THE OPTIMIZATION OF SYSTEM RELIABILITY
- COMBINE REVISED PATTERN SEARCH AND DYNAMIC PROGRAMMING

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ABSTRACT

The application of reliability optimization in the system design and development phase affects system availability, maintainability, logistics support requirement and related cost control greatly. In this research, we consider the system reliability optimization problem under different design constraints such as weight, volume, system and subsystem reliability requirements, etc. System reliability can be improved by enhancing component reliability level and/or adding redundant components. Of course, both component reliability and redundancy enhancement will definitely spend a lot of costs and resources. Therefore, there exists a trade-off between these two alternatives for resource-constrained reliability optimization.

Many optimization techniques have been used to allocate component reliability level and the number of redundancy individually. Few of them can solve reliability-redundancy allocation problems simultaneously except for some kind of combinations. The proposed model developed in this paper is based on revised H-J pattern search combined with dynamic programming approach using the concept of dominating sequences. We try to solve the system reliability-redundancy allocation problem which can handle more general system configurations which include series- parallel system, element stand-by system, and n-stage mixed series system etc.. Besides general resource capacity constraints, some specific system and subsystem performance constraints such as reliability will also be considered. Through our approach, we can get the optimal allocation of component redundancy number and reliability level to minimize the total system design and development cost

A couple of numerical examples have been demonstrated for different system configurations. The information we obtain from those is very helpful to facilitate the system design and development process. The performance comparison between this proposed approach and other reliability- redundancy allocation methods are also discussed and it turns out to be very good result. We show that this proposed approach can fit various problem types and is more realistic, flexible, even can get better results in some aspect than some traditional reliability optimization approaches.

Keywords: Reliability Optimization, Reliability-Redundancy Allocation, Revised H-J Pattern Search, Dynamic Programming, Dominating Sequences

1. INTRODUCTION

For many systems, the costs related to design and development, construction, and production, etc., are relatively well known. However, the costs occurred with the utilization, maintenance and support of the system throughout its planned life cycle are somewhat hidden. Blanchard (1998) had mentioned that a large percentage of the total life cycle cost for a given system is attributed to operating and maintenance activities and a significant part of this cost depends on the decisions made during the phases of initial design and development.

In the initial system design and development period, reliability optimization plays a key role to affect system availability, maintenance activities,logistics requirements and related costs during its planned life cycle. In the past decades, for solving reliability optimization problems, a lot of research efforts have been put on how to find the component reliability and the number of redundancy for each stage which maximize the system reliability under some resource constraints, such as weight, volume, budget, etc.. In the recent review paper, Kuo & Prasad (2000) classified papers on reliability optimization by system structure and solution method since 1977. For reliability-redundancy allocation problems, a couple of famous methods have been published, such as Tillman et al (1977) combined the H-J Pattern Search (Hooke & Jeeves, 1961) and AGM Heuristic Approach (Aggarwal et al, 1975), Kuo et al (1987) used Lagrange Multiplier and Branch and Bound Technique (LMBB), and Xu et al (1990) combined the sequential search and XKL Heuristic Approach to solve these problems. In addition, Hikita et al (1992) used a dynamic programming approach to solve single-constraint surrogate problems. Coit&Smith(1996) developed a problem-specific genetic algorithm to analyze series-parallel system and to select components and redundancy-levels to optimize some objective function, given system-level constraints on reliability, cost and weight. Reliability-redundancy allocation problems can also be applied in software reliability optimization. Chi et al (1990) formulated mixed integer nonlinear programming problems for reliability-redundancy allocation in software systems with common-cause failures and systems involving both software and hardware. Most of these reliability-redundancy optimization methods have their own specific applied area and limitations depending on different system configurations, objective functions and resource constraints.

In this paper, we try to solve reliability-redundancy allocation problem for more general system configurations such as series-parallel system, element stand-by system, and n-stage mixed series system etc. Also, the problem has been changed from different aspects compared to the template problem referred in a lot of previous studies that the system reliability and some subsystem reliability requirements were put into the constraints and the cost function was placed as objective function. It means we try to find the minimum cost solution under some constraints which include one to require the minimum system reliability and some others on subsystem reliability. The argument is, in reality, we need to meet some specific system and subsystem performance requirements at the minimum development cost as possible as we can, when a new system is designed.

In section 2, we will first describe the problem statement for this paper and do the modeling work. In section 3, an optimization procedure is developed. The Dynamic Programming and Dominating Sequences will be applied to find the optimal solution in each step of revised H-J Pattern Search modified by our model, which leads to exact optimal solution on components reliability and number of redundancies at each stage. We show some numerical examples for this proposed model in section 4 and compare performance with other reliability and redundancy optimization approaches in section 5. Finally some conclusions will be presented in section 6.

<u>Notation</u>

x_j, r_j number of components and component reliability at stage j

x_j^U, x_j^L upper bound and lower bound of x_j

n, m number of stage in the system, number of resource constraints

$\overline{R}, \overline{R}^0$ component reliability vector, initial component reliability vector

$\overline{X}$ vector of number of components used at each stage

$C_S(\overline{R}, \overline{X})$ system cost function

$C_j(r_j, x_j)$ cost function of stage j

$G_{ij}(r_j, x_j)$ the resource i required by stage j

$G_i(\overline{R}, \overline{X})$ the resource i required by system

b_i, S_j the available quantity of resource i, state variable for state j

Rs, R_s^L system reliability, lower bound of system reliability

Qs, Q_s^U system unreliability, upper bound of system unreliability

$R_j(r_j, x_j)$ reliability function of stage j

R_j^L lower bound of reliability of stage j

C set of subsystems which has lower bound in reliability

$X^*(\overline{R})$ the optimum redundancy allocation policy when reliability vector is $\overline{R}$

Wj, W weight of one component at stage j, limitation on weight

Pj product of the weight and volume per element at stage j

P limitation on product of the weight and volume

α_j, β_j constants representing the inherent reliability characteristics of each component at stage j,

2. PROBLEM STATEMENT AND MODELING

The traditional template problem applied in many studies for reliability-redundancy optimization methods is to maximize system reliability function by conforming to different resource and related cost constraints. On the contrary, in this model we use the cost function as the objective function and try to minimize it by conforming to those constraints such as system configuration, resource capacity, system and subsystem reliability requirement etc.. The systems we consider in this model include series-parallel, element stand-by and n-stage mixed series system, etc.. It means that all subsystems of this proposed system are connected in series, and these subsystems can be either or both of parallel and stand-by configurations. The number of components x_j and the component reliability r_j at the stage j are the decision variables to be determined. Therefore, the template problem in this model is defined as follows:

$$\text{Min} \quad C_s(\overline{R},\overline{X}) = \sum_{j=1}^{n} C_j(r_j, x_j) \qquad (1)$$

s.t.

$$\sum_{j=1}^{n} G_{ij}(r_j, x_j) \le b_i, \text{ for } i = 1,\ldots,m \qquad (2)$$

$$\prod_{j=1}^{n} R_j(r_j, x_j) \ge R_s^L \qquad (3)$$

$$R_j(r_j,x_j) \ge R_j^L, \text{ for some j in C} \qquad (4)$$

In the above equations, equation (1) is the objective function which will be focused on system design and development cost, equation (2) is the general form of system configuration or resource capacity constraint, and equation (3), (4) are system and subsystem reliability requirement constraints. As usual, the reliability parameters: $\overline{R}$, R_s^L, R_j^L are all real numbers between 0 and 1; $\overline{X}$ are all positive integers.

In this proposed model, the reliability constraint on equation (3) and (4) has two different forms. For the series-parallel system, $R_j(r_j, x_j)$ becomes $[1-(1-r_j)^{x_j}]$ (5)

and for the element stand-by system, $R_j(r_j, x_j)$ becomes

$$r_j[1-\ln r_j + \frac{(\ln r_j)^2}{2!} - \cdots + (-1)^{x_j-1}\frac{(\ln r_j)^{x_j-1}}{(x_j-1)!}] \qquad (6)$$

To set up the problem, several assumptions need to be made:

1. All stages is connected in series, ie., 1-out-of-N: F configuration.
2. For a parallel stage, it is a 1-out-of-x_j: G configuration and for a stand-by stage, we assume each switch from component to component is instant and successful.
3. All components in the same stage have the same probability of failure.
4. All the stages and all the components used at each stage are S-independent.
5. A short circuit failure will not be considered, that is, only a single mode of failure is assumed.
6. The stage costs, resources, unreliability are additive.
7. Rj should be large enough, such that the higher order terms can be neglected.
8. Assume all components have the characteristics of electrical component (i.e. $R(t)=e^{-\lambda t}$)

To facilitate the modeling procedure, according to assumption 7, we can simplify equation (3) for the typical series-parallel system from $\prod_{j=1}^{n}[1-(1-r_j)^{x_j}] \ge R_s^L$ to

$$\sum_{j=1}^{n}(1-r_j)^{x_j} \le Q_s^L \quad (Q_s^L = 1 - R_s^L) \qquad (7)$$

and for standby system, the equation (3) is simplified from

$$\prod_{j=1}^{n} r_j[1-\ln r_j + \frac{(\ln r_j)^2}{2!} - \cdots + (-1)^{x_j-1}\frac{(\ln r_j)^{x_j-1}}{(x_j-1)!}] \ge R_s^L$$

to

$$\sum_{j=1}^{n}[1-r_j[1-\ln r_j + \frac{(\ln r_j)^2}{2!} - \cdots + (-1)^{x_j-1}\frac{(\ln r_j)^{x_j-1}}{(x_j-1)!}] \le Q_s^L \qquad (8)$$

Then the stage variable vector of dynamic programming used in this model can be defined as:

$$\overline{S_k} = \begin{bmatrix} \sum_{j=1}^{k} G_{1j}(r_j, x_j) \\ \vdots \\ \sum_{j=1}^{k} G_{mj}(r_j, x_j) \\ \prod_{j=1}^{k} R_j(r_j, x_j) \end{bmatrix} \qquad (9)$$

Base on the above transformation which approximates the system unreliability by the addition of unreliability of all stages, the S_k is changed to

$$\overline{S_k} = \begin{bmatrix} \sum_{j=1}^{k} G_{1j}(r_j, x_j) \\ \vdots \\ \sum_{j=1}^{k} G_{mj}(r_j, x_j) \\ \sum_{j=1}^{k} [1 - R_j(r_j, x_j)] \end{bmatrix} \quad (10)$$

The recurrent formula in Dynamic Programming becomes

$$F_1(\overline{S_1}) = \min_{x_1^L \le x_1 \le x_1^U} [G_1(r_1, x_1)] \quad (11)$$

$$F_j(\overline{S_j}) = \min_{x_j^L \le x_j \le x_j^U} [G_j(r_j, x_j) + F_{j-1}(S_{j-1})] \quad for\ j = 2, \ldots, n$$

In the following section, we will apply dominating sequences concept in our dynamic programming model to find the optimal redundancy allocation under every specific component reliability level decided by revised H-J Pattern Search approach. Therefore, the above recurrent formula will be modified to only keep the dominating solutions for every step calculation.

3. AN OPTIMIZATION PROCEDURE

The combination of the revised Hooks and Jeeves pattern search and Dynamic Programming accompanied with dominating sequences is used to solve the template problem in section 2.

The main structure of this model is to apply revised H-J pattern search to find the appropriate component reliability level, and at that component reliability level, Dynamic Programming with dominating sequences is applied to get the optimal redundancy number to minimize system cost at that step. The result obtained from the computation procedure of Dynamic Programming with dominating sequences goes back to the procedure of H-J pattern search to decide if it is the optimal solution for whole model. The descriptive flow diagram is shown in Figure 1

The computation procedure of Dynamic Programming with dominating sequences for evaluating the function value of system cost, $Cs(\overline{R}, \overline{X})$ at any given point in Figure 1 are

1. Calculate x_j^U

(a) Assume $x_k = 1$, k=1,2,..., n and k≠j

(b) $x_j^U = \min_i \{\max\{x_j \mid G_i(\overline{R}, (1, \ldots, x_j, \ldots, 1)) \le b_i\}$ for $i = 1, \ldots, m\}$

2. Calculate x_j^L which satisfies (a) and (b) below:

(a) For stage j in C,
$x_j^L = \min\{x_j \mid R_j(r_j, x_j) \ge \max(R_s^L, R_j^L)\}$

(b) For other stage j not in C,
$x_j^L = \min\{x_j \mid R_j(r_j, x_j) \ge R_s^L\}$

3. Set up a matrix for the combination of stage1 and stage2

(1) The number of components, the stage cost, $G_1, \ldots\ldots, G_m$, the stage unreliability for stages 1 and 2 are presented as the column to the left of the matrix and the row above the matrix, respectively.

(2) Each entry of the matrix is a vector, which shows the system cost, $G_1, \ldots\ldots, G_m$, the system unreliability which are results of the combination of stages 1 and 2.

4. Obtain the dominating sequence by eliminating entries of matrix which are dominated by others:

(1) When any element of the entries in the matrix exceeds the constrained available resource or unreliability limit, this entry is eliminated.

(2) The dominating sequence will then be determined as follows:

a. Consider the entry having the lowest system cost, which is always one term of dominating sequences no matter what other elements of this entry are. Now compare the elements of all other entries with the elements of this entry. Eliminate all entries which have higher system costs, higher consumptions of all resources and higher unreliability.

b. Choose the entry having the next lower system cost to be the other term of dominating sequence. Compare the elements of all other entries with the elements of this entry. Eliminate all entries which have higher system cost, higher consumptions of all resources and higher unreliability.

c. Repeat step b until the dominating sequence for the combination of stage 1 and stage 2 has been finally reached.

5. Set up a matrix for the combination of stage1-2 (the dominating sequence) and stage 3 as step 3 and repeat step 4 to obtain the dominating sequence of stage 1-2-3.
6. Repeat step 5 until the matrix for the combination of stage 1-2-……-(n-1) and stage n is set up and the final dominating sequence is reached.
7. Choose the entry in the final dominating sequence which have the lowest system cost to be the optimal solution. Go back to main model (revised H-J Pattern Search model).

4. NUMERICAL EXAMPLES

One research team is assigned to develop a new system and need to meet some specific logistics requirements. It is noted that reliability optimization in the initial system design period plays a critical role to affect system availability, maintenance activities, logistics requirements and life cycle cost during system's service life. For simplifying the situation, we assume that they need to find the optimal component reliability level and redundancy number in each subsystem to minimize the system design and development cost by conforming to the constraints for two design configurations (weight, volume), system and subsystem reliability requirements. In the following we demonstrate three numerical examples which respectively represent different system configurations or require different constraints. Before proceeding our numerical examples, table 1 lists all the parameters or related information we need as follows.

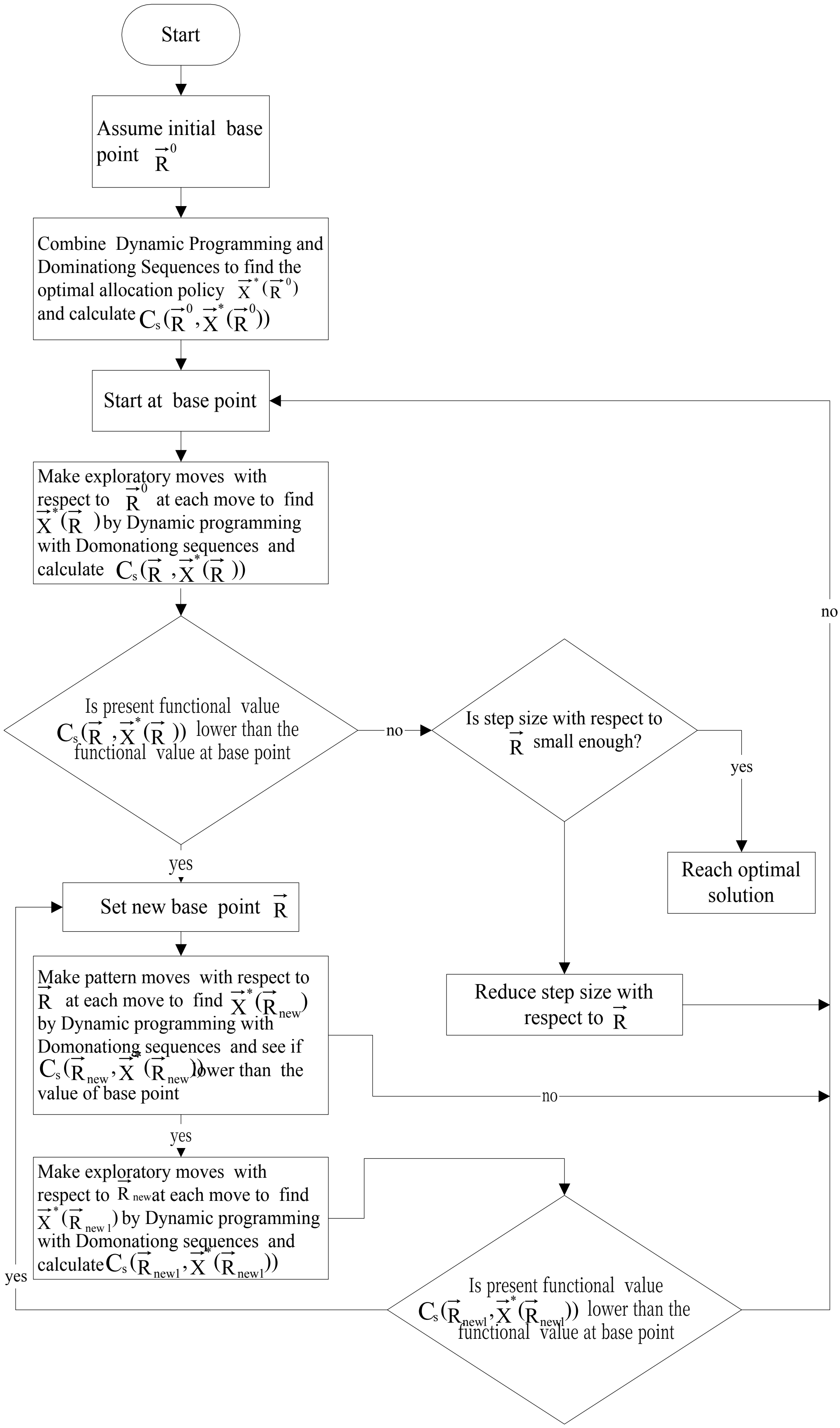
Table 1: Constant used in Examples

j	$\alpha_j (10^{-5})$	P_j	W_j
1	2.33	1	7
2	1.45	2	8
3	0.541	3	8
4	8.05	4	6
5	1.95	2	9

$\beta_j = 1.5$, j=1,2,3,4,5

t=1000

Figure 1. Descriptive Flow Diagram for Combining of Revised H-J Pattern Search and Dynamic Programming with Dominating Sequences Approach



Example 1: The typical series-parallel system is considered. There are totally 5 subsystems in this system. The limitation on product of the weight and volume is 110. The limitation on weight is 200. The low bound of system reliability is 0.9.

$$\text{Min } C_S(\vec{R}, \vec{X}) = \sum_{j=1}^{n} [\alpha_j (\frac{-t}{\ln r_j})^{\beta_j}][x_j + \exp(x_j/4)]$$

s.t

$$\sum_{j=1}^{n} p_j (x_j)^2 \le P$$

$$\sum_{j=1}^{n} w_j x_j * \exp(x_j/4) \le W \qquad (12)$$

$$\prod_{j=1}^{n} [1-(1-r_j)^{x_j}] \ge R_S^L$$

$$(\sum_{j=1}^{n} (1-r_j)^{x_j} \le 1 - R_S^L)$$

The results are shown in the following table 2.

Table 2

Optimal Solution for Example 1 (R_S^L=0.9, P=110, w=200)

Stage	1	2	3	4	5
Starting point, $\overline{R}^0$	0.8	0.8	0.8	0.8	0.8
Optimal point $\overline{R}^*$	0.83125	0.84375	0.83906	0.7	0.74844
Optimal point $\overline{X}^*$	2	2	3	3	3
System Cost	151.014378				
System Reliability	0.900021				
Used P	93				
Used W	195.534639				
Initial step size	0.05				
Final step size	0.000195				

Example 2: The typical element stand-by system is considered. There are totally 5 subsystems in this system. The limitation on product of the weight and volume is 110. The limitation on weight is 200. The low bound of system reliability is 0.9. Therefore, the whole problem is the same with example 1 except the third constraint (reliability constraint) need to be modified as follows:

$$\prod_{j=1}^{5} r_j[1-\ln r_j + \frac{(\ln r_j)^2}{2!} + \cdots\cdots + (-1)^{x_j - 1} * \frac{(\ln r_j)^{x_j-1}}{(x_j-1)!}] \ge R_S^L$$

=>

$$\sum_{j=1}^{n} \{1 - r_j[1-\ln r_j + \frac{(\ln r_j)^2}{2!} + \cdots + (-1)^{x_j-1} * \frac{(\ln r_j)^{x_j-1}}{(x_j-1)!}]\} \le 1 - R_S^L$$

The results are shown in the following table 3.

Table 3

Optimal Solution for Example 2 (R_S^L=0.9, P=110, w=200)

Stage	1	2	3	4	5
Starting point, $\overline{R}^0$	0.8	0.8	0.8	0.8	0.8
Optimal point $\overline{R}^*$	0.63438	0.63672	0.69688	0.7	0.8
Optimal point $\overline{X}^*$	3	3	3	2	2
System Cost	88.998256				
System Reliability	0.9000				
Used P	78				
Used W	195.534639				
Initial step size	0.05				
Final step size	0.000195				

Example 3: The 5-stage mixed series system with stage 2 stand-by and other four stages are parallel configurations. We also limit the reliability of subsystem 3 greater or equal to 0.95. The problem can be modified by changing reliability constraint and adding one subsystem reliability constraint as follows:

$$[1-(1-r_1)^{x_1}][r_2[1-\ln r_2 + \frac{(\ln r_2)^2}{2!} + \cdots + (-1)^{x_2-1} * \frac{(\ln r_2)^{x_2-1}}{(x_2-1)!}]] \prod_{j=3}^{5} [1-(1-r_j)^{x_j}] \ge R_S^L$$

$$\Rightarrow (1-r_1)^{x_1} + [1 - r_2[1-\ln r_2 + \frac{(\ln r_2)^2}{2!} + \cdots + (-1)^{x_2-1} * \frac{(\ln r_2)^{x_2-1}}{(x_2-1)!}]] + \sum_{j=3}^{5} (1-r_j)^{x_j} \ge 1 - R_S^L$$

and $[1-(1-r_3)^{x3}]\geq 0.95$

The results are shown in the following table 4.

Table 4

Optimal Solution for Example 3 (R_S^L =0.9, P=110, w=200)

Stage	1	2	3	4	5
Starting point, $\overline{R}^0$	0.8	0.8	0.8	0.8	0.8
Optimal point $\overline{R}^*$	0.776562	0.8	0.7975	0.78	0.79
Optimal point $\overline{X}^*$	3	2	3	2	3
System Cost	154.129294				
System Reliability	0.900008				
Used P	78				
Used W	198.588197				
Initial step size	0.05				
Final step size	0.000195				

According to the above results of table 2, table3 and table 4, the design and development cost is 151.014 for series-parallel system, 88.998 for element stand-by system, and 154.129294 for n-stage mixed series system. It is very clear that the lowest cost is incurred in element stand-by system. The reason for this is because the component reliability level of all subsystems for element stand-by system which is (0.63438, 0.63672, 0.69688, 0.7, 0.8) are almost lower than the one for series-parallel system which is (0.83125, 0.84375, 0.83906, 0.7, 0.74844), and the redundancy number for all subsystems are pretty much the same for both systems which is (3,3,3,2,2) compared to (2,2,3,3,3). The research team can easily achieve system reliability requirement (0.9) by using lower reliability component in all stand-by subsystems when they design a new system. It can save a lot of money compared to typical series-parallel system. The highest cost is incurred in n-stage mixed series system of example 3. The reason for this is intuitive that there exists a limit in subsystem 3 which reliability needs to be equal to or greater than 0.95. Therefore, the system cost for example 3 is a little higher than example 1.

5. PERFORMANCE COMPARISONS

In this section we compare the system reliability performance of this proposed method with other 11 reliability-redundancy allocation methods by conforming to the constraints for two design configurations (weight, volume), system design and development budget. In order to facilitate comparison, we apply the following template problem referred in a lot of previous studies to evaluate the performance for all compared methods.

$$\text{Max } R_s(\vec{R},\vec{X})=\prod_{j=1}^{n}[1-(1-r_j)^{x_j}] \qquad (13)$$

s.t

$$\sum_{j=1}^{n}[\alpha_j(\frac{-t}{\ln r_j})^{\beta_j}][x_j+\exp(x_j/4)]\leq 175$$

$$\sum_{j=1}^{n}p_j(x_j)^2\leq 110$$

$$\sum_{j=1}^{n}w_j x_j*\exp(x_j/4)\leq 200$$

According to the above equations, the objective function in our model needs to be changed to system reliability function. To facilitate algorithm computation process in our model, the additive property of objective and constraint function for all subsystems is required. By taking the logarithm to the above objective function, it can be modified as

$$\text{Max} \ln R_s(\vec{R},\vec{X})=\sum_{j=1}^{5}\ln[1-(1-r_j)^{x_j}]$$

The results are then shown in the following table 5.

Table 5

	System. Rel.	Unused Cost	Unused P	Unused W
The Proposed Method	0.931054	0.02655	27	7.518918
XKL Method (Xu et al)	0.93167	0.014	27	7.519
LMBB(Kuo et al)	0.92975	0.00001	27	10.572
H-J& A-G-M(Tillman et al)	0.91494	0.03373	28	1.4118
H-J(Hooke&Jeeves)& S-V (Sharma&Venkateswaran)	0.91068	0.014	32	1.412
H-J(Hooke&Jeeves)& G-A-G(Gopal et al, 1980)	0.91068	0.014	32	1.412
H-J(Hooke&Jeeves)& N-N(Nakagawa&Nakashima)	0.89829	0.036	47	25.846
H-J(Hooke&Jeeves)& K-I (Kohda&Inoue)	0.93102	0.047	27	7.519
G-A-G(Gupal et al, 1978) & S-V(Sharma&Venkateswaran)	0.89821	0.013	32	4.465
G-A-G(Gupal et al, 1978) &G-A-G(Gopal et al, 1980)	0.90446	0.025	32	4.465
G-A-G(Gupal et al, 1978) &N-N(Nakagawa&Nakashima)	0.90376	0.020	27	7.519
G-A-G(Gupal et al, 1978) &K-I(Kohda&Inoue)	0.90373	0.004	27	4.465

From the results of the above table, it shows that our proposed approach gets better system reliability performance than other methods except for XKL method. But, from the previous studies, our approach can take advantage of more general objective and constraint function than XKL method, which usually uses derivatives to define sensitivity function in its algorithm. Also, this proposed approach performs normally in spending of cost and all resources compared to other combination methods.

6. CONCLUDING REMARKS

In this paper we have developed a specific reliability-redundancy optimization approach combining the revised H-J pattern search and dynamic programming with dominating sequences. Three examples have been demonstrated for different system configurations including series-parallel, element stand-by, and 5-stage mixed series system. The results show that the lowest cost is incurred in element stand-by system, then is the series-parallel system, and the highest cost is for n-stage mixed series system. This information is very helpful when system design and development process is initiated. We also compare the performance of this proposed method with other 11 reliability-redundancy allocation methods by conforming to different constraints. It clearly shows that our proposed approach gets better system reliability performance than other methods except for XKL method.

Besides the above numerical results and comparisons, in this model the dynamic programming with dominating sequences applied at each step on the revised H-J pattern search technique seems better than traditional heuristic approach or Genetic Algorithm. It is because that dynamic programming gives more exact optimal solution. Also, by applying dominating sequence concept on dynamic programming to limit the range of redundancy, it makes our approach more efficient compared to traditional dynamic programming method and Genetic Algorithm. In addition, Genetic Algorithm seems not to be popular and efficient to solve the allocation problem of the number of redundancy and arbitrary component reliability level simultaneously. It can be applied in reliability optimization problem by selecting appropriate "commercial-off-the shelf (COTS)" items from a number of different sources of supply, though.

The reliability constraints applied in some subsystems give more tools in design process to conform to performance requirement threshold or technique bottleneck. We can also easily modify our model to include maintenance consideration to meet availability requirement. All of these show that this proposed approach is not only an efficient way to solve the reliability-redundancy optimization problem, but also more realistic and flexible than a lot of conventional approaches.

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