Abstract

Andreoni, Erard, and Feinstein (1998) suggest that imposing very high penalties for tax evasion is not possible under bankruptcy or limited liability constraints. In this paper, we complement their suggestion by showing that, in the presence of these constraints, imposing very high penalties can make an economy Pareto worse-off. This result helps provide a further insight into why governments typically do not set very high penalties for tax evasion in practice. Implications for optimal deterrence policies in the context of tax evasion are also explored.

1. Introduction

It is well known that a risk-averse person will always be deterred from accepting a gamble if its expected value is less than zero. It is easy to see that the government could readily assure that this condition is met by setting the penalty rate sufficiently high. In fact, governments typically don’t set penalties this high. This problem is often dealt with by assuming that, for a variety reasons such as bankruptcy constraints or equity considerations, penalties this high are not possible.


Under bankruptcy or the so-called “limited liability” constraints, this paper explores a counter-factual question: what if a government did impose very high penalties?
high penalties such as expropriating all of an evading taxpayer’s income or wealth? A novel finding from our study is that some taxpayers may discontinuously jump from evading some tax when penalties are moderate to evading all of their tax when penalties are raised to a sufficiently high level. This occurs mainly because limited liability may result in the complete impotence of higher penalties. On the basis of this “discontinuous” finding, we show that imposing very high penalties can make an economy Pareto worse-off. This result helps provide a further insight into why governments typically do not set penalties very high in practice.

The possibility that a very high penalty will be counterproductive in deterring crimes has been explored in several papers. Malik (1990) suggests that very high penalties may induce criminals to intensify their avoidance activities so as to offset their apprehension or conviction probability. Andreoni (1991) argues that jurors will need strong evidence to convict a criminal when penalties are very high and, as a result, very high penalties may not result in higher crime deterrence as compared to moderate penalties. Chang, Lai, and Yang (2000) show that very high penalties may increase rather than decrease crime rates if the existing corruption among enforcement officers is sufficiently widespread so that very high penalties may generate a critical mass effect to intensify the extent of corruption. Our paper contributes to this line of the literature, showing not only that setting a very high penalty may be counterproductive in deterrence, but that it can also make an economy Pareto worse-off.

2. A Standard Model of Tax Evasion

Consider an economy in which there are \( n < \infty \) taxpayers and each taxpayer is characterized by an income \( y \in (0, \infty) \). Taxpayer preferences are represented by a smooth von Neumann–Morgenstern utility function \( U(\cdot) : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) with \( U'(0) > 0, U'' < 0, U(0) = 0 \) and \( U''(0) \rightarrow \infty. \)1 A taxpayer is assumed to declare income \( x(\leq y) \) so as to maximize the expected utility:

\[
E(U) = (1 - p) U[y - T(x)] + pU[y - T(x) - (1 + \pi) [T(y) - T(x)]], \tag{1}
\]

where \( p > 0 \) is the random detection probability, \( \pi > 0 \) is the monetary penalty rate, and \( T(\cdot) \) is the tax schedule with \( T(0) = 0 \) and \( 0 < T' < 1 \) (i.e., a positive but less than 100% marginal tax rate). The setup of (1) assumes: (i) true income is unknown to the tax authority but will be discovered once evasion gets detected, and (ii) the caught evader will be fined, and a penalty will be levied on the amount of evaded tax, as is the case under most tax laws.

\[^1\]U(0) = 0 is a normalization and U'(0) \rightarrow \infty belongs to one of the so-called “Inada conditions.”
This setup has been popular in the tax evasion literature since the seminal work of Allingham and Sandmo (1972) and Yitzhaki (1974). Following Yitzhaki (1987), one can rewrite (1) and express it as:

\[ E(U) = (1 - p)U(c + g) + pU(c - \pi g), \]  

(2)

where \( c = y - T(y) \) is the after-tax true income and \( g = T(y) - T(x) \) is the tax evaded. A taxpayer is assumed to choose \( g \geq 0 \) to maximize (2). It can be shown that a taxpayer will not evade tax (i.e., \( g = 0 \)) if \( p(1 + \pi) \geq 1 \), but tax evasion will take place (i.e., \( g > 0 \)) if \( p(1 + \pi) < 1 \). The reasoning behind this result is that the condition \( p(1 + \pi) = 1 \) happens to represent a fair gamble in the standard model of tax evasion. It is well known that a risk averter takes no part of an unfavorable gamble or barely fair gamble, but always takes some part of a favorable gamble.

3. A Modified Model

In the standard model of tax evasion, all of the taxpayers are assumed to face the same detection probability. This is of course not necessarily true in the real world. In addition, a defect associated with this assumption is that either none of the taxpayers evades tax or else they all evade tax. This is the case because, given the penalty rate \( \pi \), whether a taxpayer will evade tax or not is entirely determined by the inequality \( p(1 + \pi) < 1 \) in the standard model.

To account for the real-world fact that some people evade tax while others do not, a possible way of extending the standard model is to follow the idea of Watson (1985) and Macho-Stadler and Perez-Castrillo (1997). They basically assume that the amounts of information available to the tax authority differ among taxpayers and, as a result, different taxpayers have different abilities to hide their true income in a sense (e.g., it is easier for the evasion of a wage/salary earner to be detected by the tax authority than that of someone who is self-employed). Now let \( e \in (0, 1) \) denote the tax authority’s audit effort (the higher the \( e \) the higher the effort) and \( k \in (0, 1) \) denote the taxpayer type with regard to the ability to hide the true income (the higher the \( k \) the

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2Note that the probability of detection \( p \) is exogenously given in (1). Allingham and Sandmo (1972) attempt to extend their basic model to allow for the possibility that the probability of detection depends on the amount of income a taxpayer reports. Unfortunately, as recognized by Allingham and Sandmo themselves, it is not clear whether the probability of detection is an increasing or a decreasing function of income reported. Existing models on the formation of a tax authority’s audit policy can be divided into two groups: the principal-agent model (auditing with commitment) and the game-theoretic model (auditing without commitment). Andreoni, Erard, and Feinstein (1998) point out that a crucial assumption maintained in these models is that taxpayers can correctly know or deduce the tax authority’s audit rule. However, the evidence indicates otherwise (Footnote 42, p. 833): “many taxpayers have only a rough idea of the average probability of audit in their class, and most have little idea as to how this probability changes with the level of income reported.”
lower the ability).\textsuperscript{3} Both $e$ and $k$ may vary across taxpayers. The detection probability $p$ in the standard model can be simply redefined as $p = e \cdot k$. Those taxpayers who face $p$ such that $p(1 + \pi) < 1$ will evade tax, while other taxpayers who face $p$ such that $p(1 + \pi) \geq 1$ will not evade tax. Through this reinterpretation, the standard model can be brought closer to reality.

4. Limited Liability Assumption

Each taxpayer in our modified model is essentially characterized by not only an income $y \in (0, \infty)$ but also a detection probability $p \in (0, 1)$. We impose the following assumption on the modified model:

**ASSUMPTION L:** If a taxpayer is detected as having evaded tax, the penalty plus the tax due cannot exceed his entire income.

This is the “bankruptcy” or “limited liability” assumption. In terms of our model, imposing Assumption L implies

\[ c - \pi g = 0 \quad \text{if} \quad g \geq c/\pi. \quad (3) \]

When a taxpayer is caught evading tax, his income (net of the penalty and the tax due) equals $c - \pi g$ (see Equation (2)). The term $c - \pi g$ cannot be negative under Assumption L. This is the reason for (3). In other words, when a taxpayer is protected by limited liability, he can never receive a negative income under Assumption L.\textsuperscript{4}

With the imposition of Assumption L, Equation (2) is modified to become

\[ E(U) = (1 - p)U(c + g) + pU(c - \pi g) \quad \text{if} \ 0 \leq g \leq c/\pi \quad (4.1) \]

\[ E(U) = (1 - p)U(c + g) + pU(0) \quad \text{if} \ c/\pi \leq g \leq T(y) \quad (4.2) \]

It is clear from (4) that Assumption L will become relevant only if the penalty rate $\pi$ imposed is high enough so that $c/\pi < T(y)$. If detected, this high

\textsuperscript{3}Consider the example of the wage/salary earner versus the self-employed. A possible interpretation of $k$ is that, say, 99% of audited wage/salary evaders will get caught while only 50% of audited self-employed income evaders will get caught.

\textsuperscript{4}Andreoni, Erard, and Feinstein (1998) document that the U.S. Internal Revenue Service (IRS) typically applies civil penalties at a rate of 20% of the portion of the underpayment of tax; in cases of fraud, a civil penalty may be applied at the rate of 75%. Even with these “moderate” penalties, the IRS faces some difficulties in collecting penalties from tax evaders. According to the IRS (1991), the IRS abates assessed penalties resulting from evasion detection, and these abatements equal 24.9% of the penalties from delinquency, 9.5% of the penalties from failure to pay the tax due, 9.9% of the penalties from bad checks, and 22.3% of the penalties from fraud. The taxpayers’ limited liability may be partially responsible for these penalty abatements. It is not difficult to imagine how things may get worse if penalties are set much higher.
penalty will expropriate a taxpayer’s entire income once the amount of tax he has evaded exceeds $c/\pi$.

A key property associated with (4) is that $E(U)$ is strictly increasing in $g$ when (4.2) is applicable. As a result, a taxpayer will always choose to evade all of his tax once the amount of tax evaded $g$ exceeds $c/\pi$. This key property will be utilized later.

5. Tax Evasion with Limited Liability

A taxpayer’s tax liability is normally only a fraction of his income. This section explores a counter-factual question: what if the government did impose very high penalties so that the limited liability constraint becomes relevant? Throughout the analysis, the tax schedule, $T(\cdot)$, is assumed fixed at all times.

There are two critical conditions in our model, namely, $p(1+\pi) = 1$ and $c/\pi = T(y)$. Taxpayers will not evade tax in the absence of limited liability if $p(1+\pi) \geq 1$ (i.e., $\pi \geq 1/p - 1$). The limited liability constraint will become relevant only if $c/\pi < T(y)$ (i.e., $\pi > y/T(y) - 1$). Note that the term $1/p$ in the inequality $\pi \geq 1/p - 1$ and the term $y/T(y)$ in the inequality $\pi > y/T(y) - 1$ are independent of each other. A taxpayer who faces detection probability $p$ and has income $y$ will satisfy either $1/p \leq y/T(y)$ or $1/p > y/T(y)$. In what follows we analyze these two different cases separately.

**LEMMA 1:** Consider a taxpayer who faces $p$ and has $y$ with $1/p \leq y/T(y)$. The taxpayer will choose not to evade tax before the penalty rate $\pi$ is raised to exceed $y/T(y) - 1$. Under Assumption L, the same taxpayer will still choose not to evade tax after the penalty rate $\pi$ is raised to exceed $y/T(y) - 1$.

**Proof:** In the regime where $\pi \leq y/T(y) - 1$, Assumption L is irrelevant and the taxpayer will not evade tax as long as $\pi \geq 1/p - 1$. Since $1/p \leq y/T(y)$ by assumption, the taxpayer will not evade tax for those $\pi$ that lie between $1/p - 1$ and $y/T(y) - 1$.

In the regime where $\pi > y/T(y) - 1$, we have two possibilities:

(i) Assumption L remains irrelevant and the taxpayer will not evade tax as before.

(ii) Assumption L becomes relevant and evading all of his tax gives the taxpayer the highest expected utility since $E(U)$ in (4.2) is strictly increasing in $g$.

Putting (i) and (ii) together, we only need to compare $U(c)$ (the expected utility of no evasion) with $(1-p)U(y)$ (the expected utility of complete evasion with binding limited liability) to determine the taxpayer’s choice.

Now, for any given $y$, consider the case where $1/p = y/T(y)$. This case implies the equality $(1-p)y = y - T(y)$, which in turn implies the inequality $U(c) > (1-p)U(y)$ ($U'' < 0$ and $U(0) = 0$ by assumption).
Remark 1: Lemma 1 is intuitive. Higher penalties will as a rule lower a taxpayer’s expected utility. This is true even with limited liability. However, as long as the taxpayer does not evade tax, higher penalties will become irrelevant to him and the taxpayer will still be capable of attaining the same (highest) utility as before.

Lemma 2: Consider a taxpayer who faces $p$ and has $y$ with $1/p > y/T(y)$. The taxpayer will evade some tax before the penalty rate $\pi$ is raised to exceed $y/T(y) - 1$.

Under Assumption L,

(i) if $U(c) \geq (1-p)U(y)$, then until the equality $\pi = 1/p - 1$ is reached the same taxpayer will continuously reduce the amount of tax evaded as the penalty rate is continuously raised, and he will cease to evade tax as soon as the penalty rate is raised to equal $1/p - 1$ or more;

(ii) if $U(c) < (1-p)U(y)$, then up to a certain limit the same taxpayer will continuously reduce the amount of tax evaded as the penalty rate $\pi$ is continuously raised, but he will discontinuously jump to evade all his tax as soon as the penalty rate is raised beyond the limit.

Proof: When $\pi \leq y/T(y) - 1$, Assumption L is irrelevant and the taxpayer will evade some tax as long as $\pi < 1/p - 1$. Since $1/p > y/T(y)$ by assumption, $\pi \leq y/T(y) - 1$ does imply that $\pi < 1/p - 1$.

When $\pi > y/T(y) - 1$, Assumption L becomes relevant and the objective function facing the taxpayer will be represented by (4) instead of (2). Let $EU(g^*) \equiv (1-p)U(c + g^*) + pU(c - \pi g^*)$, where $g^*$ denotes the optimal solution associated with (4.1). Since $E(U)$ in (4.2) is strictly increasing in $g$, the amount of tax evasion chosen by the taxpayer equals $\arg\max\{EU(g^*), (1-p)U(y)\}$.

The proof then involves several observations. First, since the taxpayer evades some tax when $\pi \leq y/T(y) - 1$, $EU(g^*)$ must be strictly greater than both $U(c)$ and $(1-p)U(y)$ at $\pi = y/T(y) - 1$. Secondly, $EU(g^*)$ will become smaller and smaller as $\pi$ is raised. Finally, if $U(c) \geq (1-p)U(y)$, $EU(g^*)$ will reach $EU(0) = U(c)$ before $(1-p)U(y)$ as $\pi$ is raised. This proves Lemma 2(i).

On the other hand, if $U(c) < (1-p)U(y)$, $EU(g^*)$ will again reach $U(c)$ but strictly after $(1-p)U(y)$ as $\pi$ is raised. Thus, there will exist a critical $\hat{\pi}$ such that $EU(g^*(\hat{\pi})) = (1-p)U(y)$. For all $\pi \leq \hat{\pi}$, we have $g^* = \arg\max\{EU(g^*), (1-p)U(y)\}$. For all $\pi > \hat{\pi}$, we have $T(y) = \arg\max\{EU(g^*), (1-p)U(y)\}$; that is, the taxpayer will evade

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$^5$We let the taxpayer choose to evade some tax rather than all of his tax at $\pi = \hat{\pi}$. This choice follows the convention in the principal–agent model that agents (taxpayers) are
all of his tax once the penalty rate imposed is higher than $\hat{\pi}$. This proves Lemma 2(ii). ■

Remark 2: Supposing that all taxpayers have the same income, one can then partition $p \in (0, 1)$ into three parts. Lemma 1 is applicable to taxpayers who are located in the highest part, Lemma 2(i) is applicable to taxpayers who are located in the middle part, and Lemma 2(ii) is applicable to taxpayers who are located in the lowest part.

6. A Pareto Worse-Off Outcome

This section proves a Pareto worsening result for raising penalties. For convenience, let us consider a proportional income tax so that $y/T(y) = 1/t$ for all $y$, where $t$ denotes the proportional tax rate with $0 < t < 1$. Suppose that the penalty rate $\pi$ is initially set below $1/t - 1$. In this situation, Assumption L is irrelevant and some people (those who face $p$ with $\pi < 1/p - 1$) evade tax while others (those who face $p$ with $\pi \geq 1/p - 1$) do not. This co-existence of evaders and non-evaders seems typical in the real world. Now suppose that, in order to further reduce or even eliminate tax evasion, the tax authority has decided to raise penalties sufficiently high in the sense that $\pi > 1/t - 1$ so that Assumption L becomes relevant. What will happen if such a practice is put into effect? We have the following result.

PROPOSITION: Under Assumption L, the economy can be Pareto worse-off if penalties are raised sufficiently high.

Proof: Let us call the taxpayers considered in Lemma 1, Lemma 2(i) and Lemma 2(ii) type-1, type-2, and type-3 taxpayers, respectively. With $y/T(y) = 1/t$ for all $y$, Lemma 1 is applicable to taxpayers with $p \geq t$ while Lemma 2 is applicable to taxpayers with $p < t$.

From Lemma 1, type-1 taxpayers will continue to choose not to evade tax as the penalty rate $\pi$ is raised to exceed $1/t - 1$.

From Lemma 2(i), there exists a critical penalty rate for each type-2 taxpayer such that the taxpayer will stop evading tax once the penalty rate reaches or exceeds the critical level. This critical penalty rate is defined by the equality $\pi = 1/p - 1$. Let this critical penalty rate be denoted by $\bar{\pi}(p)$ and define $\pi_2 = \max_p \{\bar{\pi}(p)\}$. It is clear that all type-2 taxpayers will stop evading tax once the penalty rate is raised to $\pi_2$ or beyond.

From Lemma 2(ii), there exists a critical penalty rate for each type-3 taxpayer such that the taxpayer will jump to evade all of his tax once the penalty rate exceeds the critical level. This critical penalty rate is defined by the equality $EU(g^*) = (1 - p)U(y)$. Let this critical penalty rate be...
denoted by $\hat{\pi}(p, y)$ and define $\pi_3 = \max_{p, y}\{\hat{\pi}(p, y)\}$. At $\pi = \pi_3$, except for the taxpayer whose critical penalty rate happens to be $\pi_3$, all type-3 taxpayers will choose to evade all of their tax.

With $\pi_2$ and $\pi_3$ at hand, we now provide a sufficient condition to guarantee that the economy will be Pareto worse-off if penalties are raised sufficiently high.

Let $\pi_3 \geq \pi_2$ and consider what will happen for different types of taxpayers if we raise the penalty rate to a level higher than $\pi_3$. First, according to Lemmas 1 and 2(i), raising penalties to a level higher than $\pi_3$ will not affect the behavior or utility of type-1 or type-2 taxpayers. These taxpayers comply completely when the penalties are set at $\pi = \pi_3$ or higher. Next, at $\pi = \pi_3$, except for the taxpayer whose critical penalty rate happens to be $\pi_3$, all type-3 taxpayers will choose to evade all of their tax. According to Lemma 2(ii), raising the penalty rate to a level higher than $\pi_3$ will affect neither these taxpayers’ behavior nor their expected utility since they will continue to evade all of their tax. As to the taxpayer whose critical penalty rate happens to be $\pi_3$, his expected utility will also remain unchanged as the penalty rate $\pi$ is raised to exceed $\pi_3$. This is true since $EU(g^*) = (1 - p) U(y)$ at $\pi_3$. However, the amount of tax he evades will discontinuously increase from $g^*$ to $T(y)$. This results in a strict loss in expected revenue.

To sum up, raising the penalty rate to a level higher than $\pi_3$ leaves each and every taxpayer’s expected utility unchanged, but it strictly reduces the expected revenue collected by the tax authority. This move is clearly Pareto worsening. ■

Remark 3: The key to our proof lies in that the amount of tax evaded displays a discontinuous increase as penalties are raised sufficiently high for type-3 taxpayers (Lemma 2(ii)). This discontinuity property allows for the possibility that raising penalties will reduce the expected revenue collected by the tax authority while, at the same time, lowering or leaving the taxpayers’ expected utility unchanged. Since the key lies in the discontinuity property, it should be clear that a proportional income tax is a convenient rather than a critical assumption in our proof.

Remark 4: Of course, what we have shown in the proof is but a sufficient condition. There may exist other sufficient conditions leading to the conclusion stated in our Proposition. For example, let $\pi_3 < \pi_2$ and consider what will happen if we raise the penalty rate to a level slightly higher than $\pi_3$. In this case everything remains the same as in the proof, except that some type-2 taxpayers will choose to evade tax at $\pi = \pi_3$ since $\pi_3 < \pi_2$. As the penalty rate is raised to a level slightly higher than $\pi_3$, the amount of tax evasion chosen by each of these type-2 taxpayers will become slightly
smaller according to Lemma 2(i). In contrast, the amount of tax evasion chosen by the type-3 taxpayer whose critical penalty rate happens to be $\pi_3$ will discontinuously increase according to Lemma 2(ii). If the expected loss in tax revenue resulting from type-3 taxpayers’ discontinuous jumps dominates the expected gain in tax revenue associated with type-2 taxpayers’ continuous behavior (say, the number of type-2 taxpayers who evade at $\pi = \pi_3$ is relatively small), then the expected revenue collected by the tax authority will decrease rather than increase as a result of raising penalties to levels slightly higher than $\pi_3$. We conclude that raising penalties to a level slightly higher than $\pi_3$ can be Pareto worsening in this case as well.

7. Discussion

Becker (1968) suggests that the optimal policy to deter crimes is to set monetary penalties at the maximum possible and the corresponding audit probabilities at the minimum possible. In the context of tax evasion, this policy prescription is to “hang tax evaders with probability zero” in the words of Kolm (1973, p. 266). This maximal penalty policy appears intuitive as long as the two instruments of tax enforcement, the monetary penalty and the audit probability, are substitutes in the sense that a reduction in one can be compensated for by an increase in the other so as to maintain deterrence. Since raising monetary penalties involves little cost while enhancing the audit intensity typically increases the use of real resources, it will be optimal in terms of economizing on enforcement costs to substitute the monetary penalty for the audit probability as far as is possible.

On the basis of our study, however, it may be desirable to prescribe a policy that runs counter to Becker’s suggestion. First, according to our Proposition, reducing penalties to a level below the maximal may be Pareto-improving for an economy. Secondly, the key reason why a type-3 taxpayer will discontinuously jump to evade all of his tax is that the expected utility from no evasion is strictly less than that from complete evasion with binding limited liability (i.e., $U(c) < (1 - p) U(y)$). If the opposite is true instead, then the taxpayer will become a type-2 taxpayer and, as our Lemma 2(i) dictates, raising penalties will in this case be effective in reducing evasion. Notice that $p$ in our modified model may be understood as $p = e \cdot k$, where $e$ denotes the tax authority’s audit effort and $k$ denotes the taxpayer type with regard to the ability to hide the true income. It is thus feasible for the tax authority to raise $p$ through an increase in $e$. By increasing $e$, the tax authority can turn a type-3 taxpayer into a type-2 taxpayer. The resulting increase in tax revenue may be

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7According to Carter (2001, p. 210), “Roughly speaking, a function is continuous if small changes in input (the independent variable) produce only small changes in output (the dependent variable).”
more than offset by the increase in cost associated with increasing $e$. Thus, instead of “hanging tax evaders with probability zero,” it may be desirable to prescribe a moderate monetary penalty together with a much higher audit effort than “probability zero” in our setup. This policy prescription seems to conform more to what we usually observe in reality.

When both monetary penalties and imprisonment are available, the standard policy prescription is that imprisonment should not be imposed unless monetary penalties are maximal. However, once limited liability is binding so that (4.2) is applicable, higher monetary penalties will be completely impotent in deterrence. In such a situation, the imposition of imprisonment can be used to effectively remove this impotence caused by limited liability. This may explain why we do observe in the real world the employment of both monetary penalties and imprisonment in deterring tax evasion even though the monetary penalties imposed are far from maximal.

Another possible way to get around the problem caused by limited liability is to seize a caught evader’s future income if his current income is insufficient to cover the penalty plus the tax due. By taking this complication into account, the limited liability constraint (3) is modified to become: $v + c - \pi g = 0$ if $g \geq (v + c)/\pi$, where $v$ denotes the present value of the tax evader’s future income stream. This modification replaces the role of $c$ with that of $v + c$ but does not change the essence of our main argument. In particular, the problem remains as penalties are raised sufficiently high. As such, unlike imprisonment that can effectively remove the problem caused by limited liability, it seems that seizing a tax evader’s future income will only mitigate, but not remove, the problem.

The detection probability $p$ is assumed to be exogenously given in our model. It is not difficult to imagine that, in the real world, the tax authority's audit effort may be a function of some observable taxpayer characteristics such as reported income type and occupation (say, wage/salary, or self-employment income). This will make the detection probability depend on these characteristics as well, leading to the so-called “audit classes” in the literature. Nevertheless, one can still make the point that, for any given audit effort within an audit class, penalties can be too high.

References


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8See Polinsky and Shavell (2000, Section 3.1.3).
9When imprisonment is available, the utility associated with the detection state is no longer lower bounded by $U(0) = 0$. 


