion was relatively high, the restricted stock other than options would be more suitable for incentive remuneration. Hall and Murphy (2002) and Jenter (2001) also mentioned that restricted stock would be a better incentive mechanism than options with non-zero striking prices. Therefore, we try to design an incentive contract involving “hedged futures”, alike the restricted stock, to solve the problem associated with the incentive-related risk.

3. FIRST-BEST SOLUTION UNDER THE INCENTIVE CONTRACT INVOLVING HEDGED FUTURES

In the past, much research has been done on option-based incentive contracts to solve the problem associated with agency cost. However, Lambert, Larcker and Verrecchia (1991) noted that the un-hedged option involved in managerial contracts could solve the agency problem associated with the manager’s excessive nonpecuniary consumption, but still had incentive risk which would decrease the manager’s valuation on incentive contracts. Hemmer (1993), the first, recommended that on the assumption that the level of agent’s effort \( \{ \alpha^h, \alpha' \} \) was dichotomy and the natural state’s \( (\varepsilon) \) distribution was symmetric and binomial, incentive contracts involving “hedged” options could solve the problem of agency cost and achieve the optimal risk-sharing. Therefore, starting from Hemmer (1993), we would like to design an incentive contract involving another type of “hedged” derivative financial commodity to investigate the effect of such an incentive contract on solving the agency problem.

In addition, we should note that on the assumption that the manager is not only risk-averse but also effort-averse, a rational manager would tradeoff benefits from decreasing efforts with losses from taking risk when pursuing self-utility maximization. Thus, considering that efforts are disutilities for the manager, a risk-and effort-averse manager would prefer “utility-maximization hedge”, not just
“risk-minimum hedge”. Also, the optimal level of effort the manager chooses would not necessarily be the highest level of effort. Therefore, we need to design an incentive mechanism for the stockholder to maximize her own returns and simultaneously induce a risk- and effort-averse manager to pursue self-utility maximization in line with the stockholder’s objectives.

Thus, the purpose of these two sections is to design an incentive contract involving “hedged futures”, putting emphasis on the hedge for the manager pursuing self-utility maximization, to solve the problem associated with agency cost so as to induce the manager in line with the stockholder, and further, to resolve the problem of incentive risk which could make the contract more effective and reduce the deadweight loss. And constructing from Holmstrom’s (1979) framework, we shall derive the optimal equilibrium of the level of effort, fixed payment and hedge ratio in an incentive contract involving “hedged futures”.

3.1. BASIC MODEL

The setting discussed in this paper is the same as that of Holmstrom (1979). The manager (i.e., the agent) is employed by the stockholder (i.e., the principal) to do a given job with the possible minimal payment. Given the contract offered, the hired manager chooses an action unobserved and waits for the output realized. After the observable output is realized, the stockholder gives the pre-specified payment to the manager. In order to induce the manager to take the action desired, the stockholder designs an incentive contract linking the compensation of the manager with the observable output statistically associated with the manager’s action. The contract designed entails trading off the benefits of imposing risk for providing incentives with the cost of imposing risk on a risk-averse manager. Thus, this principal-agent relationship produces the deadweight loss and therefore breeds positive agency cost. Besides, the manager’s action mentioned in this paper simply represents the manager’s level of effort.

The basic definitions and assumptions are as follows:

1. The level of manager’s effort \( \alpha \) is a continuous variable, and \( \alpha \in A' \subseteq R \) where \( A' \) denotes the set of feasible efforts.

2. Let the actual output be \( \tilde{u}(\alpha) = \bar{u}(\alpha) + \bar{e} \), where \( \bar{u}(\alpha) \) denotes the expected output, and \( \bar{e} \) denotes the state of nature. Besides, it is assumed that the stockholder and manager have homogenous beliefs about the probability distribution of \( \bar{e} \) and that the manager selects the level of effort \( \alpha \) before \( \bar{e} \) is known. Let \( f(u,\alpha) \) denote the probability density of the output \( \tilde{u} \), parameterized by the manager’s action \( \alpha \). That is, we assume that the output \( \tilde{u} \)
is a continuous random variable whose distribution is affected by the manager’s action.

3. Let the gains from futures be \( \bar{B} = \tilde{S} - T \), where \( \tilde{S} \) denotes the end period of stock value and \( T \) denotes the futures price of stocks. In addition, let the stock value be \( \tilde{S}(\alpha) = \bar{u}(\alpha) - D \) in which \( D \) denotes the debt cost of the company, which means that the end period of stock value is equal to the residual value of the end period of actual output (i.e., the whole company’s value after paying the money borrowed). Note that the number of company’s stock is assumed to be one share.

4. Let the manager’s compensation package be \( \bar{W} = F + \bar{P} \), a fixed payment combined with a risky portfolio, where \( F \) denotes the fixed payment. In addition, let \( \bar{P} = \bar{B} + \beta \tilde{S} \) denote the cash flow of the risky portfolio, where \( \beta \) is the hedge ratio.

5. The manager is a risk- and effort-averter. Let the manager’s utility function be \( U(\bar{W}) - V(\alpha) \), a separable von Neumann-Morgenstern utility function, where \( U(\bar{W}) \) denotes the manager’s utility of compensation and \( V(\alpha) \) denotes the manager’s disutility of effort. Suppose that \( U(.) \) is strictly increasing and concave (i.e., \( U' > 0, \ U'' < 0 \)) and \( V(.) \) is strictly increasing and convex (i.e., \( V' > 0, \ V'' > 0 \)).

6. The stockholder is risk-neutral. Let \( E[\bar{u}(\alpha) - D] - \bar{W} \) denotes the stockholder’s utility function, where \( \bar{W} \) is the expected compensation to the manager. That is, the expected utility of the risk neutral stockholder is simply the expected net profits of the company. Further, the stockholder is assumed to prefer more money to less. That is, the marginal rate of expected return would be increasing, \( \bar{u}'(\alpha) > 0 \).

3.2. FIRST-BEST SOLUTION

In this section, we simply assume that the stockholder can infer the action chosen by the manager from publicly observable output realized at the end of period. In this situation, as a benchmark for comparison, the solution to the agency problem is generally computed without the incentive problems and therefore the first-best solution can be achieved. Mathematically, the first-best solution can be expressed as the solution to the following problem:
\[
\begin{align*}
\text{Max}_{\alpha, W(.)} & \quad E[\bar{u}(\alpha) - D] - \bar{W} \\
\text{subject to} & \quad E[U(\bar{W}) - V(\alpha)] \geq \Theta \quad \text{(IR)}
\end{align*}
\]

That is, in the first-best solution, the compensation package and the action are chosen to maximize the stockholder’s expected utility conditional on guaranteeing the manager a minimal acceptable level of utility, \( \Theta \). Equation (2) represents \textit{individual rationality constraint} (IR), where (IR) constraint indicates the manager will not continue staying at the company until he receives at least the payment of \( \Theta \), or he would rather work for other companies to ensure to obtain the expected utility \( \Theta \). Therefore, \( \Theta \) denotes the manager’s reservation utility. Note that no incentive compatibility constraint presents here since the manager cannot privately choose the action to maximize self-utility under the situation where the action chosen will be completely inferred by the stockholder.

Substituting the manager’s compensation package \( \bar{W} = F + [\bar{B} + \beta \bar{S}] \) into Equations (1) and (2), we can re-express the problem as:

\[
\begin{align*}
\text{Max}_{\alpha, \beta, F} & \quad E\{\bar{u}(\alpha) - D\} - F - [\bar{B} + \beta \bar{S}] \\
\text{subject to} & \quad E\{U[F + [\bar{B} + \beta \bar{S}]] - V(\alpha)\} \geq \Theta
\end{align*}
\]

Further, substituting \( \bar{B} + \beta \bar{S} = (1 + \beta)(\bar{u} - D) - T \) into the above equations and letting \( \lambda \) be the Lagrange multiplier on the individual rationality constraint, the first-best solution can be written as the solution to the stockholder’s problem, as follows:

\[
\begin{align*}
\text{Max}_{\alpha, \beta, F} \int \{\bar{u}(\alpha) - D - [F + (1 + \beta)(\bar{u} - D) - T ]\} f(u, \alpha) du \\
+ \lambda \left\{ U \{F + (1 + \beta)(\bar{u} - D) - T ] - V(\alpha)\} f(u, \alpha) du - \Theta \right\}
\end{align*}
\]

The optimal contract is characterized by taking the derivative of the objective function with respect to \( F \) and \( \beta \), respectively, for each possible value of \( \bar{u} \). First, the first-order condition on \( F \) satisfies

\[
(-1) \cdot f(u, \alpha) + \lambda \cdot U[\bar{W}] \cdot f(u, \alpha) = 0
\]

which can be re-written as \( 1/U[\bar{W}] = \lambda \). Next, the first-order condition on \( \beta \) satisfies:

\[
\{(-1) \cdot f(u, \alpha) + \lambda \cdot U[\bar{W}] \cdot f(u, \alpha)\} \cdot (\bar{u} - D) = 0
\]
which can be re-expressed as $1/U[\tilde{W}] = \lambda$ as Equations (6) and (7) indicate that the optimal contract satisfies $1/U[\tilde{W}] = \lambda$.

Since $\lambda$ is a constant, this implies that the optimal contract pays the manager a fixed payment independent of the output and the risky portfolio as well, i.e., $\tilde{W} = F_*$. Further, let $W$ denote the reservation wage to the manager\(^2\), and $F$ can be chosen to make the individual rationality constraint (IR) binding. That is, the stockholder can retain the manager with a minimal acceptable level of wage, $W$, when the manager chooses the action desired. Thus, the optimal contract is $\tilde{W} = F_* = W$. That is, since a risk averse manager prefers to bear no risk, the optimal contract completely shields the manager from any risk. Therefore, the optimal contract can achieve the optimal risk sharing.

On the other hand, interestingly, the optimal contract could be designed as a risk-free\(^3\) incentive contract only if the manager chooses the action desired by the stockholder. Since the stockholder can perfectly infer the first-best action chosen or not, the optimal risk sharing contract would be effectively achieved. That is, the stockholder can design the incentive contract involving the hedge ratio $\beta_{FB}^* = -1$, (if and) only if the first-best action has been chosen. And $F_{FB}^*$ can be chosen to make the (IR) constraint an equality, as follows:

$$\{U \left[ F_{FB}^* + (1 + \beta_{FB}^*)(\tilde{u} - D) - T \right] - V(\alpha_{FB}^*) \} f(u, \alpha) du = \Theta$$

Further, letting $W$ denote the reservation wage to the manager, we can express the expected compensation package as:

$$E[\tilde{W}(\tilde{u})] = F_{FB}^* + (1 + \beta_{FB}^*)(\tilde{u} - D) - T = W$$

Moreover, substituting $\beta_{FB}^* = -1$ into the manager’s compensation package, we can derive the optimal fixed payment, as follows:

$$F_{FB}^* = W + T$$ (8)

The optimal fixed payment derived includes the reservation wage combined with the futures price. The higher the reservation wage negotiated at the managerial labor market, the more the optimal fixed payment paid to the manager. Also, the higher the futures price of stocks the company sets, the more the optimal fixed payment the manager rewards. Since the higher futures price specified causes the less gains from futures obtained, the stockholder needs to compensate the manager

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\(^2\) The reservation wage means that the wage paid to the manager given the first-best action chosen under the reservation utility exactly held, i.e. $U[W] - V(\alpha_{FB}^*) = \Theta$.

\(^3\) The risk-free contract here has a complete analysis in Appendix.
for higher fixed payment. Importantly, this first-best incentive contract \( \{F_{FB}^*, \beta_{FB}^*\} \) can be effectively realized when the manager would be penalized substantially given any action selected other than the first-best.

For exploring more meaningful implications, we follow Holmstrom and Milgrom’s (1987) approach and introduce a more tractable formulation\(^4\) of the agency model, i.e., linear-exponential-normal (LEN) formulation. First, the manager’s utility function is assumed to be negative exponential, \( U(\tilde{Y}) = -e^{-c\tilde{Y}} \), where \( c \) is the coefficient of absolute risk aversion, and more definitively, \( \tilde{Y} = \tilde{W} - \upsilon(\alpha) \) is the manager’s “net” revenue after subtracting the “cost” of effort. Specifically, \( \upsilon \) represents the monetary equivalent of the manager’s disutility of effort. Consistent with the assumption that the manager’s disutility of effort is strictly increasing and convex, let \( \upsilon(\alpha) = (1/2)\alpha^2 \) denote the manager’s cost of effort. Second, the output \( \tilde{u} \) is assumed to be a normal distribution. The manager’s action is assumed to have an impact on the expected value but not the variance of the output. For simplifying computation, the expected value of \( \tilde{u} \) is assumed to be a linear function of the manager’s effort. This linear form assumed here is not important, which is just easier for computation. Let the output function be \( \tilde{u} = s_1\alpha + s_2 + \tilde{e} \), where \( s_1 \) represents the marginal rate of return from the manager’s efforts and \( s_2 \) represents, on average, the minimum output of the company when the manager makes no efforts. Let \( \sigma^2 \) denote the variance of \( \tilde{u} \), i.e., \( \tilde{e} \sim N(0,\sigma^2) \). Finally, as described above, the compensation contract is assumed to be a linear function of the observed output, i.e.,

\[
\tilde{W}(\tilde{u}) = F + \left[ \tilde{B} + \beta \tilde{S} \right] = F + \left[ (1 + \beta)(\tilde{u} - D) - T \right]
\]

where \( E[\tilde{W}(\tilde{u})] = F + \left[ (1 + \beta)(s_1\alpha + s_2 - D) - T \right] \)

\[
Var[\tilde{W}(\tilde{u})] = (1 + \beta)^2 \cdot \sigma^2
\] (9)

Note that the manager’s action only has an impact on the expected value but not the variance of the compensation package. Proposition 1 verifies that under LEN formulation, the stockholder can design an incentive contract without sacrificing optimal risk sharing.

**Proposition 1.** Given the assumptions in section 3.1 and the above LEN formulation, under the situation where the action chosen will be completely inferred by the stockholder, an incentive contract \( \{ \beta_{FB}^* = -1 \, , \, F_{FB}^* = \tilde{W} + (l/2)s_1^2 + T \} \) can effectively achieve the risk-free effect, i.e., the optimal risk sharing rule.

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\(^4\) We appreciate the review’s suggestion on a more tractable formulation required.
Proof: The synthesis of the above assumptions of the negative exponential utility, normal distribution, and linear contract connotes that the certainty equivalent (CE) of the manager’s expected utility can be simply written in the following form:

\[
CE = E\left[\hat{W}(\hat{u})\right] - \frac{1}{2} \cdot c \cdot Var\left[\hat{W}(\hat{u})\right] - \nu(\alpha) \\
= F + \left[(1+\beta)(s_1\alpha + s_2 - D) - T\right] - \frac{1}{2} \cdot c \cdot \left[(1+\beta)^2 \cdot \sigma^2\right] - \frac{1}{2} \alpha^2 
\]  

(10)

That is, the certainty equivalent of the manager’s expected utility is equal to the expected value of his compensation package after deducting the costs of making effort and bearing risk dependent on his risk averse degree and the variance of his compensation. With this construction, we can express the stockholder’s problem as:

\[
Max_{\alpha, \beta, F} E\left[\hat{u}(\alpha) - D - \left[F + (1+\beta)(\hat{u} - D) - T\right]\right] 
\]  

subject to

\[
F + \left[(1+\beta)(s_1\alpha + s_2 - D) - T\right] - \frac{1}{2} \cdot c \cdot \left[(1+\beta)^2 \cdot \sigma^2\right] - \frac{1}{2} \alpha^2 \geq CE(\Theta) = \hat{W} 
\]  

(12)

The stockholder maximizes her expected profits after deducting the debt cost and the manager’s compensation. The manager’s individual rationality constraint (Equation (12)) requires the contract designed by the stockholder to ensure the incentive attractive enough to offer the manager a minimal acceptable level. Here, the manager’s expected utility is expressed with its certainty equivalent. Specifically, the certainty equivalent of the manager’s reservation utility, \(CE(\Theta)\), is denoted by \(\hat{W}\). Note that there is no incentive compatibility constraint under the first-best situation, i.e., the stockholder can perfectly infer the first-best action chosen by the manager from the observed output.

The fixed payment of the manager’s compensation package, \(F\), can be chosen to make the individual rationality constraint (IR) an equality, i.e.,

\[
F^*_{FB} + \left[(1+\beta)(s_1\alpha + s_2 - D) - T\right] = \hat{W} + \frac{1}{2} \cdot c \cdot \left[(1+\beta)^2 \cdot \sigma^2\right] + \frac{1}{2} \alpha^2 
\]  

(13)

Thus, we can directly substitute the resulting term into the objective function to remove the (IR) constraint (Equation (12)), as follows:
Eventually, we can characterize the first-best action and the optimal contract by taking derivative of the resulting objective function with respect to $\alpha$ and $\beta$, respectively. Further, without loss of generality, we assume that the coefficient of absolute risk aversion ($c$) and the variance of the normal distribution ($\sigma^2$) are positive. Thus, the first-best action derived satisfies

$$\alpha_{FB}^* = s_i$$

(15)

and the optimal contract derived satisfies

$$\beta_{FB}^* = -1$$

(16)

Further, directly substituting $\beta_{FB}^* = -1$ and $\alpha_{FB}^* = s_i$ into the binding (IR) constraint (Equation (13)), we can derive the optimal fixed payment, i.e.,

$$F_{FB}^* = \hat{W} + \frac{1}{2}s_i^2 + T$$

(17)

Besides, substituting $\beta_{FB}^* = -1$ (Equation (16)) and $F_{FB}^*$ (Equation (17)) into the manager’s compensation package (Equation (9)) yields

$$\hat{W}^* = F_{FB}^* + \left[1 + \beta_{FB}^*(\bar{u} - D) - T\right] = \hat{W} + \frac{1}{2}s_i^2$$

(18)

Equation (18) illustrates that the optimal contract can pay the manager a constant, $\hat{W}^* = \hat{W} + (1/2)s_i^2$, which is independent of the output and the manager’s effort. As described above, a risk averse manager prefers not to bear any risk, and therefore, the optimal contract can achieve the optimal risk sharing.

On the other hand, more interestingly, the stockholder can design a risk-free incentive contract $\{F_{FB}^*, \beta_{FB}^*\}$ involving the hedge ratio, $\beta_{FB}^* = -1$ (if and) only if the first-best action has been chosen. Note that this risk-free incentive contract $\{F_{FB}^*, \beta_{FB}^*\}$ can be effectively achieved when the manager would be penalized substantially given any action selected other than the first-best, i.e., $\alpha_{FB}^* = s_i$. Therefore, this risk-free incentive contract also can achieve the optimal risk sharing rule.

Under the proof of Proposition 1, Equation (15) illustrates that given the contract offered, the first-best action chosen by the manager is equal to the marginal rate of return, $\alpha_{FB}^* = s_i$. The higher the marginal rate of return contributed from manager’s effort, the greater the first-best action desired by the stockholder.
Equation (17) shows that the optimal fixed payment depends on the certainty equivalent of the manager’s reservation utility (\( \hat{W} \)), the marginal rate of return (\( s_1 \)), and the futures price of stocks (\( T \)). When the certainty equivalent of the manager’s reservation utility (\( \hat{W} \)) or the futures price of stocks (\( T \)) heightens, the optimal fixed payment compensated to the manager will build up. Also, when the marginal rate of return contributed from manager’s effort (\( s_1 \)) raises, the optimal fixed payment to the manager will increase to motivate higher efforts and similarly, from Equation (15), the optimal level of manager’s effort will increase.

Next, we turn to the situation where only the output (\( \tilde{u} \)) is observable and the stockholder cannot perfectly infer the manager’s action from the realized output.

4. SECOND-BEST SOLUTION UNDER THE INCENTIVE CONTRACT INVOLVING HEDGED FUTURES

4.1. SECOND-BEST SOLUTION

In this section, we embark on analyzing the model under the situation where the manager’s action can’t be completely inferred. The resulting second best solution produces the deadweight loss relative to first best one. In this second best solution, since the stockholder cannot perfectly infer the manager’s action, she needs to design an incentive mechanism not only to maximize her own returns but also to induce a risk- and effort-averse manager to pursue self-utility maximization in line with the stockholder’s objectives. As noted above, considering that efforts are disutility for the manager, a risk- and effort-averse manager would more prefer “utility-maximization hedge” than “risk-minimum hedge”. Thus, following Holmstrom’s (1979) framework, we mainly explore an incentive contract involving “hedged futures”, putting emphasis on the hedge for the manager pursuing self-utility maximization, to solve the problem associated with agency cost so as to induce the manager in line with the stockholder, and further, to resolve the problem of incentive risk which could make the contract more effective and reduce the deadweight loss.

Following the definitions of section 3.1, we can construct the stockholder’s problem as:
In the second-best solution, we maximize the stockholder’s expected net return subject to providing the manager an acceptable level of expected utility and allowing the manager to pursue self-utility maximization. Note that here the stockholder’s problem includes the incentive compatibility constraint (IC), indicating that the manager desires to choose the action that maximizes his expected utility given the incentive contract offered.

As assumed above, \( f(u, \alpha) \) denotes the probability density of the output \( u \) affected by the manager’s action. Further, it is assumed that the expected output will increase in the level of effort; particularly, higher level of effort will shift the probability distribution of the output to the right in the sense of first-order stochastic dominance.

As to the resolution methods for (IC) constraint, there are a number of different mathematical approaches used. Following Holmstrom’s (1979) analysis, we solve (IC) constraint with F.O.C. method. Assuming the manager’s expected utility is a strictly concave function of his effort and the optimal effort is in the interior of the action set, the first-order condition will be satisfied for only one action, and that action will be the global maximum for the manager.

Substituting \( \tilde{B} + \beta \tilde{S} = (1 + \beta)(\tilde{u} - D) - T \) into the above equations and letting \( \lambda_1 \) be the Lagrange multiplier for the individual rationality constraint and \( \lambda_2 \) be the Lagrange multiplier for the manager’s first-order condition on effort, we can express the stockholder’s problem as:

\[
Max_{\alpha, \beta, F} E\left[\tilde{u}(\alpha) - D - [F + (\tilde{B} + \beta \tilde{S})]\right] \tag{19}
\]

subject to

\[
E\left[U\left[F + (\tilde{B} + \beta \tilde{S})\right] - V(\alpha)\right] \geq \Theta \text{ (IR)} \tag{20}
\]

\[
\alpha \in \arg\max_{\alpha \in A'} E\left[U\left[F + (\tilde{B} + \beta \tilde{S})\right] - V(\alpha')\right] \tag{IC} \tag{21}
\]

\[5\] See Holmstrom’s (1979) discussion that two approaches were used, including the state-space approach mentioned by Spence and Zeckhauser (1971), Ross (1973), and Harris and Raviv (1979), and Mirrlees’ (1976) approach. Others, like Grossman and Hart (1983), introduced another discussion of discrete action agency models.

\[6\] See several agency theory researchers’ discussion of substituting this first-order condition on effort for the incentive compatibility constrain, which develop conditions to ensure that the agent’s expected utility is a strictly concave function of his effort (e.g. Grossman and Hart 1983; Rogerson 1985; Jewitt 1988).
Thus, the optimal contract can be derived by differentiating the objective problem with respect to $F$ and $\beta$, respectively, for each value of output $\tilde{u}$. First, the derivative of the objective problem with respect to $F$ is
\[(1) \cdot f(u, \alpha) + \lambda_1 \cdot U'[\tilde{W}] \cdot f(u, \alpha) + \lambda_2 \cdot U'[\tilde{W}] \cdot f_\alpha(u, \alpha) = 0\]
which can be re-expressed as:
\[
\frac{1}{U'[\tilde{W}]} = \lambda_1 + \lambda_2 \frac{f_\alpha(u, \alpha)}{f(u, \alpha)}
\]
Next, the derivative of the objective problem with respect to $\beta$ is
\[(\tilde{u} - D) \cdot \{ -(1) \cdot f(u, \alpha) + \lambda_1 \cdot U'[\tilde{W}] \cdot f(u, \alpha) + \lambda_2 \cdot U'[\tilde{W}] \cdot f_\alpha(u, \alpha) \} = 0\]
which can be re-expressed as:
\[
\frac{1}{U'[\tilde{W}]} = \lambda_1 + \lambda_2 \frac{f_\alpha(u, \alpha)}{f(u, \alpha)}
\]
Equations (23) and (24) indicate that the optimal contract satisfies
\[
\frac{1}{U'[\tilde{W}]} = \lambda_1 + \lambda_2 \frac{f_\alpha(u, \alpha)}{f(u, \alpha)}
\]
Note that if $\lambda_2 = 0$, this reduces to the optimal risk sharing contract in first-best solution of section 3.2, i.e., the contract pays the manager a fixed payment, $\tilde{W} = F^*$, which makes the manager’s compensation independent of the output and his effort. However, under such a contract, a risk- and effort-averse manager has no incentive to work hard, so he chooses the minimal possible effort. Thus, under second best solution, if the stockholder would like to motivate the manager to increase his effort, $\lambda_2$ should not be zero. As proved by Holmstrom (1979), $\lambda_2 > 0$ as long as the stockholder wants to motivate more than the lowest possible level of effort. Thus, with $\lambda_2 > 0$, the optimal contract trades off the benefits of imposing risk for providing sufficient incentives with the cost of imposing risk on a risk-averse manager.
In order to analyze more meaningful economic implications, as described above, we follow Holmstrom and Milgrom’s (1987) approach and investigate second best solution under the linear-exponential-normal (LEN) formulation constructed in section 3.2.

With this formulation derived in section 3.2, the stockholder’s problem can be expressed in the certainty equivalent form, as follows:

$$\text{Max}_{\alpha, \beta, F} \ E\{\tilde{u}(\alpha) - D - \tilde{W}(\tilde{u})\}$$

subject to

$$E[\tilde{W}(\tilde{u})] - \frac{1}{2} \cdot c \cdot \text{Var}[\tilde{W}(\tilde{u})] - v(\alpha) \geq CE(\Theta) = \tilde{W} \quad \text{(IR)}$$

$$\alpha \in \arg\max_{\alpha' \in A'} E[\tilde{W}(\tilde{u})] - \frac{1}{2} \cdot c \cdot \text{Var}[\tilde{W}(\tilde{u})] - v(\alpha') \quad \text{(IC)}$$

The stockholder maximizes her expected net return contingent on offering the manager an acceptable level of expected utility and allowing the manager to pursue his best interests given the contract offered. Here, the manager’s expected utility is expressed in its certainty equivalent form, and specifically, the certainty equivalent of the manager’s reservation utility, $CE(\Theta)$, is denoted by $\tilde{W}$. Also, under the situation where the stockholder cannot completely infer the action chosen, the stockholder’s problem requires to include the incentive compatibility constraint indicating that the manager has his incentive to pursue self-utility maximization.

**Proposition 2.** Given the assumptions in section 3.1 and the above LEN formulation, under the situation where the manager’s action cannot be perfectly inferred by the stockholder, the optimal contract $\{F^*_SB, \beta^*_SB\}$ trades off benefits from providing incentives with the cost of imposing risk on a risk-averse manager. The higher the optimal level of manager’s effort to motivate, the fewer the optimal hedge ratio required to induce the manager to work harder, ceteris paribus.

**Proof:** Replacing the manager’s compensation package $\tilde{W}(\tilde{u})$ with Equation (9) of section 3.2, we can re-express the stockholder’s problem as:

$$\text{Max}_{\alpha, \beta, F} \ E\{\tilde{u}(\alpha) - F - [(1 + \beta)(\tilde{u}(\alpha) - D) - T]\}$$

subject to

$$F + [(1 + \beta)(s_1\alpha + s_2 - D) - T] - \frac{1}{2} \cdot c \cdot [(1 + \beta)^2 \cdot \sigma^2] - \frac{1}{2} \cdot \alpha^2 \geq \tilde{W} \quad \text{(25)}$$

$$\alpha \in \arg\max_{\alpha' \in A'} \ F + [(1 + \beta)(s_1\alpha' + s_2 - D) - T] - \frac{1}{2} \cdot c \cdot [(1 + \beta)^2 \cdot \sigma^2] - \frac{1}{2} \cdot (\alpha')^2 \quad \text{(26)}$$
With the certainty equivalent (CE) form of the manager’s expected utility (Equation (26)), the manager’s first order condition on his action simplifies to

\[(1 + \beta) \cdot s_1 - \alpha = 0 \tag{27}\]

This formulation satisfies the condition requiring that the manager’s expected utility is a strictly concave function of his effort since the output function is a linear function of the manager’s effort and the cost of effort is convex (i.e., $V'>0$, $V''>0$). Thus, this allows us to substitute the first-order condition for the incentive compatibility constraint.

As described above, the fixed payment of the manager’s compensation package, $F$, can be chosen to make the individual rationality constraint an equality, i.e.,

\[F^*_F + [(1 + \beta) (s_1 \alpha + s_2 - D) - T] = \hat{W} + \frac{1}{2} \cdot c \cdot [(1 + \beta)^2 \cdot \sigma^2] + \frac{1}{2} \alpha^2 \tag{28}\]

And the first-order condition on the manager’s action (Equation (27)) can be re-expressed as:

\[\alpha = (1 + \beta) \cdot s_1 \tag{29}\]

After replacing the output function with the linear form, i.e., $\tilde{u} = s_1 \alpha + s_2 + \bar{e}$, we can substitute Equations (28) and (29) directly into the objective function and therefore, eliminate these two constraints (IR) and (IC), as follows:

\[
Max_{\alpha, \beta, F} \left\{ s_1 [(1 + \beta) \cdot s_1] + s_2 - D \right\}
- \left\{ \hat{W} + \frac{1}{2} \cdot c \cdot [(1 + \beta)^2 \cdot \sigma^2] + \frac{1}{2} [(1 + \beta) \cdot s_1]^2 \right\} \tag{30}
\]

Therefore, the optimal contract can be obtained by differentiating the objective function with respect to $\beta$. And, as described above, without loss of generality, the coefficient of absolute risk aversion ($c$) and the variance of the normal distribution ($\sigma^2$) are assumed to be positive. The resulting first-order condition on $\beta$ is:

\[(1 + \beta^*_{SB}) = \frac{s_1^2}{c \cdot \sigma^2 + s_1^2} \tag{31}\]
which can be re-expressed as:

$$\beta_{SB}^* = -1 + \frac{s_1^2}{c \cdot \sigma^2 + s_1^2} \tag{32}$$

Since the marginal rate of return ($s_1$), the coefficient of absolute risk aversion ($c$) and the variance of the normal distribution ($\sigma^2$) are positive, the resulting optimal hedge ratio will be greater than -1. The resulting first-order condition on $s_1$ illustrates that when the marginal rate of return ($s_1$) increases, the optimal hedge ratio ($\beta_{SB}^*$) will become less negative. That is, the optimal hedge ratio will shift toward unhedging with the result that the portfolio will become more risky and thus more chance to earn more.

Besides, we can derive the optimal level of manager’s effort (i.e., the second-best action) by substituting Equation (32) directly into the first-order condition on the manager’s action (Equation (29)), as follows:

$$\alpha_{SB}^* = \frac{s_1^3}{c \cdot \sigma^2 + s_1^2} \tag{33}$$

That is, the optimal second-best action depends on the coefficient of absolute risk aversion ($c$), the marginal rate of return ($s_1$), and the variance of the distribution of the output ($\sigma^2$). The resulting first-order condition on $s_1$ indicates that the optimal level of manager’s effort will increase in the marginal rate of return. Combined with the resulting first-order condition of Equation (32), we verify that ceteris paribus, the higher the optimal level of manager’s effort to motivate, the fewer the optimal hedge ratio required to induce the manager to work harder.

Finally, the optimal fixed payment can be obtained by substituting the optimal hedge ratio (Equation (32)) and the optimal level of effort (Equation (33)) into the binding (IR) constraint (Equation (28)). Thus, the resulting optimal fixed payment is

$$F_{sb}^* = T + \hat{W} + \frac{s_2^2}{c \cdot \sigma^2 + s_1^2} \cdot (D - s_2) + \left[ \frac{s_1^2}{c \cdot \sigma^2 + s_1^2} \right] \cdot \left[ \left( -\frac{1}{2} \right) \cdot s_2^2 + \frac{1}{2} \cdot c \sigma^2 \right]$$

$$= F(c, \sigma^2, s_1, s_2, D, T, \hat{W}) \tag{34}$$

where $F()$ is the function of exogenous variables {$c, \sigma^2, s_1, s_2, D, T, \hat{W}$}. Therefore, the set of the optimal contract {$F_{sb}^*, \beta_{SB}^*$} (i.e., second best solution) can be
obtained under the information asymmetry situation where the stockholder cannot perfectly infer the action chosen by the manager from the realized output.

Under the proof of Proposition 2, Equation (32) shows that the optimal hedge ratio \((\beta_{SB}^*)\) depends on the coefficient of absolute risk aversion \((c)\), the marginal rate of return \((s_1)\), and the variance of the distribution of the output \((\sigma^2)\). With the comparative analysis, we take the derivative of Equation (32) with respect to \(c\), \(s_1\), and \(\sigma^2\), respectively. As discussed by Holmstrom (1979, corollary 1), when the marginal rate of return increases, the stockholder will provide higher incentive, such as a more risky portfolio here, to induce the manager to work harder. Further, when the coefficient of absolute risk aversion \((c)\) or the variance of the distribution of the output \((\sigma^2)\) increases, the optimal hedge ratio \((\beta_{SB}^*)\) will become more negative (i.e., the optimal hedge ratio will shift toward risk-free hedge ratio). That is, when the manager is more risk averse or faces more uncertainty, the stockholder needs to reduce the risk of the compensation package, such as increasing the hedge ratio in the risky portfolio.

Moreover, we take the derivative of Equation (33) with respect to \(c\), \(s_1\), and \(\sigma^2\), respectively. The resulting first-order conditions illustrate that when the marginal rate of return \((s_1)\) increases, the optimal level of effort selected by the manager will rise. Further, when the coefficient of absolute risk aversion \((c)\) or the variance of the distribution of the output \((\sigma^2)\) increases, the optimal level of effort selected by the manager will fall.

Equation (34) indicates that the optimal fixed payment \((F_{SB}^*)\) depends on the coefficient of absolute risk aversion \((c)\), the variance of the distribution of the output \((\sigma^2)\), the marginal rate of return \((s_1)\), the minimum expected output \((s_2)\), the debt cost \((D)\), the futures price of stocks \((T)\), and the certainty equivalent of the reservation utility \((\hat{W})\). With the comparative analysis, the results illustrate that when the minimum output \((s_2)\) decreases, the debt cost \((D)\) increases, the futures price the company specifies \((T)\) increases, or the CE of the reservation utility \((\hat{W})\) increases, the stockholder needs to compensate the manager higher fixed payment to ensure that the manager can obtain exactly the reservation utility.

### 4.2. EQUILIBRIUMS COMPARISON

With the comparison between the first-best and second-best solutions of manager’s effort, from Equations (33) and (15), we can observe that the second-best action chosen is less than the marginal rate of return, i.e., \(\alpha_{SB}^* < s_1 = \alpha_{FB}^*\). Consistent with the analysis of Holmstrom (1979), the second-best action is strictly inferior to the first-best action. Note that observing from the relationship of
\( \alpha_{SB}^* = (1 + \beta_{SB}^*) \cdot s_1 \) (Equation (29)), we can find that ceteris paribus, the less negative the optimal hedge ratio, the higher the optimal level of effort. That is, if the stockholder desires to motivate the manager to work harder, he can decrease the hedge ratio in the compensation package, ceteris paribus, but it depends in part on how risk averse the manager is. Furthermore, if the optimal hedge ratio becomes un-hedged (i.e., \( \beta_{SB}^* = 0 \)), the optimal level of manager’s effort satisfies \( \alpha_{SB}^* = s_1 \) where the marginal rate of return \( s_1 \) is exactly equal to the first-best action derived in section 3.2. That is, when the stockholder totally gives the whole share to the manager, alike the company is “sold” to the manager, the manager bears all risk. But the incentive problem is internalized, and thus, the manager will choose the optimal level of effort equal to the first-best action (i.e., \( \alpha_{SB}^* = s_1 = \alpha_{FB}^* \)). Besides, we can verify the illustration of Equation (32) that if the stockholder designs a risk-free incentive contract (i.e., the risk-free hedge ratio \( 1 - b = b \)), the optimal level of effort the manager chooses is zero.

As for the comparison between the first-best and second-best solutions of the hedge ratio, from Equation (32), the second-best hedge ratio will be greater than \(-1\). That is, under the situation where the stockholder cannot perfectly infer the action chosen by the manager, the resulting optimal hedge ratio (\( \beta_{SB}^* \)) (i.e., the second-best solution) will not be the risk-free hedge ratio (\( \beta = -1 \)) (i.e., the first-best solution). Otherwise, a risk and effort averse manager will have no incentive to work hard, and thus he will choose the minimal possible effort.

Finally, from Equations (17) and (34), the comparison between the first-best and second-best solutions of the fixed payment shows that the second-best fixed payment depends on more complicated factors, such as the minimum expected output without any effort, the cost of debt, the setting of the futures price of stocks, and the situation of the managerial labor market.

5. SUMMARY

In the Principal-Agent theory, the firm is considered as a nexus for contracting relationships, and does not have the unique objective for maximizing the company’s profit. Contrarily, both sides of the contracting relationship pursue self-utilities maximization, and therefore, this agency relationship breeds what is called the “agency problem”. And further, the source of agency cost is that the agent would like to transfer the principal’s wealth to himself through his dysfunctional behavior.

The goal of the research attempts to explore another type of incentive mechanism to make the objectives of both sides coincident, so as to maximize the
firm’s profit. What the mechanism is to design incentive contracts involving a set of derivative financial commodities (e.g. stock futures), in light of the stockholder, in order to motivate the manager to choose the optimal level of effort. Specifically, we design an incentive contract involving “hedged futures”, putting emphasis on the hedge for the manager pursuing self-utility maximization. This emphasis is resulting from considering that a risk- and effort-averse manager trades off benefits from decreasing efforts with losses from taking risk when pursuing self-utility maximization. Thus, a rational manager prefers “utility-maximization hedge”, not just “risk-minimum hedge”. Constructing from Holmstrom’s (1979) framework, we derive the manager’s optimal level of effort, optimal hedge ratio, and the optimal fixed payment under the incentive contract involving “hedged futures”, including first best solution and second best solution. Following linear-exponential-normal (LEN) formulation of Holmstrom and Milgrom’s (1987) framework, we investigate the comparative static analysis after deriving the optimal solution, and obtained many meaningful economic implications.

The results show that under the situation where the stockholder can perfectly infer the manager’s action, the optimal risk sharing contract can be effectively achieved by two ways. First, the optimal contract pays the manager a fixed payment to a minimal acceptable level of wage, \( \hat{W} = F^* = W \). Second, the optimal contract can be designed as a risk-free incentive contract (if and) only if the manager chooses the action desired by the stockholder. This risk-free incentive contract \( \{ F_{FB}^* = W + T + s_1, \beta_{FB}^* = -1 \} \) can be effectively realized when the manager will be penalized substantially given any action selected other than the first-best. These two contracts both can achieve the optimal risk sharing since a risk neutral stockholder doesn’t not care about bearing all the risk, while a risk averse manager prefers bearing no risk, ceteris paribus.

Moreover, with LEN formulation, the results indicate that given the contract offered, the first-best action chosen by the manager is equal to the marginal rate of return (i.e., \( \alpha_{FB}^* = s_1 \)), and the optimal contract derived satisfies \( \{ \beta_{FB}^* = -1, F_{FB}^* = \hat{W} + (1/2) s_1^2 + T \} \). That is, analogous to the analysis without LEN form: first, the optimal contract can pay the manager a constant, \( \hat{W}^* = \hat{W} + (1/2) s_1^2 \), the function of the certainty equivalent of the manager’s reservation utility (\( \hat{W} \)) and the marginal rate of return (\( s_1 \)), which is independent of the output and the manager’s effort. Second, the stockholder can design a risk-free incentive contract \( \{ F_{FB}^*, \beta_{FB}^* \} \) involving the hedge ratio, \( \beta_{FB}^* = -1 \) (if and) only if the first-best action has been chosen. Besides, the optimal fixed payment \( F_{FB}^* \) to the manager depends on the certainty equivalent of the manager’s reservation utility (\( \hat{W} \)), the marginal rate of return (\( s_1 \)), and the futures price of stocks (\( T \)). When the certainty equivalent of the
manager’s reservation utility ($\hat{W}$) or the futures price of stocks ($T$) heightens, the optimal fixed payment compensated to the manager will build up. In addition, when the marginal rate of return contributed from manager’s effort ($s$) raises, the optimal fixed payment to the manager will increase to motivate higher efforts and meanwhile, the optimal level of manager’s effort will also increase. This risk-free incentive contract \( \{F_{FB}^*, \beta_{FB}^*\} \) can be effectively achieved when the manager would be penalized substantially given any action selected other than the first-best, i.e., \( \alpha_{FB}^* = s_1 \).

On the other hand, under the situation where the stockholder cannot perfectly infer the manager’s action, if the stockholder would like to motivate the manager to increase his effort, as discussed by Holmstrom (1979), the optimal contract trades off the benefits of imposing risk to provide sufficient incentives with the cost of imposing risk on a risk averse manager. Furthermore, with LEN formulation, the results illustrate that the resulting optimal hedge ratio ($\beta_{SB}^*$) (i.e., the second-best solution) will not be the risk-free hedge ratio ($\beta = -1$), depending on the coefficient of absolute risk aversion ($c$), the marginal rate of return ($s$), and the variance of the distribution of the output ($\sigma^2$). In addition, the optimal second-best action depends on the coefficient of absolute risk aversion ($c$), the marginal rate of return ($s$), and the variance of the distribution of the output ($\sigma^2$). Further, consistent with the analysis of Holmstrom (1979), the second-best action is strictly inferior to the first-best one (i.e., $\alpha_{SB}^* < s_1 = \alpha_{FB}^*$). However, if the stockholder “sells” the company to the manager, the incentive problem is internalized, and thus, the manager will choose the optimal level of effort equal to the first-best action (i.e., $\alpha_{SB}^* = s_1 = \alpha_{FB}^*$).

With the comparative analysis, the results indicate that when the marginal rate of return ($s$) increases, the coefficient of absolute risk aversion ($c$) decreases, or the variance of the distribution of the output ($\sigma^2$) decreases, the optimal level of effort selected by the manager will rise. Further, when the marginal rate of return ($s$) increases, the optimal hedge ratio ($\beta_{SB}^*$) will become less negative (i.e., shift toward un-hedging). This is consistent with Holmstrom’s (1979, corollary 1) discussion that when the marginal rate of return increases, the stockholder will provide more incentive to induce the manager to work harder. When the coefficient of absolute risk aversion ($c$) or the variance of the distribution of the output ($\sigma^2$) increases, the optimal hedge ratio ($\beta_{SB}^*$) will become more negative (i.e., shift toward risk-free hedge ratio). That is, when the manager is more risk averse or faces more uncertainty, the stockholder needs to reduce the risk of the compensation package, such as increasing the hedge ratio in the risky portfolio.
Besides, the resulting optimal fixed payment \( (F^*_SB) \) depends on the coefficient of absolute risk aversion \((c)\), the variance of the distribution of the output \((\sigma^2)\), the marginal rate of return \((s_1)\), the minimum expected output \((s_2)\), the debt cost \((D)\), the futures price of stocks \((T)\), and the certainty equivalent of the reservation utility \((\tilde{W})\). When the minimum output \((s_2)\) decreases, the debt cost \((D)\) increases, the futures price the company specifies \((T)\) increases, or the certainty equivalent of the reservation utility \((\tilde{W})\) increases, the stockholder needs to compensate the manager higher fixed payment to ensure that the manager can obtain exactly the reservation utility. Furthermore, our result notes that ceteris paribus, the less negative the optimal hedge ratio, the higher the optimal level of effort. That is, if the stockholder desires to motivate the manager to work harder, he can decrease the hedge ratio in the compensation package, ceteris paribus, but it depends in part on how risk averse the manager is.

In this paper, we follow Holmstrom and Milgrom’s (1987) approach and introduce linear-exponential-normal (LEN) formulation since it’s a more tractable formulation of the agency model. In the future research, it could be extended to other types of formulation to investigate the differences between various formulations. With regard to applying derivative financial commodities in incentive contracts for solving agency problems, the comparison between various derivative financial commodities could be done in the future research, such as options and futures. Particularly, because futures should be exercised at the expiration date, but options don’t have to be exercised at the expiration date, it may be possible that the optimal level of effort the manager chooses may be higher under an incentive contract involving “futures” than that involving “options”. This could be strictly explored in the future research.

**APPENDIX**

Here, we shall illustrate that the risk-free effect of the incentive contract involving “hedged futures”. Definitions and assumptions are the same as those in section 3.1. Particularly, the manager’s compensation package is assumed to be \( \tilde{W} = \tilde{F} + \tilde{P} \) where \( \tilde{P} \) is the cash flow of the risky portfolio, and \( \tilde{F} \) is the fixed payment. Let the cash flow of the risky portfolio be \( \tilde{P} = \tilde{B} + \beta \tilde{S} \), where \( \tilde{B} \) is the gain from futures, \( \beta \) is the hedge ratio, and \( \tilde{S} \) is the end period of stock value. Further, the gain from futures is assumed to be \( \tilde{B} = \tilde{S} - T \), where \( T \) is the futures price of stocks. And the end period of stock value is assumed to be \( \tilde{S}(\alpha) = \tilde{u}(\alpha) - D \) in which \( D \) denotes the debt cost, which indicates that the end period of stock value is equal to the whole company’s value after paying the money borrowed. And
note that the number of company’s stock is assumed to be one share.

Substituting these definitions into the manager’s compensation package, we can re-express the compensation package as:

\[ \tilde{W} = F + \tilde{P} = F + [\tilde{B} + \beta \tilde{S}] = F + [(1 + \beta)(\tilde{u} - D) - T] \]

That is, as discussed by Hemmer (1993), the compensation package can be expressed as risky portfolios related to the company’s output for providing incentives combined with a fixed payment that can be chosen to guarantee a minimal acceptable level of expected compensation. Note that such a contract involving “hedged futures” can be re-expressed as a linear function of the output.

In addition, when \( \beta = -1 \) is held, the compensation package paid to the manager is the fixed payment, i.e., \( \tilde{W} = F \), in which involves no risk. Therefore, such \( \beta \) is called the risk-free hedge ratio, and thus, an incentive contract involving the hedge ratio \( \beta = -1 \) is called a risk-free incentive contract.

Moreover, under linear-exponential-normal (LEN) formulation defined in section 3.2, more economic implications can be derived. Specifically, the output \( \tilde{u} \) is assumed to be a normal distribution and for simplifying computation, the expected value of \( \tilde{u} \) is assumed to be a linear function of the manager’s efforts. Mathematically, the output function is expressed as \( \tilde{u} = s_1 \alpha + s_2 + \tilde{e} \) where \( s_1 \) represents the marginal rate of return from the manager’s efforts, \( s_2 \) represents, on average, the minimum output of the company when the manager makes no efforts, and \( \tilde{e} \) denotes the state of nature with zero mean. Let \( \sigma^2 \) denote the variance of \( \tilde{u} \), i.e., \( \tilde{e} \sim N(0, \sigma^2) \) and thus, \( \tilde{u} \sim N(s_1 \alpha + s_2, \sigma^2) \). Note that the manager’s action is assumed to have an impact on the expected value but not the variance of the output.

As described above, one unit cash flow of risky portfolio can be re-expressed as:

\[ \tilde{P}(\tilde{u}) = \tilde{B} + \beta \tilde{S} = (1 + \beta)(\tilde{u} - D) - T \] \hspace{1cm} (A.1)

where \( E[\tilde{P}(\tilde{u})] = (1 + \beta)(s_1 \alpha + s_2 - D) - T \) \hspace{1cm} (A.2)

\[ Var[\tilde{P}(\tilde{u})] = (1 + \beta)^2 \times \sigma^2 \] \hspace{1cm} (A.3)

The risky portfolio is a linear function of the observed output, where the manager’s effort affects the mean but not the variance of the risky portfolio. With the discussion of Equations (A.2) and (A.3), we can investigate the range of the hedge ratio. Here, as described in section 4.1, more effort is assumed to increase the
expected value of the portfolio, i.e., first-order stochastic dominance. Thus, by differentiating the expected value of the portfolio (Equation (A.2)) with respect to manager’s effort, the resulting first-order condition satisfies $(1 + \beta) \cdot s_i \geq 0$.

Since the marginal rate of expected return is assumed to be increasing (i.e., $\pi'(\alpha) > 0$), the marginal rate of return ($s_i$) is positive with the result that the hedge ratio is no less than -1 (i.e., $1 + \beta \geq 0$). Further, observing the variance of the portfolio (Equation (A.3)), we can obtain the range of hedge ratio is from 0 to -1. That is, the stockholder can provide an incentive contract involving the hedge ratio ranging from un-hedging (i.e., $\beta = 0$) to, at most, the risk-free hedge ratio (i.e., $\beta = -1$).

REFERENCES


Harris, M., and A. Raviv. 1979. Optimal incentive contracts with imperfect


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