PROCESSES CONTROL FOR TWO FAILURE MECHANISMS

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ABSTRACT

This paper considers the economic statistical process control for two dependent processes with two failure mechanisms which obey Weibull shock models and have increasing failure rates. We construct individual economic statistical X control chart to monitor the in-coming quality produced by the first process, and use the cause-selecting control chart to monitor the specific quality produced by the second process with minimal cost and required statistical properties. By using the proposed control charts, we can effectively and economically distinguish which process is out of control. The renewal theorem approach is extended to construct the cost model for our proposed control charts, and optimization method is used to determine the optimal design parameters of the proposed control charts. Finally, we give an example to show how to construct and apply the proposed control charts.

Keywords: processes, weibull distribution, control charts, renewal theorem

1. INTRODUCTION

Control charts were first proposed by Shewhart [18], and have become important tools for statistical quality control. Before using a control chart, we have to determine three design parameters: the sample size n, the length of the sampling interval h, and the control limit coefficient k. Duncan[5]first proposes the design of control chart from economic viewpoint. He derives a single assignable-cause economic model to determine the optimal design parameters of control chart. However, he assumes the occurrence of assignable cause may influence only the process mean, while the process variance maintains constant. Following Duncan's approach, Saniga[15] has developed the joint economic design of $\overline{X}$ and R charts, Saniga and Montgamery [17], Jones and Case [10], Rahim [12]. Rahim, Lashkari and Banerjee [13] discuss the joint economic design of $\overline{X}$ and S^2 control charts. Collani and Sheil [4] propose the economic design of S chart when the assignable cause affects only the process variance. All the above papers consider a single assignable cause only. However, in reality, there may have multiple assignable causes in the production process. Duncan [6] develops the economic design of $\overline{X}$ chart with double assignable causes. Tagaras and Lee [19] propose the economic design of control charts with different control limits for different assignable causes. Chung [3] proposes the approximate approach to determineing the optimal design parameters of attribute control charts with multiple assignable causes. Yang [24] presents the joint economic design of $\overline{X}$ and R charts with multiple assignable causes using Markov chain approach.

One disadvantage of the economic control chart is that it is difficult to estimate the process costs [7]. Taguchi et al. [20] indicate that a quadratic approximation function represents sufficiently the economic losses due to the deviation of quality characteristic from its target. Kacker [11] indicates that the concept of quadratic loss emphasizes the importance of continuous reducing performance variation. Various quality evaluation systems using the loss function are presented by Chen and Kapur [2]. The loss function as a rational approach to minimizing process variation has been widely accepted. Elsayed and Chen [7] present a new economic design of $\overline{X}$ chart based on Taguchi's loss function for the process with a single assignable cause and continuous operation. Yang [25] presents an economic design of joint $\overline{X}$ and S control charts using asymmetric quadratic loss function when the process has two assignable causes.

Woodall [22][23] suggests that in many economic designs the probability of Type I error of a control chart is much higher than that in a statistical design, and thus resulting in more false alarms than expected. A higher Type I error probability can also cause process over-adjustment, which leads to an increase in the variance of the distribution of the quality characteristic. Saniga [16] presents a method for improving the economic control charts by bounding Type I and Type II error probabilities and limiting the

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average time to signal (ATS) an expected shift. These improvements are in accord with industry's demand for low-process variability and long-term quality, and posses other advantages as well. The design maintains the effectiveness of economic designs, while minimizing probability of a false alarm and an incorrect adjustment. The design is called economic statistical design. Yang [26] presents the economic statistical design of S control charts using Taguchi's loss function.

In all the above studies, it is assumed that the time to occurrence of assignable cause is described by exponential distribution with constant hazard rate. This is the reason behind a constant sampling interval. However this assumption is not always appropriate for those processes, which deteriorate over time. For this reason, Hu [8] presents an economic design of chart but he presents a uniform sampling scheme when the time to occurrence of assignable cause follows a Weibull distribution. It is counterintuitive; hence, Banerjee and Rahim [1] propose a cost model in which the length of the sampling interval varies with time. They indicate that the cost of their model under non-uniform sampling intervals is lower than Hu's model.

Today, many industrial products are produced by several dependent processes, not just one process. Consequently, it is not appropriate to monitor these processes with a control chart for each individual process; what is needed is an appropriate method for controlling the processes. Zhang [27] proposes the simple cause-selecting chart to monitor the second process of the two dependent processes. Wade and Woodall [21] review the basic principles of the cause-selecting chart for two dependent processes and suggest a modification to the use of simple cause-selecting chart. They also examine the relationship between the simple cause-selecting chart and the multivariate T^2 chart. In their opinion, the simple cause-selecting control chart has some advantages over the T^2 chart. However, economic-statistical dependent processes control has not been addressed. In this paper, we consider two dependent processes with two failure mechanisms, which follow independent Weibull distributions with increasing failure rates. The occurrence of one failure mechanism would change the mean of in-coming quality produced by the first process, and the occurrence of another failure mechanism would change the mean of out-going quality produced by the second process. To effectively distinguish which process is out of control, we construct the individual economic statistical X control chart to monitor the in-coming quality, and the cause-selecting control chart to monitor the specific quality with minimal cost and required statistical properties. In Section 2, we describe the process behavior, and construct two control charts to monitor two dependent processes. In Section 3, we derive the cost model of the proposed control charts by the renewal theory approach. In Section 4, a numerical example is given to show the construction and application of the proposed control charts. Finally, we give some conclusions and suggestions.

2. THE PROCESS BEHAVIOR

In this paper, we consider two dependent processes, which may have two failure mechanisms. The times until the occurrence of the two independent failure mechanisms follow independent Weibull distributions with increasing failure rates. One of the failure mechanisms occurs only in the first process and causes the mean shift of the in-coming quality, while the other occurs only in the last process and causes the mean shift of the out-going quality. Two economic statistical control charts will be derived to effectively distinguish and monitor the process states of the two processes. Before describing how to derive the economic statistical control charts, the assumptions of the production process behavior are given as follows.

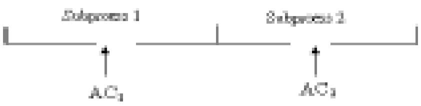
2.1 Assumptions and notation

Assumptions

(1) The production has two processes. The first process is called the subprocess 1 and the second process is called the subprocess 2. The subprocess 1 and the subprocess 2 are dependent. So the in-coming quality X produced by the subprocess 1 will affect the out-going quality Y produced by the subprocess 2. When the process is in control the distributions of X and Y are $N(\mu_{00x}, \sigma_x^2)$ and $N(\mu_{00y}, \sigma_y^2)$, respectively.

(2) Two failure mechanisms may occur in the two dependent processes. One failure mechanism (or assignable cause1), say AC_1, occurs only in the subprocess 1 and influences the mean shift of in-coming quality, while the other failure mechanism (assignable cause 2), say AC_2 , occurs only in the subprocess 2 and influences the mean shift of out-going quality (see Figure 1). When the process is influenced by AC_1 the distributions of X and Y are $N(\mu_{10x}, \sigma_x^2)$ and $N(\mu_{10y}, \sigma_y^2)$, respectively, where $\mu_{10x} = \mu_{00x} + \delta_{10}\sigma_x$, $\delta_{10} \neq 0$, and $\mu_{10y} \neq \mu_{00y}$. When the process is influenced by AC_2 the distributions of X and Y are $N(\mu_{00x}, \sigma_x^2)$ and $N(\mu_{01y}, \sigma_y^2)$, respectively, where $\mu_{01y} = \mu_{00y} + \delta_{01}\sigma_y$. When the process is influenced by AC_1 and AC_2 the distributions of

X and Y are $N(\mu_{10x}, \sigma_x^2)$ and $N(\mu_{11y}, \sigma_y^2)$, respectively, where $\mu_{11y} \neq \mu_{01y} \neq \mu_{00y}$.



Figue1. Two processes

(3) The times (T_{AC_1}, T_{AC_2}) until the occurrence of failure mechanisms follow two independent Weibull distributions. Their probability density functions are, respectively, given by

$$f(t_1) = \lambda_1 \theta \ t_1^{(\theta-1)} \exp\left\{-\lambda_1 t_1^{\theta}\right\}, \ t_1 > 0 \ , \theta \geq 1 \ , \ \lambda_1 > 0 \tag{2.1}$$

$$f(t_2) = \lambda_2 \theta \ t_2^{(\theta-1)} \exp\left\{-\lambda_2 t_2^{\theta}\right\}, \ t_2 > 0 \ , \theta \geq 1 \ , \ \lambda_2 > 0 \tag{2.2}$$

(4) Two processes are monitored by drawing a random sample with one paired observation (X,Y) from the end of the suprocess 2 at time h_1, (h_1+h_2), $(h_1+h_2+h_3)$, ⋯⋯, for keeping the probability of a shift in an interval constant for all sampling interval, where

$$h_i = \left[i^{1/\theta} - (i-1)^{1/\theta} \right] h_1 \ , \ i = 2,3,\cdots \tag{2.3}$$

(5) The time to sampling and charting one item is negligible.
(6) Production ceases during the search and adjust any failure mechanism.
(7) We assume that the process is composed of independent and identical cycles [5](Figure2).

Figure2. The process denited by cycles

A cycle length is composed of in control time, out of control time and search and adjustment time until the next starting cycle (see Figure 3).

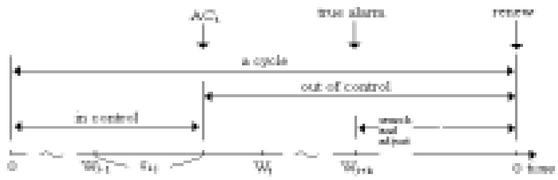


Figure 3. a cycle

The notation used is described as follows.

Process parameters:

$\sigma_{y|x}$ = the standard deviation of the conditional distribution $Y|X$.

$\delta_{10}\,\sigma_x$ = the mean shift scale for in-coming quality, when AC1 occurs in the subprocess 1.

$\delta_{01}\,\sigma_{y|x}$ = the mean shift scale for out-going quality, when AC2 occurs in the subprocess 2.

M = the quantity of output per unit production time.

Random variables:

X_{00i} = the i^{th} observation of the in-coming quality when the subprocess 1 is in control, where i = 1, 2,⋯.

X_{10i} = the i^{th} observation of the in-coming quality when the AC_1 occurs in the subprocess 1, where i = 1, 2,⋯.

X_{01i} = the i^{th} observation of the in-coming quality when the AC_2 occurs in the subprocess 1, where i = 1, 2,⋯.

X_{11i} = the i^{th} observation of the in-coming quality when the AC_1 occurs in the subprocess 1 and AC_2 occurs in the subprocess 2, where i = 1, 2,⋯.

Y_{00i} = the i^{th} observation of the out-going quality when the process is in control, where i = 1, 2,⋯.

Y_{10i} = the i^{th} observation of the out-going quality when the AC_1 occurs in the subprocess 1, where i = 1, 2,⋯.

Y_{01i} = the i^{th} observation of the out-going quality when the AC_2 occurs in the subprocess 2, where i = 1, 2,⋯.

Y_{11i} = the i^{th} observation of the out-going quality when the AC_1 occurs in the subprocess 1 and AC_2 occurs in the subprocess 2, where i = 1, 2,⋯.

Time parameters:

W_j = the time until we take the j^{th} sample, W_0=0, $W_j = \sum_{i=1}^{j} h_i$, j =1,2,....

$E(T)$ = the expected cycle length

$E(T_j)$ = the expected residual length in the cycle beyond time W_j given that the process is in control at time W_j, where j=1,2,….

τ_{ij} = the expected time until the occurrence of AC_i, given that it occurs in time interval (W_{j-1}, W_j), where i=1,2, j = 1, 2,….

$$\tau_{ij} = E\left(T_{AC_i} - W_{j-1} \middle| W_{j-1} < T_{AC_i} < W_j\right) \tag{2.4}$$

$\tau_{(i)j}$ = the expected time until the occurrence of the i[th] arrived assignable cause given that both AC_1 and the AC_2 occur in time interval (W_{j-1}, W_j), where j = 1, 2,….

$$\tau_{(i)j} = E\left(T_{AC_{(i)}} - W_{j-1} \middle| W_{j-1} < T_{AC_1}, T_{AC_2} < W_j\right),\ i=1,2\ldots \tag{2.5}$$

Ta = target value.

T_f = the expected search time for at least one false alarm.

T_{sr} = the expected time to search and adjust AC_i or both, i = 1, 2…

Cost parameters:

A_i = the coefficient of asymmetric loss function, where i = 0 when outgoing quality Y is smaller than Ta, and i=1 when Y is greater than Ta.

$E(C)$ = the expected cycle cost.

$E(C_j)$ = the expected residual cost in the cycle beyond time W_j given that the process is in control at time W_j, j=1,2,…

D_{00} = the expected loss per unit product when the process is in control.

D_{10} = the expected loss per unit product when only AC_1 occurs in the subprocess 1.

D_{01} = the expected loss per unit product when only AC_2 occurs in the subprocess 2.

D_{11} = the expected loss per unit product when both AC_1 and AC_2 occur in the processes.

$D^{(j)}$ = the expected loss per unit product when it is produced in time interval ($\tau_{(1)j}$, $\tau_{(2)j}$), where j= 1, 2, ⋯.

D = $D^{(j)}$, since $D^{(j)}$ is independent of j.

b = the sample cost.

C_f = the search cost for at least one false alarm.

C_{sr} = the cost to search and adjust the AC_i or both, i = 1, 2.

Probability notation:

P_{ij} = the probability that the AC_i occurs between times W_{j-1} and W_j, given that the process is in control before time W_{j-1}, where i = 1, 2, j=1,2,3,….

$$P_{ij} = P\left(T_{AC_i} < W_j \middle| W_{j-1} < T_{AC_i}\right) = 1 - \exp\left(-\lambda_i h_1^{\theta}\right) \tag{2.6}$$

Since P_{ij} is independent of time, hence we let $P_{ij} = P_i$.

α : the probability that there is at least one false alarm released by the two control charts given that both subprocesses are in control.

β_{10} : The probability that there are no alarms released by the two charts given that the AC_1 occurs in the subprocess 1.

β_{01} : The probability that there are no alarms released by the charts given that AC_2 occurs in the subprocess 2.

β_{11} : The probability that there are no alarms released by the charts given that AC_1 occurs in the subprocess 1and AC_2 occurs in the subprocess2.

2.2 The possible distributions of *x* and *y*

When we draw a random sample of size one from the end of the subprocess 2 at sampling time W_j (j = 1, 2,⋯), we can get a pair of observations (x_{mni}, y_{mni}), m ,n = 1or 0. Since X_{mni} affects Y_{mni}, the model relating the two variables can take many forms. One of the most useful models is the simple linear regression model:

$E[Y_{mni} | X_{mni}] = a_0 + a_1 X_{mni}$, where a_0 *and* a_1 are constants. (2.7)

However, the model needs not be linear for application of our proposed approach. The possible distributions of X_{mni} , $Y_{mni} | X_{mni}$ *and* Y_{mni} for in-control and out-of-control processes are illustrated in the following (see Table 1).

2.3 The construction of individual x control chart and cause-selecting control chart

In order to distinguish if AC2 or/and AC1 occurs/occur in the subprocess 2 or/and the subprocess 1, we construct two control charts to monitor the two dependent processes. The individual X control chart is constructed to monitor the subprocess 1 based on the in-control distribution of in-coming quality. To monitor the subprocess 2, it is incorrect to construct the control chart based on the distribution of out-going quality since out-going quality is affected by in-coming quality. The proposed approach is to monitor the specific quality in the subprocess 2 by adjusting the effect of Y from X; that is the specific quality is presented by the cause-selecting value ($Z_{mni} = \frac{(Y_{mni} \mid X_{mni}) - \mu_{mni}}{\sigma_{y|x}}$, m, n = 0 or 1). When the specific quality is in control, the cause-selecting control chart is constructed by the in-control distribution of specific quality. The distribution of in-control in-coming quality is $X_{00i} \sim N(\mu_{00x}, \sigma_x^2)$, so the individual X chart has upper control limit (UCL) = ($\mu_{00x} + k_1\sigma_x$), central line (CL) = μ_{00x}, and lower control limit (LCL) = ($\mu_{00x} - k_1\sigma_x$) (Figure4)

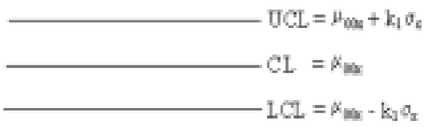


Figure 4. The Individual X Chart

The distribution of in-control specific quality or the cause-selecting random variable is a standardized normal distribution, Z_{00i} ~N(0,1), so the cause-selecting control chart has UCL = k_2, CL = 0, and LCL=-k_2 (Figure5).

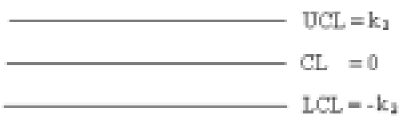


Figure 5. The Cause-selecting Chart

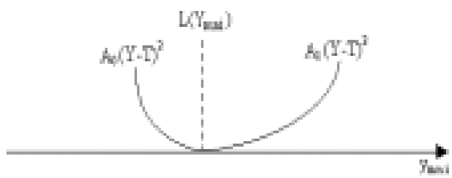
Once the optimal design parameters (h_1, k_1 and k_2) of the individual X chart and the cause-selecting chart are determined, the proposed control charts can be used to effectively monitor the two processes with minimal cost. That is, after each sampling and testing, we chart the value of X on the individual X chart and chart the value of Z on the cause-selecting chart. When the value of X falls outside the control limits of the individual X chart, it indicates that AC_1 occurs in the subprocess 1. When the value of Z falls outside the control limits of the cause-selecting chart, it indicates that AC_2 occurs in the subprocess 2. The process engineer should take action to search and adjust the AC_i, i = 1,2. After the adjustment of the process, the process would be renewed. When both the value of X and the value of Z fall inside the control limits of the individual X chart and the cause-selecting chart, respectively, it indicates that the process is in control, no action is taken and the process continues.

3. THE COST MODEL

Since the design parameters (h_1, k_1 and k_2) of the proposed control charts influence the process cost, so we have to derive the process cost model before determining the optimal design parameters. In this section, we proposed the renewal theory approach to obtain the process cost model, which is the ratio of the expected cycle cost and the expected cycle length. Consequently, the optimal design parameters of the proposed charts can be obtained by minimizing the cost model, and the economic-statistical design of the individual X chart and the cause-selecting chart can be constructed.

3.1 Asymmetric loss function

Conventionally, quality loss is considered as the cost incurred when the quality characteristic is not within the specification limits. A symmetric quadratic approximation loss function sufficiently representing economic losses due to the deviation of quality characteristic from its target is indicated by Taguchi et al.[20]. However, a symmetric quadratic approximation loss function is always not true in reality. A more appropriate asymmetric loss function is considered to evaluate the quality cost of the process in this paper. The asymmetric loss function $L(Y)$ used on the distribution of out-going quality Y is described as follows (Figure 6 and equation (3.1)).



$T_a = \mu_{00y}$

Figure 6. The Asymmetric Loss Function

$$L(Y_{mni}) = \begin{cases} A_1\left[Y_{mni} - E(Y_{00i})\right]^2, & if \quad Y_{mni} \ge Ta \\ A_0\left[Y_{mni} - E(Y_{00i})\right]^2, & if \quad Y_{mni} < Ta \end{cases},$$

$$m = 0,1;\ n = 0,1;\ i = 1,2,\cdots \tag{3.1}$$

The process mean of the out-going quality is assumed to be the target value when the process is in control. The expected loss per unit product when the process is in control or out of control is illustrated as follows.

(1) The in-control expected loss per unit product:

$$D_{00} = E\left(L(Y_{00i})\right) = E\left(A_j\left(Y_{00i} - E(Y_{00i})\right)^2\right) = [(A_0 + A_1)/2]\sigma_y^2 \tag{3.2}$$

(2) The expected loss per unit product, when AC_1 occurs in the subprocess 1:

$$\begin{aligned} D_{10} &= E(L(Y_{10i})) = E(A_j(Y_{10i} - E(Y_{00i}))^2) \\ &= \{A_0\Phi(-a_1\delta_{10}\sigma_x / \sigma_y) + A_1[1-\Phi(-a_1\delta_{10}\sigma_x / \sigma_y)]\} \\ &\times\ [\sigma_y^2 + (a_1\delta_{10}\sigma_x)^2] \end{aligned} \tag{3.3}$$

(3) The expected loss per unit product, when AC_2 occurs in the subprocess 2:

$$\begin{aligned} D_{01} &= E(L(Y_{01i})) = E(A_j(Y_{01i} - E(Y_{00i}))^2) \\ &= \{A_0\Phi(-\delta_{01}\sigma_{y|x} / \sigma_y) + A_1[1-\Phi(-\delta_{01}\sigma_{y|x} / \sigma_y)]\} \\ &\times\ [\sigma_y^2 + (\delta_{01}\sigma_{y|x})^2] \end{aligned} \tag{3.4}$$

(4) The expected loss per unit product, when AC_1 occurs in the subprocess 1 and AC_2 occurs in the subprocess 2:

$$\begin{aligned} D_{11} &= E[L(Y_{11i})] = E[A_j(Y_{11i} - E(Y_{00i}))^2] \\ &= \{A_0\Phi((-a_1\delta_{10}\sigma_x - \delta_{01}\sigma_{y|x}) / \sigma_y) \\ &+\ A_1[1-\Phi((-a_1\delta_{10}\sigma_x - \delta_{01}\sigma_{y|x}) / \sigma_y)]\} \\ &\times\ [\sigma_y^2 + (a_1\delta_{10}\sigma_x + \delta_{01}\sigma_{y|x})^2] \end{aligned} \tag{3.5}$$

(5) The expected loss per unit product, when product is produced from time interval ($\tau_{(1)j}$ and $\tau_{(2)j}$), j=1,2,…,

$$D^{(j)} = \frac{\left(\frac{D_{10}\lambda_1 + D_{01}\lambda_2}{\lambda_1+\lambda_2}\right) - [D_{10}(1-P_2) + D_{01}(1-P_1)] + \left(\frac{D_{10}\lambda_2 + D_{01}\lambda_1}{\lambda_1+\lambda_2}\right)(1-P_1)(1-P_2)}{P_1P_2}$$

Since $D^{(j)}$ is independent of j, so we let $D^{(j)} = D$. (3.6)

3.2 Expected cycle length

A renewal theory approach is used to derive an expression for the expected cycle length *E(T)*. We divide the cycle into the following three components: (1) the in-control period; (2) the time to obtain a true alarm given that the process is out of control; and (3) the time to search for and adjust the assignable causes. Before deriving the expected cycle length, we have to study the possible states of the process at the end of the first sampling and testing time. Depending on the process states, the probability, the expected residual cycle length (the expected length in the cycle beyond the first sampling and testing time) and the expected residual cycle cost (the expected cost in the cycle beyond the first sampling and testing time) for each process state are computed. These values lead us to formulate the renewal equation. There are eight possible process states, which are defined as follows (see Table 2).

State 1 : The production is in control, and there are no false alarms released by the two charts.
State 2 : The production is in control, but there is at least one false alarm released by the two charts.
State 3 : AC_1 occurs in the subprocess 1, but there are no alarms released by the two charts.
State 4 : AC_1 occurs in the subprocess 1, and there is at least one alarm released by the two charts.
State 5 : AC_2 occurs in the subprocess 2, but there are no alarms released by the two charts.
State 6 : AC_2 occurs in the subprocess 2, and there is at least one alarm released by the two charts.
State 7 : Both AC_1 and AC_2 occur in the process, but there are no true alarms released by the two charts.
State 8 : Both AC_1 and AC_2 occur in the process, and there is at least one true alarm released by the two charts.

Table 3 displays the expected residual cycle length associated with the probability of being in each respective state at the end of the first sampling and testing time (W_1). Hence, the renewal equation for *E(T)* is

$$E(T) = h_1 + Pr_1 E(T_1) + Pr_2\left[T_f + E(T_1)\right] + \sum_{i=3}^{8} Pr_i R_i$$

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$$E(T) = h_1 + Pr_1 E(T_1) + Pr_2\left[T_f + E(T_1)\right] + \sum_{i=3}^{8} Pr_i R_i$$

Simplifying this, we get

$$\begin{aligned} E(T) &= h_1 + \alpha T_f(1-P_1)(1-P_2) + T_{sr}(P_1 + P_2 - P_1P_2) + P_1(1-P_2)\left(\sum_{i=1}^{\infty} h_{i+1}\beta_{10}{}^{i}\right) \\ &+ (1-P_1)P_2\left(\sum_{i=1}^{\infty} h_{i+1}\beta_{01}{}^{i}\right) + P_1P_2\left(\sum_{i=1}^{\infty} h_{i+1}\beta_{11}{}^{i}\right) + (1-P_1)(1-P_2)E(T_1) \end{aligned} \tag{3.7}$$

To derive *E(T₁)*, the same approach is used.

The expected cycle length is obtained by solving a recursive system (see Appendix 1 for the proof), that is

$$E(T) = h_1 \sum_{i=1}^{\infty}\left[i^{1/\theta} - (i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1} + \alpha T_f \sum_{i=1}^{\infty}(1-P_1)^{i}(1-P_2)^{i}$$

$$+h_1\left(\frac{P_1}{1-P_1}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{10}^i\right\}$$

$$+h_1\left(\frac{P_2}{1-P_2}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{01}^i\right\}$$

$$+h_1\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{11}^i\right\}$$

$$+T_{sr}(P_1+P_2-P_1P_2)\sum_{i=1}^{\infty}(1-P_1)^{i-1}(1-P_2)^{i-1} \qquad (3.8)$$

Note that when all parameters are given, we can calculate *E(T)*. Since there are six infinite series

$$\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1},$$

$$\sum_{i=1}^{\infty}(1-P_1)^i(1-P_2)^i,$$

$$\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{10}^i,$$

$$\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{01}^i,$$

$$\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{11}^i,$$

and $\sum_{i=1}^{\infty}(1-P_1)^{i-1}(1-P_2)^{i-1}$ in *E(T)*, so we have to calculate their approximate values. The algorithm used to calculate the infinite series, is described as follows.

Step 1: The values of P_1, P_2, θ, and β_{10} are known. Set the value of tolerance TOL=10^{-5} and the sum of the infinite series is SUM.

Step 2: Set *j*=1, SUM1=0, SUM2=0.

Step 3: Set *i*=1,

$$SUM=(1-P_1)^j(1-P_2)^j\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{10}^i$$

Step 4: Set *i*=*i*+1,

$$SUM=SUM+(1-P_1)^j(1-P_2)^j\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{10}^i$$

Step 5: If ∣SUM-SUM1∣< TOL, then go to Step 6. Otherwise set SUM1=SUM, and go to Step 4.

Step 6 : If ∣SUM-SUM2∣< TOL, then print out SUM and stop calculating. Otherwise, set SUM2=SUM, *j*=*j*+1, and go to Step 3.

The approximate values of other infinite series can be obtained using a similar approach.

3.3 Expected cycle length cost

The approach to derive the expected cycle cost is similar to derive the expected cycle length. To obtain an expression for the expected cycle cost *E(C)*, we divide the cycle cost into the following two components: (1) the cost incurred in the first sampling and testing time, and (2) the expected residual cost beyond time W_1 given that the process is in control at time W_1. We present the possible states of the system, their costs incurred in the first sampling and testing interval, and the expected residual costs in Table 4. Hence, the renewal equation is

$$E(C)=Pr_1\left[\ (b+D_{00}Mh_1)\ +E(C_1)\ \right]+ Pr_2\left[(b+D_{00}Mh_1)+C_f+E(C_1)\right]+\sum_{i=3}^{8}Pr_i R_i^{'}$$

Simplifying this, we get

$$E(C)=b+(1-P_1)(1-P_2)\alpha C_f+P_1(1-P_2)\left(\frac{b\beta_{10}}{1-\beta_{10}}\right)$$
$$+P_1(1-P_2)D_{10}M\left(\sum_{i=1}^{\infty}h_{i+1}\beta_{10}^i\right)$$
$$+P_1(1-P_2)M\tau_{11}(D_{00}-D_{10})+(P_1+P_2-P_1P_2)C_{sr}$$
$$+(1-P_1)P_2\left(\frac{b\beta_{01}}{1-\beta_{01}}\right)$$
$$+(1-P_1)P_2D_{01}M\left(\sum_{i=1}^{\infty}h_{i+1}\beta_{01}^i\right)$$
$$+(1-P_1)P_2M\tau_{21}(D_{00}-D_{01})+P_1P_2\left(\frac{b\beta_{11}}{1-\beta_{11}}\right)$$
$$+P_1P_2D_{11}M\left(\sum_{i=1}^{\infty}h_{i+1}\beta_{11}^i\right)+P_1P_2M\tau_{(1)1}(D_{00}-D)$$
$$+P_1P_2M\tau_{(2)1}(D-D_{11})+D_{00}h_1M$$
$$+(D_{10}-D_{00})h_1MP_1+(D_{01}-D_{00})h_1MP_2$$
$$+(D_{00}+D_{11}-D_{10}-D_{01})h_1MP_1P_2$$
$$+(1-P_1)(1-P_2)E(C_1)$$

(the derivation of D, see Appendix 2)

(3.9)

We provide a set of recursive systems in the forms of *E(C)*, $E(C_1)$, $E(C_2)$, …..,and so on. The system can be solved to obtain an expression for *E(C)* (see Appendix 3 for the proof).

$$E(C)=\left(\frac{b}{P_1+P_2-P_1P_2}\right)+\alpha C_f\left(\frac{(1-P_1)(1-P_2)}{P_1+P_2-P_1P_2}\right)$$
$$+\left(\frac{b\beta_{10}}{1-\beta_{10}}\right)\left(\frac{P_1(1-P_2)}{P_1+P_2-P_1P_2}\right)+C_{sr}$$
$$+D_{10}h_1M\left(\frac{P_1}{1-P_1}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{10}^i\right\}$$
$$+(D_{00}-D_{10})M\left(\frac{P_1}{1-P_1}\right)\sum_{i=1}^{\infty}(1-P_1)^j(1-P_2)^j\tau_{1i}$$
$$+\left(\frac{b\beta_{01}}{1-\beta_{01}}\right)\left(\frac{P_2(1-P_1)}{P_1+P_2-P_1P_2}\right)$$

$$+D_{01}h_1M\left(\frac{P_2}{1-P_2}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{01}^i\right\}$$
$$+(D_{00}-D_{01})M\left(\frac{P_2}{1-P_2}\right)\sum_{i=1}^{\infty}(1-P_1)^j(1-P_2)^i\tau_{2i}+\left(\frac{b\beta_{11}}{1-\beta_{11}}\right)\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)$$
$$+D_{11}h_1M\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)\bullet$$
$$\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{11}^i\right\}$$
$$+(D_{00}-M\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)\sum_{i=1}^{\infty}(1-P_1)^i(1-P_2)^i\tau_{(1)i}$$
$$+(D-D_{11}\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)\sum_{i=1}^{\infty}(1-P_1)^i(1-P_2)^i\tau_{(2)i}$$
$$+D_{00}h_1M\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$
$$+(D_{10}-D_{00})h_1MP_1\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$
$$+(D_{01}-D_{00})h_1MP_2\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$
$$+(D_{00}+D_{11}-D_{10}-D_{01})h_1MP_1P_2\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right]\bullet$$
$$(1-P_1)^{i-1}(1-P_2)^{i-1} \tag{3.10}$$

In the expression of $E(C)$, there are five infinite series. We can obtain the approximate values of these infinite series by using an algorithm described in Section 3.2.

3.4 Determination of the optimal design parameters for the proposed control charts

The approximate expected cost per unit time in the long run can be expressed as the ratio of the expected cycle cost and the expected cycle length with probability 1 [14]. The expected cost per unit production time is a function of the design parameters h_1, k_1 and k_2. By minimizing the objective function, the optimal design parameters can be determined by using an optimization technique. That is the objective function =

$$\frac{E(C)}{E(T)}=\frac{right\ \ side\ \ of\ \ (3.10)}{right\ \ side\ \ of\ \ (3.8)} \tag{3.11}$$

For the proposed control charts to have the desired statistical properties, we let the upper bounds of α, β_{10}, β_{01}, and β_{11} be α_U, β_{10U}, β_{01U} and β_{11U} respectively. The upper bounds of h_1, k_1 and k_2 are set to be 8 hours, 4 and 4, respectively. Hence, the optimization model is expressed as follows (see equation (3.12)).

Minimize the objective function $=\frac{E(C)}{E(T)}$

Subject to $0<h_1\le 8$, $0<k_1\le 4$, $0<k_2\le 4$,
$0<\alpha\le\alpha_U$, $0<\beta_{10}\le\beta_{10U}$
$0<\beta_{01}\le\beta_{01U}$, $0<\beta_{11}\le\beta_{11U}$

(3.12)

4. A NUMERICAL EXAMPLE

In this section, we give an example to illustrate how the proposed method can be used to solve the real process control problem.

Assume that a factory produces cotton yarn in two dependent processes. The subprocess 1 produces fiber length (in-coming quality *X*), while the subprocess 2 produces skein strength (out-going quality *Y*). The process model is the same as described in Section 2, and the relationship between *X* and *Y* when the process is in control is known as $E\left[Y_{00i}|X_{00i}\right]=11+1.1X_{00i}$. The distributions of X_{00i}, $Y_{00i}|X_{00i}$, and Y_{00i} are illustrated as follows, using history in-control data.

$$X_{00i}\sim N(77.05,\ 4.8^2) \tag{4.1}$$

$$Y_{00i}\mid X_{00i}\sim N(11+1.1X_{00i},\ 6.01^2) \tag{4.2}$$

$$Y_{00i}\sim N(95.755,\ 8^2) \tag{4.3}$$

Two out-of-control machines, say machine 1 and machine 2, may shift the process means of the subprocess 1 and subprocess 2, respectively. That is, it would cause a change in the mean of X_{ooi} to be 91.45, and the mean of $Y_{00i}\mid X_{00i}$ to be 29.03+1.1 X_{00i}. The time until machine *i* out of control has increasing failure rate and follows a Weibull distribution with parameters (λ_1=0.002, λ_2=0.001, θ=3), *i*=1, 2.

To effectively monitor the two dependent process steps with minimal cost and required statistical property, two economic-statistical control charts should be constructed. To determine the two economic-statistical control charts, the set of process and cost parameters are estimated as λ_1=0.002 λ_2=0.001,*b*=$25per sample, C_f=$400 per hour, Csr=$1000 per hour, $\theta=3$, $\delta_{10}=3$, $\delta_{01}=3$, A_0=1.2 per unit product $T_f=0.1$ hours, $T_{sr}=0.4$ hours The

Table1. The Possible Distributions of In-coming Quality (X_{mni}) and Out-going Quality (Y_{mni})

Process state	Distribution of in-coming quality	Conditional distribution of out-going quality	Distribution of out-going quality
in control processes	$X_{00i} \sim N(\mu_{00x}, \sigma_x^2)$	$Y_{00i} \vert X_{00i} \sim N(a_0 + a_1 X_{00i}, \sigma_{y\vert x}^2)$ $= N(\mu_{00i}, \sigma_{y\vert x}^2)$	$Y_{00i} \sim N(a_0 + a_1 \mu_{00x}, a_1^2 \sigma_x^2 + \sigma$ $= N(\mu_{00y}, \sigma_y^2)$
out-of-control subprocess 1	$X_{10i} \sim N(\mu_{00x} + \delta_{10}\sigma_x, \sigma_x^2)$ $= N(\mu_{10x}, \sigma_x^2)$	$Y_{10i} \vert X_{10i} \sim N(a_0 + a_1 X_{10i}, \sigma_{y\vert x}^2)$ $= N(\mu_{10i}, \sigma_{y\vert x}^2)$	$Y_{10i} \sim N(a_0 + a_1 \mu_{10x}, a_1^2 \sigma_x^2 + \sigma$ $= N(\mu_{10y}, \sigma_y^2)$
out-of-control subprocess 2	$X_{01i} = X_{00i} \sim N(\mu_{00x}, \sigma_x^2)$	$Y_{01i} \vert X_{01i} \sim N(\mu_{00i} + \delta_{01}\sigma_{y\vert x}, \sigma_{y\vert x}^2)$ $= N(\mu_{01i}, \sigma_{y\vert x}^2)$	$Y_{01i} \sim N(a_0 + a_1\mu_{00x} + \delta_{01}\sigma_{y\vert x}, a_1^2\sigma_x^2 + \sigma_{y\vert x}^2)$ $= N(\mu_{01y}, \sigma_y^2)$
two out-of-control subprocesses	$X_{11i} = X_{10i} \sim N(\mu_{10x}, \sigma_x^2)$	$Y_{11i} \vert X_{11i} \sim N(\mu_{10i} + \delta_{01}\sigma_{y\vert x}, \sigma_{y\vert x}^2)$ $= N(\mu_{11i}, \sigma_{y\vert x}^2)$	$Y_{11i} \sim N(a_0 + a_1\mu_{10x} + \delta_{01}\sigma_{y\vert x}, a_1^2\sigma$ $= N(\mu_{11y}, \sigma_y^2)$

Table 2. Definition of the Eight Process States

State	Subprocess 1 in control ?	Subprocess 2 in control ?	At least one alarm for the two charts?
1	Yes	Yes	No
2	Yes	Yes	Yes
3	No	Yes	No
4	No	Yes	Yes
5	Yes	No	No
6	Yes	No	Yes
7	No	No	No
8	No	No	Yes

Table 3. Probability and Expected Residual Cycle Length for Each State at Time W_1

State	Probability	Expected residual cycle length
1	$Pr_1 = (1-P_1)(1-P_2)(1-\alpha)$	$R_1 = E(T_1)$
2	$Pr_2 = (1-P_1)(1-P_2)\alpha$	$R_2 = T_f + E(T_1)$
3	$Pr_3 = P_1(1-P_2)\beta_{10}$	$R_3 = (1-\beta_{10})\sum_{i=1}^{\infty}(w_{i+1}-h_1)\beta_{10}^{i-1} + T_{sr}$
4	$Pr_4 = P_1(1-P_2)(1-\beta_{10})$	$R_4 = T_{sr}$
5	$Pr_5 = (1-P_1)P_2\beta_{01}$	$R_5 = (1-\beta_{01})\sum_{i=1}^{\infty}(w_{i+1}-h_1)\beta_{01}^{i-1} + T_{sr}$
6	$Pr_6 = (1-P_1)P_2(1-\beta_{01})$	$R_6 = T_{sr}$
7	$Pr_7 = P_1P_2\beta_{11}$	$R_7 = (1-\beta_{11})\sum_{i=1}^{\infty}(w_{i+1}-h_1)\beta_{11}^{i-1} + T_{sr}$
8	$Pr_8 = P_1P_2(1-\beta_{11})$	$R_8 = T_{sr}$

Table 4. Cost in the First Sampling and Testing Interval and Expected Residual Cost beyond time W_1

State	Cost in the first sampling and testing + Expected residual costand testing
1	$R_1^{'} = b + D_{00}Mh_1 + E(C_1)$
2	$R_2^{'} = b + D_{00}Mh_1 + C_f + E(C_1)$
3	$R_3^{'} = b + D_{00}M\tau_{11} + D_{10}M(h_1 - \tau_{11}) + \frac{b}{(1-\beta_{.10})} + D_{10}M(1-\beta_{10})\sum_{i=1}^{\infty}(w_{i+1} - h_1)\beta_{10}^{\ i-1} + C_{sr}$
4	$R_4^{'} = b + D_{00}M\tau_{11} + D_{10}M(h_1 - \tau_{11}) + C_{sr}$
5	$R_5^{'} = b + D_{00}M\tau_{21} + D_{01}M(h_1 - \tau_{21}) + \frac{b}{(1-\beta_{.01})} + D_{01}M(1-\beta_{01})\sum_{i=1}^{\infty}(w_{i+1} - h_1)\beta_{01}^{\ i-1} + C_{sr}$
6	$R_6^{'} = b + D_{00}M\tau_{21} + D_{01}M(h_1 - \tau_{21}) + C_{sr}$
7	$R_7^{'} = b + D_{00}M\tau_{(1)1} + DM(\tau_{(2)1} - \tau_{(1)1}) + D_{11}M(h_1 - \tau_{(2)1}) + \frac{b}{(1-\beta_{.11})} + D_{11}M(1-\beta_{11})\sum_{i=1}^{\infty}(w_{i+1} - h_1)\beta_{11}^{\ i-1} + C_{sr}$
8	$R_8^{'} = b + D_{00}M\tau_{(1)1} + DM(\tau_{(2)1} - \tau_{(1)1}) + D_{11}M(h_1 - \tau_{(2)1}) + C_{sr}$

Figure 7. The Economic-statistical Individual X Chart

Figure 8. The Economic-statistical Cause-selecting Chart

optimal design parameters are determined by the Fortran program, and IMSL subroutine (**dnconf**) [9] with constraints $\alpha_U = 0.1$, $\beta_{10U} = 0.3$, $\beta_{01U} = 0.3$ and $\beta_{11U} = 0.3$. Hence $(h_1, k_1, k_2) =$ (1.478, 2.255, 1.764), α = 0.1, $1-\beta_{10} = 0.79$, $1-\beta_{01} = 0.894$, $1-\beta_{11} = 0.975$ and the expected cost per unit production time is $4945.93. The economic-statistical individual *X* chart has UCL = (77.05+10.824) = 87.87, CL = 77.05, and LCL = (77.05-10.824) = 66.23 (see Figure 7); while the cause-selecting chart has UCL = 1.76, CL = 0, and LCL = -1.76 (see Figure 8).

To monitor the process, a sample with one paired observation (x_{mni}, y_{mni}) is taken and the plotted points x_{mni} and $z_{mni} = \frac{y_{mni} - (11 + 1.1x_{00i})}{6.01}$ are tested on the proposed charts after 1.478 hours from the starting process. If there are no alarms, the second sample (x_{mni}, y_{mni}) is taken and the plotted points x_{mni} and z_{mni} are tested on the charts at time $h_2 = (2^{1/3} \cdot 1.478)$ hours = 1.862 hours. If there is still no alarm at W_2, the third sample (x_{mni}, y_{mni}) is taken hours, and so on. When the x_{mni} falls outside the control limits of the individual *X* chart or/and z_{mni} falls outside the control limits of the cause-selecting chart, it indicates that the subprocess 1 is out of control
or/and the subprocess 2 is out of control. The processes are stopped and the process engineer should search and adjust AC_1 or/and AC_2. Once the processes are adjusted, the process would be renewed. The economic-statistical individual *X* chart and cause-selecting chart have test powers 0.79, 0.894, and 0.975 when the subprocess 1 is out of control, the subprocess 2 is out of control, and both the subprocesses are out of control, respectively. It is obviously that the proposed control charts are powerful on detecting which process is out of control.

5. CONCLUSIONS AND SUGGESTIONS

So far, two dependent processes monitoring for multiple failure mechanisms with increasing failure rates has not been addressed. In this study, we construct the individual *X* control chart and the cause-selecting control chart to monitor the two dependent processes effectively and economically. The derived cost model for the two processes, involving Taguchi's loss function, overcomes the difficulty in cost estimation and reveals quality variation. The optimal design parameters of the proposed control charts are determined by using the Fortran and IMSL subroutines. Using the proposed charts, we can distinguish effectively and economically whether the subprocess 1 or/and the subprocess 2 is/are in control or not. Furthermore, the advantage of using the economic-statistical design is that it improves the economic design to achieve desirable powers and type I error probability. An example was given to illustrate the application of the proposed economic-statistical individual *X* control chart and the cause-selecting control chart. In reality, a production system is always including multiple sequential processes, so the proposed process control approach is quite important and useful. The number of the dependent processes may not only two and the failure mechanisms may be more than two. However, our proposed approach can easily be extended to construct the economic-statistical individual X control chart and multiple cause-selecting control charts, which may effectively distinguish which of the multiple processes is out of control.

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Appendix 1: Calculate *E(T)*

By equation (3.7),

$$E(T) = h_1 + \alpha T_f (1-P_1)(1-P_2) + T_{sr}(P_1 + P_2 - P_1P_2) + P_1(1-P_2)\left(\sum_{i=1}^{\infty} h_{i+1}\beta_{10}{}^{i}\right)$$

$$+ (1-P_1)P_2\left(\sum_{i=1}^{\infty} h_{i+1}\beta_{01}{}^{i}\right) + P_1P_2\left(\sum_{i=1}^{\infty} h_{i+1}\beta_{11}{}^{i}\right) + (1-P_1)(1-P_2)E(T_1)$$

and for $E(T_1), E(T_2)$, …, and so on,

$$E(T_{j-1}) = h_j + \alpha \quad T_f (1-P_1)(1-P_2) + T_{sr}(P_1 + P_2 - P_1P_2) + P_1(1-P_2)\left(\sum_{i=1}^{\infty} h_{i+j}\beta_{10}{}^{i}\right)$$

$$+(1-P_1)P_2\left(\sum_{i=1}^{\infty}h_{i+j}\beta_{01}^{\,i}\right)+P_1P_2\left(\sum_{i=1}^{\infty}h_{i+j}\beta_{11}^{\,i}\right)+(1-P_1)(1-P_2)E(T_j),\ j=2,3,\cdots$$

By solving the recursive system, we may obtain the expected cycle length, that is

$$E(T)=h_1+(1-P_1)(1-P_2)h_2+(1-P_1)^2(1-P_2)^2h_3+\cdots$$

$$+\alpha T_f(1-P_1)(1-P_2)+\alpha T_f(1-P_1)^2(1-P_2)^2+\cdots$$

$$+\left(\frac{P_1}{1-P_1}\right)\left[(1-P_1)(1-P_2)\sum_{i=1}^{\infty}h_{i+1}\beta_{10}^{i}+(1-P_1)^2(1-P_2)^2\sum_{i=1}^{\infty}h_{i+2}\beta_{10}^{i}+\cdots\right]$$

$$+\left(\frac{P_2}{1-P_2}\right)\left[(1-P_1)(1-P_2)\sum_{i=1}^{\infty}h_{i+1}\beta_{01}^{i}+(1-P_1)^2(1-P_2)^2\sum_{i=1}^{\infty}h_{i+2}\beta_{01}^{i}+\cdots\right]$$

$$+\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left[(1-P_1)(1-P_2)\sum_{i=1}^{\infty}h_{i+1}\beta_{11}^{i}+(1-P_1)^2(1-P_2)^2\sum_{i=1}^{\infty}h_{i+2}\beta_{11}^{i}+\cdots\right]$$

$$+T_{sr}(P_1+P_2-P_1P_2)\left[1+(1-P_1)(1-P_2)+(1-P_1)^2(1-P_2)^2+\cdots\right]$$

$$=\sum_{i=1}^{\infty}h_i(1-P_1)^{i-1}(1-P_2)^{i-1}+\alpha T_f\left[\sum_{i=1}^{\infty}(1-P_1)^i(1-P_2)^i+T_{sr}P\sum_{i=1}^{\infty}(1-P)^{i-1}\right]$$

$$+\left(\frac{P_1}{1-P_1}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^i(1-P_2)^i\sum_{i=1}^{\infty}h_{i+j}\beta_{10}^{i}\right]$$

$$+\left(\frac{P_2}{1-P_2}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^i(1-P_2)^i\sum_{i=1}^{\infty}h_{i+j}\beta_{01}^{i}\right]$$

$$+\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^i(1-P_2)^i\sum_{i=1}^{\infty}h_{i+j}\beta_{11}^{i}\right]$$

$$+T_{sr}(P_1+P_2-P_1P_2)\sum_{j=1}^{\infty}(1-P_1)^{i-1}(1-P_2)^{i-1}$$

By equation (2.3),

$$E(T)=h_1\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$

$$+\alpha T_f\left[\sum_{i=1}^{\infty}(1-P_1)^i(1-P_2)^i+T_{sr}P\sum_{i=1}^{\infty}(1-P)^{i-1}\right]$$

$$+h_1\left(\frac{P_1}{1-P_1}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^i(1-P_2)^i\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{10}^{i}\right\}$$

$$+h_1\left(\frac{P_2}{1-P_2}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^i(1-P_2)^i\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{01}^{i}\right]$$

$$+h_1\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^i(1-P_2)^i\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{11}^{i}\right]$$

$$+T_{sr}(P_1+P_2-P_1P_2)\sum_{j=1}^{\infty}(1-P_1)^{i-1}(1-P_2)^{i-1}$$

Appendix 2: Calculate $D^{(j)},\ j=1,2,\ldots$

$$D^{(j)}=D_{10}P\left(T_{AC1}<T_{AC2}\,\middle|\,w_{j-1}<T_{AC1},T_{AC2}<w_j\right)+D_{01}P\left(T_{AC2}<T_{AC1}\,\middle|\,w_{j-1}<T_{AC1},T_{AC2}<w_j\right)$$

$$=D_{10}\left[\int_{w_{j-1}}^{w_j}\int_{w_{j-1}}^{t_2^{(j)}}\left(\frac{f(t_1)}{\int_{w_{j-1}}^{w_j}f(t_1)dt_1}\right)\left(\frac{f(t_2)}{\int_{w_{j-1}}^{w_j}f(t_2)dt_2}\right)dt_1^{(j)}dt_2^{(j)}\right]$$

$$+D_{01}\left[\int_{w_{j-1}}^{w_j}\int_{w_{j-1}}^{t_1^{(j)}}\left(\frac{f(t_1)}{\int_{w_{j-1}}^{w_j}f(t_1)dt_1}\right)\left(\frac{f(t_2)}{\int_{w_{j-1}}^{w_j}f(t_2)dt_2}\right)dt_2^{(j)}dt_1^{(j)}\right]$$

$$=\frac{\left(\dfrac{D_{10}\lambda_1+D_{01}\lambda_2}{\lambda_1+\lambda_2}\right)-\left[D_{10}(1-P_2)+D_{01}(1-P_1)\right]+\left(\dfrac{D_{10}\lambda_2+D_{01}\lambda_1}{\lambda_1+\lambda_2}\right)(1-P_1)(1-P_2)}{P_1P_2}$$

$$=D$$

Appendix 3: Calculate *E(C)*

By equation (3.9),

$$E(C)=b+(1-P_1)(1-P_2)\alpha C_f+P_1(1-P_2)\left(\frac{b\beta_{10}}{1-\beta_{10}}\right)$$

$$+P_1(1-P_2)D_{10}M\left(\sum_{i=1}^{\infty}h_{i+1}\beta_{10}^{i}\right)$$

$$+P_1(1-P_2)M\tau_{11}(D_{00}-D_{10})+(P_1+P_2-P_1P_2)C_{sr}+(1-P_1)P_2\left(\frac{b\beta_{01}}{1-\beta_{01}}\right)$$

$$+(1-P_1)P_2D_{01}M\left(\sum_{i=1}^{\infty}h_{i+1}\beta_{01}^{i}\right)+(1-P_1)P_2M\tau_{21}(D_{00}-D_{01})+P_1P_2\left(\frac{b\beta_{11}}{1-\beta_{11}}\right)$$

$$+P_1P_2D_{11}M\left(\sum_{i=1}^{\infty}h_{i+1}\beta_{11}^{i}\right)+P_1P_2M\tau_{(1)1}(D_{00}-D)+P_1P_2M\tau_{(2)1}(D-D_{11})$$

$$+D_{00}h_1M+(D_{10}-D_{00})h_1MP_1+(D_{01}-D_{00})h_1MP_2$$

$$+(D_{00}+D_{11}-D_{10}-D_{01})h_1MP_1P_2+(1-P_1)(1-P_2)E(C_1),$$

and for $E(C_1),E(C_2)$, …, and so on,

$$E(C_{j-i})=b+(1-P_1)(1-P_2)\alpha C_f+P_1(1-P_2)\left(\frac{b\beta_{10}}{1-\beta_{10}}\right)$$

$$+P_1(1-P_2)D_{10}M\left(\sum_{i=1}^{\infty}h_{i+j}\beta_{10}^{i}\right)$$

$$+P_1(1-P_2)M\tau_{1j}(D_{00}-D_{10})+(P_1+P_2-P_1P_2)C_{sr}+(1-P_1)P_2\left(\frac{b\beta_{01}}{1-\beta_{01}}\right)$$

$$+(1-P_1)P_2D_{01}M\left(\sum_{i=1}^{\infty}h_{i+j}\beta_{01}^{i}\right)+(1-P_1)P_2M\tau_{2j}(D_{00}-D_{01})+P_1P_2\left(\frac{b\beta_{11}}{1-\beta_{11}}\right)$$

$$+P_1P_2D_{11}M\left(\sum_{i=1}^{\infty}h_{i+j}\beta_{11}^{i}\right)+P_1P_2M\tau_{(1)j}(D_{00}-D)+P_1P_2M\tau_{(2)j}(D-D_{11})$$

$$+D_{00}h_jM+(D_{10}-D_{00})h_jMP_1+(D_{01}-D_{00})h_jMP_2$$

$$+(D_{00}+D_{11}-D_{10}-D_{01})h_jMP_1P_2$$

$$+(1-P_1)(1-P_2)E(C_j),\ j=2,3,\cdots\cdots$$

By solving the recursive system, we may obtain the expected cycle cost, that is

$$E(C)=b\left[1+(1-P_1)(1-P_2)+(1-P_1)^2(1-P_2)^2+\cdots\cdots\right]$$

$$+\alpha C_f\left[(1-P_1)(1-P_2)+(1-P_1)^2(1-P_2)^2+\cdots\cdots\right]$$

$$+b\left(\frac{\beta_{10}}{1-\beta_{10}}\right)\left(\frac{P_1}{1-P_1}\right)\left[(1-P_1)(1-P_2)+(1-P_1)^2(1-P_2)^2+\cdots\cdots\right]$$

$$+D_{10}M\left(\frac{P_1}{1-P_1}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}h_{i+j}\beta_{10}^{i}\right]$$

$$+(D_{00}-D_{10})M\left(\frac{P_1}{1-P_1}\right)\left[(1-P_1)(1-P_2)\tau_{11}+(1-P_1)^2(1-P_2)^2\tau_{12}+\cdots\cdots\right]$$

$$+C_{sr}(P_1+P_2-P_1P_2)\left[1+(1-P_1)(1-P_2)+(1-P_1)^2(1-P_2)^2+\cdots\cdots\right]$$

$$+b\left(\frac{\beta_{01}}{1-\beta_{01}}\right)\left(\frac{P_2}{1-P_2}\right)\left[(1-P_1)(1-P_2)+(1-P_1)^2(1-P_2)^2+\cdots\cdots\right]$$

$$+D_{01}M\left(\frac{P_2}{1-P_2}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}h_{i+j}\beta_{01}^{i}\right]$$

$$+(D_{00}-D_{01})M\left(\frac{P_2}{1-P_2}\right)\left[\begin{array}{l}(1-P_1)(1-P_2)\tau_{21}+\\(1-P_1)^2(1-P_2)^2\tau_{22}+\cdots\cdots\end{array}\right]$$

$$+b\left(\frac{\beta_{11}}{1-\beta_{11}}\right)\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left[(1-P_1)(1-P_2)+(1-P_1)^2(1-P_2)^2+\cdots\cdots\right]$$

$$+D_{11}M\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left[\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}h_{i+j}\beta_{11}^i\right]$$

$$+(D_{00}-D)M\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left[(1-P_1)(1-P_2)\tau_{(1)1}+(1-P_1)^2(1-P_2)^2\tau_{(1)2}+\cdots\cdots\right]$$

$$+(D-D_{11})M\left(\frac{P_1}{1-P_1}\right)\left(\frac{P_2}{1-P_2}\right)\left[(1-P_1)(1-P_2)\tau_{(2)1}+(1-P_1)^2(1-P_2)^2\tau_{(2)2}+\cdots\cdots\right]$$

$$+D_{00}M\left[\sum_{i=1}^{\infty}h_i(1-P_1)^{i-1}(1-P_2)^{i-1}\right]+M(D_{10}-D_{00})P_1\left[\sum_{i=1}^{\infty}h_i(1-P_1)^{i-1}(1-P_2)^{i-1}\right]$$

$$+M(D_{01}-D_{00})P_2\left[\sum_{i=1}^{\infty}h_i(1-P_1)^{i-1}(1-P_2)^{i-1}\right]$$

$$+(D_{00}+D_{11}-D_{10}-D_{01})MP_1P_2\left[\sum_{i=1}^{\infty}h_i(1-P_1)^{i-1}(1-P_2)^{i-1}\right]$$

By equation(2.3),

$$E(C)=\left(\frac{b}{P_1+P_2-P_1P_2}\right)+\alpha C_f\left(\frac{(1-P_1)(1-P_2)}{P_1+P_2-P_1P_2}\right)+\left(\frac{b\beta_{10}}{1-\beta_{10}}\right)\left(\frac{P_1(1-P_2)}{P_1+P_2-P_1P_2}\right)+C_{sr}$$

$$+D_{10}h_1M\left(\frac{P_1}{1-P_1}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{10}^i\right\}$$

$$+(D_{00}-D_{10})M\left(\frac{P_1}{1-P_1}\right)\sum_{i=1}^{\infty}(1-P_1)^j(1-P_2)^i\tau_{1i}+\left(\frac{b\beta_{01}}{1-\beta_{01}}\right)\left(\frac{P_2(1-P_1)}{P_1+P_2-P_1P_2}\right)$$

$$+D_{01}h_1M\left(\frac{P_2}{1-P_2}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{01}^i\right\}$$

$$+(D_{00}-D_{01})M\left(\frac{P_2}{1-P_2}\right)\sum_{i=1}^{\infty}(1-P_1)^j(1-P_2)^i\tau_{2i}+\left(\frac{b\beta_{11}}{1-\beta_{11}}\right)\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)$$

$$+D_{11}h_1M\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)\left\{\sum_{j=1}^{\infty}(1-P_1)^j(1-P_2)^j\sum_{i=1}^{\infty}\left[(i+j)^{1/\theta}-(i+j-1)^{1/\theta}\right]\beta_{11}^i\right\}$$

$$+(D_{00}-D)M\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)\sum_{i=1}^{\infty}(1-P_1)^i(1-P_2)^i\tau_{(1)i}$$

$$+(D-D_{11})M\left(\frac{P_1P_2}{(1-P_1)(1-P_2)}\right)\sum_{i=1}^{\infty}(1-P_1)^i(1-P_2)^i\tau_{(2)i}$$

$$+D_{00}h_1M\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$

$$+(D_{10}-D_{00})h_1MP_1\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$

$$+(D_{01}-D_{00})h_1MP_2\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$

$$+(D_{00}+D_{11}-D_{10}-D_{01})h_1MP_1P_2\sum_{i=1}^{\infty}\left[i^{1/\theta}-(i-1)^{1/\theta}\right](1-P_1)^{i-1}(1-P_2)^{i-1}$$

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兩個失效機械的製程管制

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摘要

本文提出相依子製程有兩個遞增失效率機械的製程管制方法。我們分別建立經濟統計X管制圖及選控圖以追蹤第一子製程的品質特性及第二子製程的品質特性。使用此二管制圖可以有效且經濟的區別那個子製程失控。我們提出以更新理論方法推導出製程成本模式而且以最適化方法決定管制圖之最適設計參數值。最後，我們舉例說明如何建立這些管制圖及其在製程上的應用。

關鍵詞: 製程，韋伯分配，控制圖，更新理論
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