The Optimal Product Mix for Hedging Longevity Risk in Life Insurance Companies

H.C. Huang, Sharon S. Yang, Jennifer L. Wang, Jeff T. Tsai

National Cheng-chi U. /Soochow U./ National Cheng-chi U./National Taiwan U.

Taiwan

Outline

- Introduction
- Literature review
- Immunization Strategy
- Modeling Longevity Risk
- Numerical Illustration
- Conclusion and further research

Introduction

- Hedging longevity risks has taken on an increasingly important role for life insurance companies.
- According to the concept of natural hedging, life insurance can serve as a dynamic hedge vehicle against unexpected mortality risk.
- To help life insurers achieve a better natural hedging effect, we propose an immunization model that incorporates a stochastic mortality to calculate the optimal level of a product mix to effectively reduces longevity risks for insurance companies.

Introduction-Con't

Research Purpose

- Using **US mortality experience**, we demonstrates that our proposed model can lead to calculate the optimal product mix and thus effectively reduce longevity risks for insurance companies.

Literature Review

- Mortality risk and pricing issues for annuity products
  - Friedman and Warshawsky (1990)
  - Frees, Carriere, and Valdez (1996)
- Mortality derivatives and survival bonds
  - Blake and Burrows (2001)
  - Charupat and Milevsky (2001)
  - Lin and Cox (2005)
  - Dowd, Blake, Cairns, Dawson (2006)
  - Denuit, Devolder, Godernaiaux (2007)

Literature Review-Con't

- Stochastic mortality
  - Lee and carter (1992)
  - Marceau and Gaillardetz (1999)
  - Lee (2000) and Yang (2000)
  - Milevsky and Promislow (2001, 2002)
  - Renshaw and Haberman (2003)
  - Pitacco (2004)
  - Cairns et. al. (2006)
  - Schrager (2006)
- Natural hedging
  - Lin and Cox (2004)

Literature Review-Con’t

- No discussion in the insurance literature so far addresses product strategies for natural hedging.
- This paper attempts to fill this gap.

Immunization Strategy

- The total liability V of the insurer equals the sum of the liabilities for different business

$$V = V^{life} + V^{annuity}$$

- To achieve that the effect of changing mortality on total liability is immunized.

$$\frac{dV}{d\mu} = 0$$

Immunization Strategy-Con't

- Under the assumption of constant force of mortality, we define mortality duration for insurance and annuity as follows

$$D_{\mu}^{life} = -\frac{dV^{life}}{d\mu} \cdot \frac{1}{V^{life}}$$

$$D_{\mu}^{annuity} = -\frac{dV^{annuity}}{d\mu} \cdot \frac{1}{V^{annuity}}$$

Immunization Strategy-Con't

- Therefore, we can achieve $\dfrac{dV}{d\mu} = 0$ by setting

the mortality duration of total liability equal to 0.

$$D_\mu = 0$$

- where $D_\mu = D_\mu^{life} \cdot \omega_{life} + D_\mu^{annuity} \cdot \omega_{annuity}$

.

Immunization Strategy-Con't

- The optimal product mix of liability proportions:

$$\omega_{life} = \frac{D_{\mu}^{annuity}}{D^{annuity} - D_{u}^{life}}$$

Immunization Strategy-Con't

- The concept of duration employs assumptions of constant cash flows, flat yield curves, and parallel shifts in interest rates. However, these assumptions may not be realistic in practice.
- Kalotay, Williams, and Fabozzi (1993), David, Merrill, and Panning (1997), and Gajek, Ostaszewski, and Zwiesler (2005) propose effective duration as an alternative risk measure, which also applies to measuring mortality risk.

Immunization Strategy-Con't

- Effective Mortality Duration

$$D_{eu}^{life} = \frac{V^{life-} - V^{life+}}{2 \times V^{life} \times \Delta\mu}$$

$$D_{eu}^{annuity} = \frac{V^{annuity-} - V^{annuity+}}{2 \times V^{annuity} \times \Delta\mu}$$

Modeling Longevity Risk

Stochastic Mortality Model

- Lee Carter Model

$$\ln m_{x,t} = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}$$

$$\text{s.t} \qquad \sum_x \beta_x = 1 \quad and \quad \sum_t \kappa_t = 0$$

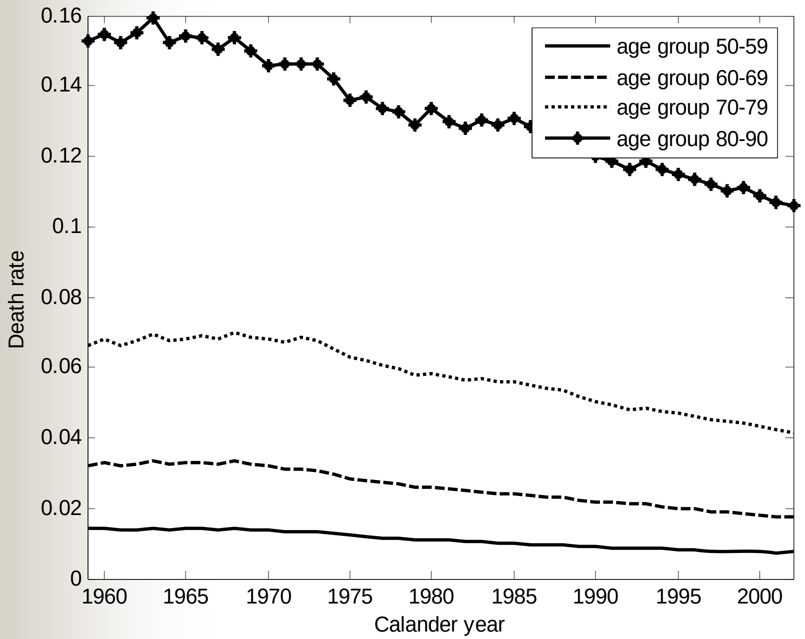
  - $m_{x,t}$: the central death rate for age x in year t
  - $\exp(\alpha_x)$: the general shape of the mortality schedule
  - β_x : rates decline rapidly and which slowly over time in response to change in κ_t.
  - κ_t : is a stochastic process

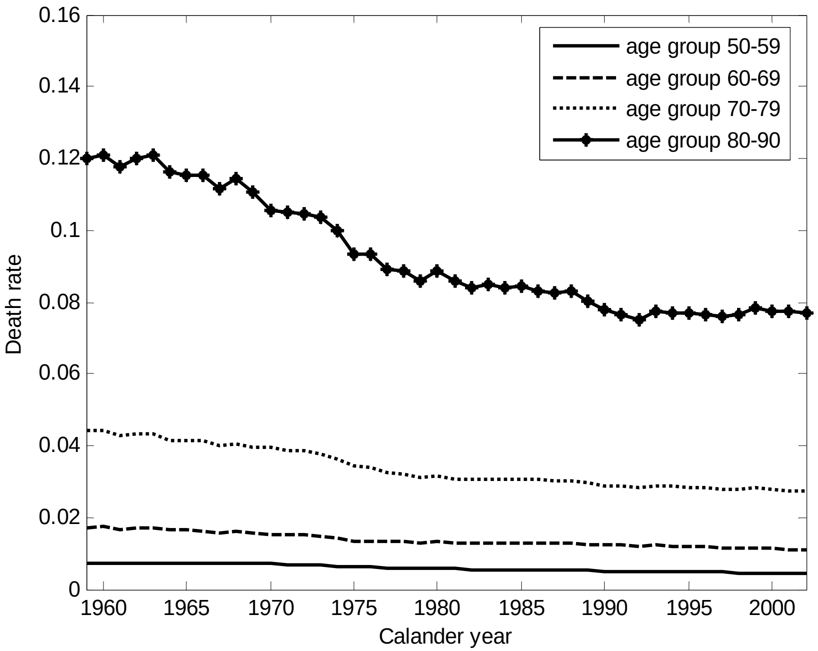
Practical Issues

- Mortality experience for life insurance product is different to that for annuity product.
- Mortality experience is different to countries.
- Model risk and parameter risk are important in dealing with natural hedging.(Melnikov and Romaniuk 2006; Koissi, Shapiro and Hognas 2006)

Mortality Investigation

- **US mortality experience** obtained from HMD data base.
- Data period: US aged 25–100 from 1959 to 2002
- Trend of Probabilities of Death for 10-Year Age Cohorts, (left: male; right: female)





Model Fitting

- SVD method (Estimation method has been discussed in
- Parameter Estimates of α_x and β_x in Lee-Carter model

	Female			Male	
age	α_x	β_x	age	α_x	β_x
25	-7.35118	0.00938	25	-6.36892	0.00712
26	-7.31421	0.00910	26	-6.36916	0.00613
27	-7.27519	0.00881	27	-6.35958	0.00576
28	-7.20874	0.00883	28	-6.32864	0.00532
⋮	⋮	⋮	⋮	⋮	⋮
98	-1.25084	0.00338	98	-1.14248	0.00188
99	-1.18752	0.00306	99	-1.09254	0.00163
100	-1.12742	0.00276	100	-1.04501	0.00141
101	-1.07054	0.00247	101	-0.99989	0.00120

Model Fitting-Con't

- Parameter Estimates of κ_t in Lee-Carter model

	Female	Male
Year t	κ_t	κ_t
1959	26.64288	19.19884
1960	27.19024	19.59866
1961	23.80692	16.70521
⋮	⋮	⋮
1999	-23.11967	-29.11703
2000	-24.34012	-30.69297
2001	-24.70004	-32.10778
2002	-25.53304	-32.83798

Model Fitting-Con't

- **Mean Absolute Percentage Error (MAPE)**

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{\left|X_t - \hat{X}_t\right|}{X_t}$$

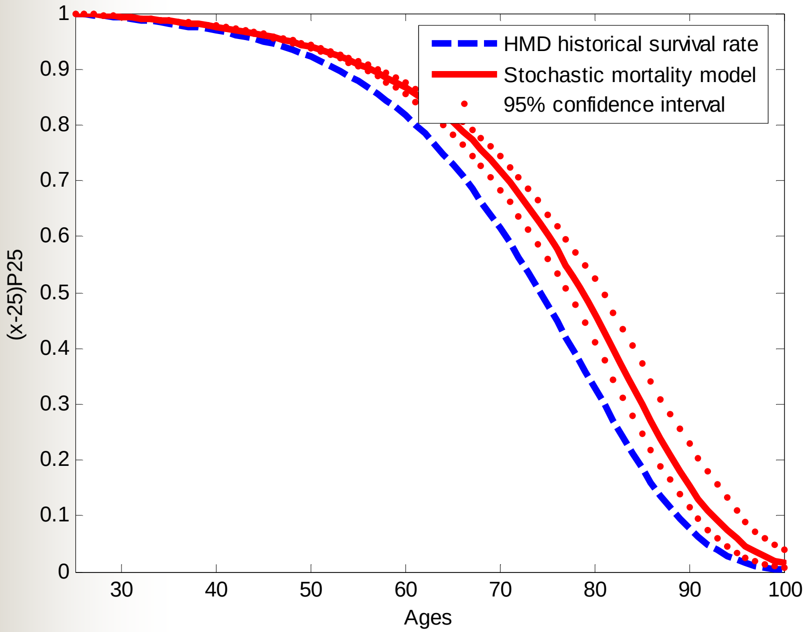
where $\hat{X}_t$: forecasted value

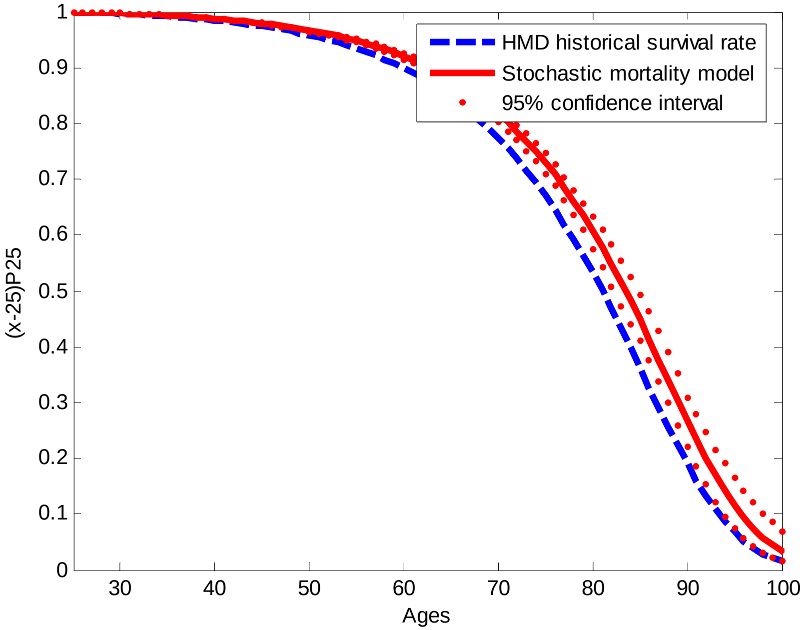
X_t : observed value

	Female	Male
MAPE	0.0682	0.0893

Forecasting Survival Probability

- Estimated Confidence Interval of Simulated Survival Rate (left: male, right: female)





Numerical Illustration

Assumptions of Numerical Settings

Product	Deferred Annuity	Life insurance
Age of insured	25	25
Gender	Female	Female
Coverage/payout benefit	US$10,000 (per year)	US$1,000,000
Coverage /payout benefit period (years)	Whole life	Whole life
Method of paying premium	Single	Single
Interest rate	4%	4%
Deferred period	30	None
Pricing mortality basis	HMD, 2002	HMD, 2002
Forecasted mortality basis	Stochastic mortality model	Stochastic mortality model

Insurance Premiums of illustrated insurance products with different mortality estimate

	Pricing Mortality Basis		Forecasted Mortality Basis by Lee-Carter (expected)		Forecasted Mortality with 10% shock (unexpected)	
Coverage /payout benefit period	30-year Deferred Annuity	Life	30-year Deferred Annuity	Life	30-year Deferred Annuity	Life
20-year term	37,026	12,836	37,043	11,959	37,495	10,772
30-year term	44,149	24,027	44,212	22,848	45,001	20,606
Whole life	46,749	129,328	46936	128,121	48,105	122,667

Optimal Product Mix Ratio

- The K-ratio implies that if an insurance company sells one unit of an annuity policy, it should sell K units of life insurance policy to achieve the hedging effect and immunize itself against longevity risk.

$$K = \frac{w^{life}}{w^{annuity}} \cdot \frac{P_{annuity}}{P_{life}} = -\frac{D_{e\mu}^{annuity}}{D_{e\mu}^{life}} \cdot \frac{P_{annuity}}{P_{life}}$$

Product Mix Proportion and K-Ratio:
(Women, single premium)

Coverage /payout benefit period	20-Year Term Life	Whole Life
20-year term annuity	**11.6%**	**22.9%**
	(0.380)	(0.085)
30-year term annuity	**16.0%**	**30.1%**
	(0.660)	(0.150)
Whole life annuity	**20.7%**	**37.1%**
	(0.950)	(0.210)

* In parentheses represent the K-ratios.

Product Mix Proportion and K-Ratio:

(Men, single premium)

(

Coverage /payout benefit period	20-Year Term Life	Whole Life
20-year term annuity	**16.0%**	**31.0%**
	(0.260)	(0.096)
30-year term annuity	**20.7%**	**38.1%**
	(0.420)	(0.150)
Whole life annuity	**24.1%**	**42.8%**
	(0.530)	(0.190)

* In parentheses represent the K-ratios.

Product Mix Proportion and K-Ratio: (Deferred Period)

Product Mix		Males	Females
Whole life	Whole life annuity (deferred 20 years)	**31.8%**	**26.8%**
		0.220	0.230
Whole life	Whole life annuity (deferred 30 years)	**42.8%**	**37.1%**
		0.190	0.210
Whole life	Whole life annuity (deferred 40 years)	**55.5%**	**49.5%**
		0.150	0.180

* In parentheses represent the K-ratios.

Product Mix Proportion and K-Ratio:

(Different issued Age)

Product Mix		Males	Females
Whole life	Whole life annuity (issued at age of 25)	**42.8%**	**37.1%**
		(0.190)	(0.210)
Whole life	Whole life annuity (issued at age of 35)	**56.8%**	**50.2%**
		(0.170)	(0.190)
Whole life	Whole life annuity (issued at age of 45)	**71.3%**	**64.7%**
		(0.130)	(0.150)

* In parentheses represent the K-ratios.

Product Mix Proportion and K-Ratio:

(Between Gender and Age)

Product Mix		Males	Females
Whole life (Male, 35)	Deferred life annuity (issued at age of 35, Attend age 65)	56.8%	49.8%
		(0.170)	(0.160)
Whole life (Male, 35)	Deferred life annuity (issued at age of 45, Attend age 65)	55.6%	48.9%
		(0.250)	(0.230)
Whole life (Male, 35)	Whole life annuity (issued at age of 55, Attend age 65)	44.5%	34.9%
		(0.528)	(0.467)
Whole life (Female, 35)	Deferred life annuity (issued at age of 35, Attend age 65)	57.3%	50.2%
		(0.210)	(0.190)
Whole life (Female 35)	Deferred life annuity (issued at age of 45, Attend age 65)	56.0%	49.3%
		(0.300)	(0.280)
Whole life (Female, 35)	Whole life annuity (issued at age of 55, Attend age 65)	44.5%	34.9%
		(0.532)	(0.472)

Product Mix Proportion and K-Ratio:

(with mortality shift 25%)

	20-Year Term Life	Difference in Mortality Curve Shift	Whole Life	Difference in Mortality Curve Shift
20-year term annuity	**11.6%**	**0%**	**22.7%**	**0%**
	(0.380)	0.076	(0.084)	-0.018
30-year term annuity	**16.0%**	**0%**	**29.9%**	**0%**
	(0.660)	0.185	(0.150)	-0.014
Whole life annuity	**20.9%**	**0%**	**37.1%**	**0%**
	(0.960)	0.395	(0.210)	0.018

* The proportions have little difference to 10%-shift, because of the convexity to the productions are not significant. We have a good approximation in linear hedging.

Conclusion and Further Research

- This paper investigate the optimal strategy for hedging the longevity risk of an annuity by using life insurance products.
- The proposed immunization model incorporates stochastic mortality dynamics to calculate an optimal product mix.
- The results strongly demonstrate that the proposed model can lead to an optimal product mix and effectively reduce longevity risks for life insurance companies.

Thank you for your attention.