

Continuous-Time Modeling of CRMA Protocol for High Speed LANs and MANs

管郁君

Eugenia Y. Huang

Department of Management Information Systems

National Chengchi University

Taipei, Taiwan, R.O.C.

Abstract

Since the emergence of Cyclic Reservation Multiple Access (CRMA) protocol for high-speed Local Area Networks (LANs) and Metropolitan Area Networks (MANs), there were only limited analysis results based on simulation and simplistic mathematical analysis. The channel access control method grants channel access to stations through channel reservations. It operates under service cycles and reserve cycles, and can be adopted in both folded and dual bus. The service commands and reserve commands signal the beginning of service cycles and reserve cycles, respectively. In this paper, an improved CRMA is described, based on direct inspection. Then, a D/Er/1 queueing model is developed to capture the essence of the mean cycle waiting time (MCWT) characteristics. The result shows the inter-relationship among MCWT, input traffic intensity, the number of stations, and the interarrival time of reserve commands. The probability of no waiting *Pr.(no waiting)*, which indicates the chance that a reservation's service starts as soon as it enters the global queue, is also calculated. This work gives a clear picture of how the stations of a CRMA network would perform collectively and qualitatively.

1. Introduction

Among the various recently proposed MAN Medium Access Control (MAC) layer protocols,

Distributed Queue Dual Bus (DRAB) [1, 2, 3] and Cyclic Reservation Multiple Access (CRMA) protocols have received the most attention due to their innovative designs. The functionality of MAC pro-

protocols correspond to the lower subset of the Data Link Control layer in the ISO (Organization for International Standards) Open System Interconnection Reference Model [4]. As an earlier proposal for MAN, DQDB has been extensively studied [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Thus, its behavior of unfairness in both channel capacity allocation and mean waiting time distribution is well known. Though there were a few proposed bandwidth (capacity) balancing mechanisms [18, 19, 20, 21], they are either too complex or not effective. This was the background of the emergence of CRMA. The proposal of CRMA protocol first appeared in [22], and limited analysis results can be found in [23, 24, 25]. The dual bus structure that is used in both DQDB and CRMA is described in many literatures. It will not be repeated in this paper. The popularity of DQDB and CRMA protocols is partly due to the powerful driving forces --- Bell Operating Companies for DQDB, and IBM for CRMA. This paper is by no means an advocate for CRMA. Before any meaningful comparisons can be made, comparable amount of analysis for CRMA, as for DQDB, should be conducted. This paper presents an elegant mathematical analysis using a continuous-time model. CRMA is designed for a slotted-time network, thus the analysis is not exact. However, never has been any mathematical analysis meant to describe the network performance exactly. The developed model, nevertheless, is probably the closet approximation one could obtain. The only difference between continuous- and slotted-time is that for continuous time, the transmission does not have to conform to the time slot boundary.

The improved CRMA protocol is described in Section 2. In Section 3, the definitions and as-

sumptions for the analysis are lay out, followed by the derivation of cycle service time distribution. One of the assumptions is that the reserve commands arrive at fixed time intervals, i.e., the arrival follows a deterministic (D) process. It turned out that the cycle service time is Erlang (Er) distributed. The MCWT and the *Pr.(no waiting)* expression are then derived using the properties of a D/Er/1 queue, in terms of two parameters. The solutions of these two parameters are further established in Section 4. Section 5 presents the result of MCWT distribution and *Pr.(no waiting)* versus the input traffic intensity, the number of stations, and the interarrival time of reserve commands. Brief conclusion, including the discussion of the implications of the analysis result, will be offered in Section 6.

2. Protocol Description

The access method for a slotted-time dual bus where each message is subdivided to equal-size packets for transmission is illustrated in Figure 1 and described as follows:

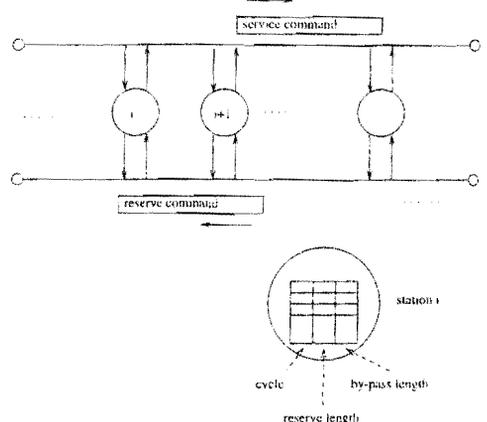


Figure 1: Cyclic Reservation Multiple Access protocol.

1. The tail end issues reservation commands periodically with cycle length set to zero. A reservation command consists of a preamble, cycle number, and cycle length.
2. As each station sees a reservation command, it can reserve slots for use in the reverse channel by augmenting the cycle length. The station records the cycle number as *cycle*, the number of slots it reserved as *reserve length*, and the cycle length prior to its own reservation as *by-pass length*.
3. At the head end, upon the arrival of a reservation command, the cycle number and the cycle length are entered into a global reservation queue.
4. The head end serves the global reservation queue according to the FIFO discipline by issuing service commands. A service command consists of a preamble, the cycle number, and is followed by as many empty slots as the current cycle length indicates.
5. A station, upon recognizing a matching cycle number with *cycle*, leaves "by-pass length" number of empty slots for downstream stations and then transmits as many packets in as many empty slots as the *reserve length* indicates.

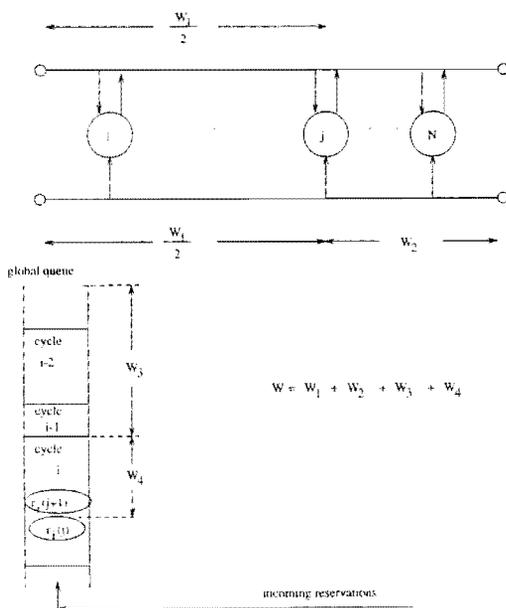


Figure 2. Mean message waiting time components.

2. the reservation delay, which is the time it takes for an arriving message to successfully reserve slots in a cycle,
3. the mean cycle waiting time at the global queue,
4. the by-pass delay, which is from the time the head end starts serving the corresponding cycle till the first empty slot that is designated to the station starts to leave the head end.

The second and third components are self-explained. The first component, the delay of round-trip time, arises from the fact that once a station reserved slots in a cycle, the reserve command has to propagate to the head end in order to enter the global queue, and when this cycle is being served by the head end, it will take an equal

As illustrated in Figure 2, the mean message waiting time at a station (e.g. station j) consists of the following four components:

1. the propagation delay, which is the round-trip time from the station to the head of the bus,

amount of time for the service command to reach the station. Clearly, the stations that are further downstream suffer longer propagation delays. To compensate this effect, we let the downstream stations be "served" first. In other words, the order of service is according to the order of reservation. The notion of "service" is from the view point of the head end. This service discipline is easily implemented by letting each station leave the recorded "by-pass length" number of slots for downstream stations at each service command. The total effect of the first and the fourth components will come to a balance at some point, because the former favors upstream stations, while the latter favors downstream stations. The merit of this arrangement of service is not that the mean message waiting times is equal at certain point for uniform traffic distribution among stations, but that the worst mean message waiting time is kept as small as possible.

There are two design parameters in this protocol. Namely, the interarrival time of reserve commands and the maximum number of slots each station is allowed to reserve in a cycle. The latter plays a secondary role in preventing heavy stations hogging the channel bandwidth. An arbitrary reasonable limit will serve this secondary control mechanism well. In view of preserving message integrity, this limit can be set to one message size, whatever the message size is. The access fairness induced by this one-message limit is subject to further study.

3. Analysis Model

A continuous-time model is first considered,

where the transmission does not have to conform to the time slot boundary. Namely, it can use any fraction of a time slot, and the remaining fraction of time slot can further be utilized.

Let

- $r_i(j)$ = the local reservation length of station j at cycle i , and
- t_i = interarrival time between reserve commands $i - 1$ and i ,
- x_i = service time for cycle i ,
- y_i = waiting time for cycle i ,
- ρ_j = input traffic intensity to station j ,
- ρ = total traffic intensity to the bus,
- n = the number of stations.

Define

$$u_i = x_i - t_{i+1}$$

and let $A(t)$, $B(x)$, $C(u)$ be the probability distribution functions of t_i , x_i , u_i , $W(y)$ be the limiting distribution function of y_i as $i \rightarrow \infty$, and $a(\cdot)$, $b(x)$, $c(u)$, $w(y)$, the corresponding probability density functions.

Assume that the reserve commands arrive at every fixed interval t , i.e. the interarrival process is deterministic with interarrival time $t_i = t$ for all i . Also assume that the reservation is exhaustive at every reserve command, i.e. each station reserves as much channel bandwidth as its local queue requires so that the local reservation length is independent from cycle to cycle. Now if the input traffic is uniformly distributed over all stations, then $r_i(j)$ are *i.i.d.* random variables. Assume

that $r_i(j)$ has exponential distribution with *p.d.f.*

(probability density function) $f(r) = \mu e^{-\mu r}, r \geq 0$.

Since

$$x_i = \sum_{j=1}^n r_i(j), \quad (1)$$

x_i 's are also *i.i.d.* random variables.

The *p.d.f.* for x_i , is the convolution of the *p.d.f.*'s for $r_i(j)$'s. That is, $b(x)$ is the convolution of $f(r)$:

$$b(x) = f(x) \otimes f(x) \otimes \dots \otimes f(x) \quad (2)$$

Taking the Laplace transforms of both sides gives

$$\mathcal{L}\{b(x)\} = \left(\frac{\mu}{\mu + s} \right)^n \quad (3)$$

Then $b(x)$ is easily obtained by inverting (See Laplace Transform Table [26]) the right hand side of Equation 3.

$$b(x) = \frac{\mu^n}{(n-1)!} x^{n-1} e^{-\mu x}, x \geq 0 \quad (4)$$

That is, the cycle service time x is Erlang distributed with mean $\frac{n}{\mu}$ and variance $\frac{n}{\mu^2}$.

By recognizing that the distance between two consecutive Poisson points is exponentially distributed, this Erlang density function can also be derived by determining the *p.d.f.* for the distance between two Poisson points that are $n-1$ Poisson points apart [27].

With the above assumptions, the global queue forms a *D/Er/1* queue with degree n , where *D* signifies that the interarrival process is deterministic, and *Er* signifies that the service time is Erlang distributed. The waiting time distribution for the cycles at the global queue can be derived by using Lindley's integral equation [28, 29] for *GI/G/1* queue, since *D/Er/1* queue is a special case of a *GI/G/1* queue.

Lindley's integral equation states that

$$w(y) = \begin{cases} -\int_{z=0}^{\infty} w(z) dC(y-z) & y \geq 0 \\ 0 & y < 0 \end{cases} \quad (5)$$

where $C(u) = \int_{t=0}^{\infty} B(u+t) dA(t) \quad (6)$

In the case of a *D/Er/1* queue, $t = t_0$, where t_0 is a constant,

$$\frac{dA(t)}{dt} = \delta(t - t_0)$$

Hence,

$$\begin{aligned} C(u) &= \int_{t=0}^{\infty} B(u+t) \delta(t - t_0) dt \\ &= B(u + t_0) \end{aligned} \quad (7)$$

Substituting Equation (7) into Equation (5) gives

$$w(y) = \begin{cases} -\int_{z=0}^y w(y-z) dB(z) & y \geq t_0 \\ 0 & y < t_0 \end{cases} \quad (8)$$

The mean waiting time distribution is obtained by direct substitution of the assumed solution

$W(y) = \sum_{s=0}^n \alpha_s e^{-\beta_s y}$ into Equation (8). This results in the following solution [29]:

$$W(y) = \sum_{s=0}^n \alpha_s e^{-\beta_s y}, \quad \alpha_0 = 1, \beta_0 = 1, \Re(\beta_s) \geq 0, \quad (9)$$

where β_s satisfies

$$\left(\frac{\mu}{\mu - \beta}\right)^n e^{-\beta t} = 1, \quad (10)$$

and α_s satisfies

$$\sum_{s=0}^n \frac{\alpha_s}{(\mu - \beta_s)} = 0, \quad k = 1, 2, \dots, n. \quad (11)$$

Let \bar{y} be the mean waiting time for the cycles at the global queue. Then, \bar{y} can be obtained as follows:

$$\begin{aligned} \bar{y} &= \int_{y=0}^{+\infty} y w(y) dy \\ &= \int_{y=0}^{+\infty} y \frac{dW(y)}{dy} dy \\ &= \int_{y=0}^{+\infty} y \left[\sum_{s=1}^n \alpha_s (-\beta_s) e^{-\beta_s y} \right] dy \\ &= - \sum_{s=1}^n \alpha_s \int_{y=0}^{+\infty} y \beta_s e^{-\beta_s y} dy \\ &= - \sum_{s=1}^n \alpha_s \left(\frac{1}{\beta_s} \right) \end{aligned}$$

Therefore we have

$$\bar{y} = - \sum_{s=1}^n \frac{\alpha_s}{\beta_s}. \quad (12)$$

Moreover, since by definition $W(y=0) = Prob.(y \leq 0)$, and $y \geq 0$, we have that $W(y=0) =$

$Prob.(y = 0)$. Since $W(y=0)$ is the probability of no waiting at the global queue, we have

$$Pr.(no\ waiting) = 1 + \sum_{s=1}^n \alpha_s \quad (13)$$

The fact that a probability is not greater than one tells us that the non-real α_s must come in complex conjugate pairs, and that the real part of α_s must be negative. Now recall that the local reservation length $r_i(j)$ has average $\frac{1}{\mu}$. Intuitively, $\frac{1}{\mu}$ is proportional to t and the input intensity ρ_j . For a uni-

form traffic model, $\rho_j = \frac{\rho}{n}$, this suggests that

$$\frac{1}{\mu} = \frac{t\rho}{n}, \quad (14)$$

and this $D/Erl/1$ queue is characterized by three parameters n, t , and ρ .

4. Solution

To solve for the β_s 's, we need to find the roots of the nonlinear transcendental equation (10). This type of nonlinear equations do not have explicit solution, it can only be solved numerically. However the off-the-shelf software packages do not provide means to solve a nonlinear equation with multiple roots directly. Therefore we use the following steps to reduce Equation (10) to $n \times 2$ nonlinear systems with the real part and imaginary part of β being the unknowns, and solve these n nonlinear systems.

Rewrite Equation (10) as

$$\left(1 - \frac{\beta}{\mu}\right)^n e^{\beta t} = 1,$$

and let $z = 1 - \frac{\beta}{\mu}$, i.e. $\beta = \mu(1-z)$. Equation (10)

then becomes

$$z^n e^{j\mu(1-z)} = 1.$$

Therefore, the roots are Z_k 's, $k = 0, \dots, n-1$, which satisfy

$$z_k e^{\frac{j\mu}{n}(1-z_k)} = e^{j\frac{2\pi k}{n}}. \tag{15}$$

If Z_k is expressed in the Cartesian complex form

$$z_k = g_k + jh_k,$$

where g_k is the real part and h_k is the imaginary part of Z_k , then equation (15) becomes

$$(g_k + jh_k) e^{\frac{j\mu}{n}(1-g_k)} e^{-j\frac{\mu}{n}h_k} = e^{j\frac{2\pi k}{n}}.$$

By multiplying both sides by $e^{j\frac{\mu}{n}h_k}$, and then equating separately the real part and imaginary part of both sides, Equation (15) is reduced to a 2 x 2 nonlinear system as follows, for $k = 1, \dots, n$.

$$g_k e^{\frac{j\mu}{n}(1-g_k)} = \cos \frac{2\pi k + \mu h_k}{n}, \tag{16}$$

$$h_k e^{\frac{j\mu}{n}(1-g_k)} = \sin \frac{2\pi k + \mu h_k}{n}.$$

Careful examination of the above equations reveals that the roots β_s 's come in complex conjugate pairs, except for real roots. These 2 x 2 nonlinear systems are solved by using the International Math and Statistics Library, which implements the Secant method [30] to solve a system of nonlinear equations. The β_s 's are then substituted in the $n \times n$ linear system, Equation (11), to solve for α 's.

Express Equation (11) in the matrix form

$$\underline{A}\alpha = \underline{v}:$$

$$\begin{bmatrix} \frac{1}{\mu-\beta_1} & \frac{1}{\mu-\beta_2} & \dots & \frac{1}{\mu-\beta_n} \\ \frac{1}{(\mu-\beta_1)^2} & \frac{1}{(\mu-\beta_2)^2} & \dots & \frac{1}{(\mu-\beta_n)^2} \\ \vdots & \vdots & \dots & \vdots \\ \frac{1}{(\mu-\beta_1)^n} & \frac{1}{(\mu-\beta_2)^n} & \dots & \frac{1}{(\mu-\beta_n)^n} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} -\frac{1}{\mu} \\ -\frac{1}{\mu^2} \\ \vdots \\ -\frac{1}{\mu^2} \end{bmatrix}$$

Note that the matrix \underline{A} is a Vandermonde matrix [31], and this linear system is a Vandermonde system. \underline{A} is nonsingular if and only if all β_s 's, $s=1, \dots, n$ are distinct. It is known that a Vandermonde matrix becomes increasingly ill conditioned as the dimension n increases [32], ill conditioned meaning that relatively small changes in the entries of \underline{A} can cause relatively large changes in the solution. This characteristic is a hinder in solving a high order system by Gaussian Elimination method, since accuracy is lost at each iteration. Hence, to obtain the numerical solutions, the explicit solution has to be found.

The determinant of \underline{A} is [31]

$$\det(\underline{A}) = \prod_{i>j} \frac{\beta_i - \beta_j}{(\mu - \beta_i)(\mu - \beta_j)} \tag{17}$$

Let \underline{A}_k be the matrix obtained by replacing the k th column of \underline{A} with the vector \underline{v} . Note that replacing the i th column of \underline{A} is equivalent to setting $\beta_k = 0$ in \underline{A} .

Therefore

$$\det(\underline{A}_k) = \det(\underline{A})|_{\beta_k=0}$$

Cramer's rule gives

$$\alpha_k = \frac{\det(\underline{A}_k)}{\det(\underline{A})} \quad k = 1, 2, \dots, n.$$

So, after simplification,

$$\alpha_k = - \left(\frac{\mu - \beta_k}{\mu} \right)^n \prod_{i \neq k} \frac{\beta_i}{(\beta_i - \beta_k)} \quad (18)$$

For large n , to prevent the computer's computation overflows, the calculation is carried out as

$$\alpha_k = - \frac{\mu - \beta_k}{\mu} \prod_{i \neq k} \frac{\beta_i (\mu - \beta_k)}{(\beta_i - \beta_k) \mu} \quad (19)$$

Figure (3) and (4) show the examples of the solutions α 's and β 's on the complex plane for $\rho = 0.9, t = 100$, and $n = 101$. Existence of solutions is proved in [29].

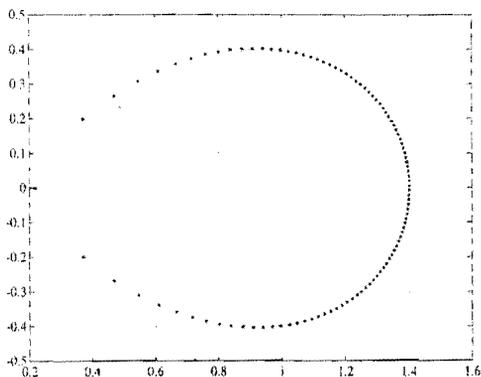


Figure 3: The β 's for $t=100, n=101$, and $\rho = 0.9$.

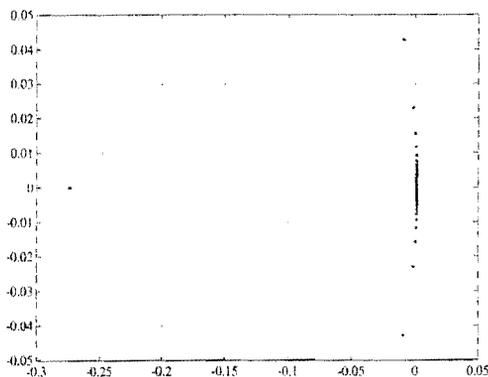


Figure 4: The α 's for $t=100, n=101$, and $\rho = 0.9$.

5. Analysis Result

The calculated results using the model developed in Section 3 and the solution derived in Section 4 are organized concisely, and plotted in Figures 5 to 8. The network performance indicators here are the mean cycle waiting time, \bar{y} , and the probability of no waiting, $Pr.(no\ waiting)$. These two quantities are plotted with varying n, t , and ρ , (the number of stations, inter-arrival time of successive reserve commands, and the total traffic intensity). For easier presentation in two-dimensional plot, in each Figure, one of the three parameters n, t , and ρ , is fixed, and the two performance indicators are plotted in the same figure for three values of another parameter, while the remaining parameter varying. In Figures 5 and 6, ρ is fixed at 0.9, in Figure 7, n at 50, and in Figure 8, t at 200. From Figure 5, it is observed that as n increases, $Pr.(no\ waiting)$ increases steadily, while the mean waiting time drops sharply. Moreover, though $Pr.(no\ waiting)$ stays the same for different values of t , the mean cycle waiting time increases as t increase. Figure 6 describe similar phenomenon using different illustration (reverting n and t). It is clear in Figure 6 that $Pr.(no\ waiting)$ is a constant with respect to t , and that the increase of \bar{y} with respect to t is linear. In Figure 7, again, different values of t give the same $Pr.(no\ waiting)$ curve. Comparing Figure 7 with Figure 3, it is easily seen that the effects of ρ on the two performance indicators, \bar{y} and $Pr.(nowaiting)$, are just opposite to that of n . When ρ is beyond 0.85, \bar{y} grows and $Pr.(no\ waiting)$ drops rapidly. Lastly, in Figure 8, t is fixed instead of n . $Pr.(no\ waiting)$ decreases, and \bar{y} increases, as n decreases and ρ increases. Three $Pr.(no\ waiting)$

and three \bar{y} curves appear for three values of n , with larger probability and smaller \bar{y} being for larger n .

6. Conclusion

From the result presented in Section 5, \bar{y} is dependent on n , t , and ρ , but $Pr(\text{no waiting})$ is uniquely determined by n and ρ . Among all the relationships of \bar{y} and $Pr(\text{no waiting})$ versus n , t , and ρ , it is easy to understand that \bar{y} increases, and $Pr(\text{no waiting})$ decreases, as ρ increases. The rest are not intuitively apparent, and this is the significance of this paper's contribution.

From the result, we know that to keep \bar{y} small, it is desirable to have large number of stations (n); however, to keep high $Pr(\text{no waiting})$, n has to be small. There is a tradeoff between \bar{y} and $Pr(\text{no waiting})$. One can argue that $Pr(\text{no waiting})$ may not be as important, as long as the waiting time is small. Consequently, this network supports large number of stations well. Since $Pr(\text{no waiting})$ does not depend on t , given n and ρ , and that \bar{y} decreases as t decreases, this network operates better at small t . Of course, one could further explore the possibility of changing t adaptively according to the traffic pattern or intensity, through simulation. But the rule of thumb is to keep t small.

Note that the analysis presented in this paper is for the most non-trivial component that contributes to the mean message waiting time at the local stations. Other three components are the propagation delay, reservation delay, and by-pass

delay. Propagation delay is a physical property that only depends on the bus length, and as long as small t value is used, the by-pass delay is not likely to overcompensate the propagation delay. (Please refer to the discussions in Section 2.). When the network is not overloaded, the reservation delay is bounded by t . Therefore, if the guideline of keeping t relatively small is followed, the mean message waiting time performance will be kept benign.

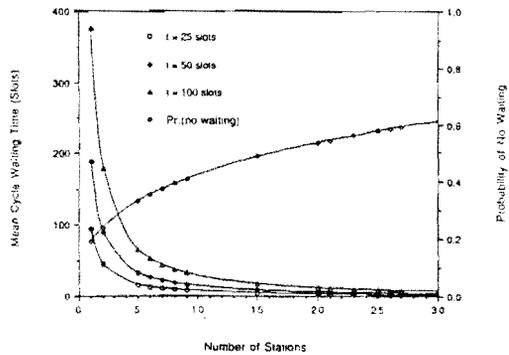


Figure 5: Mean waiting time and probability of no waiting versus the number of stations.

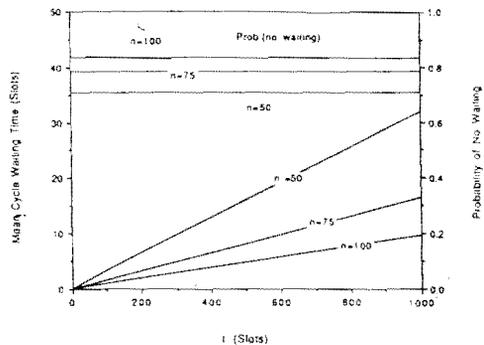


Figure 6: Mean waiting time and probability of no waiting versus the interarrival time.

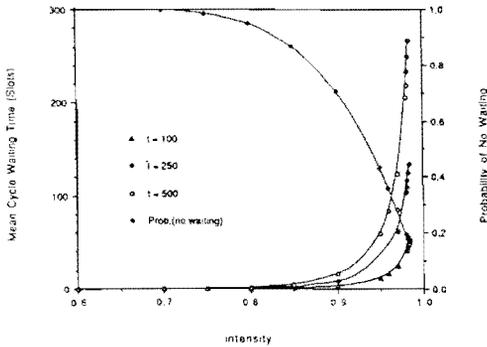


Figure 7: Mean waiting time and probability of no waiting versus the intensity, with $n = 50$.

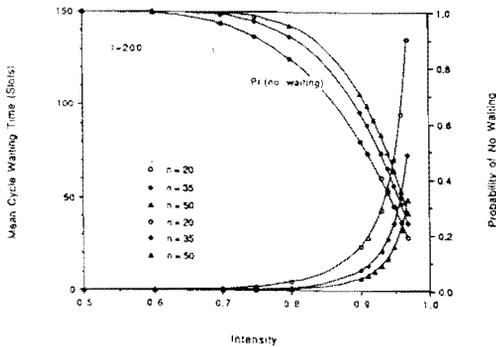


Figure 8: Mean waiting time and probability of no waiting versus the intensity, with $t = 200$.

References

[1] Z. L. Budrikis, R. M. Newman and J. L. Hullet, "QPSX: a queued packet and synchronous circuit exchange," in *ICCC'86 Proceedings*, pp. 288-293, 1986.

[2] R. M. Newman and J. L. Hullet, "Distributed queueing: a fast and efficient packet access protocol for QPSX," *ICCC'86 Proceedings*, pp. 294-299, 1986.

[3] *IEEE Standard 820.6*, "Distributed queue dual bus (DQDB) - Subnetwork of a metropolitan area network (MAN)," 1991.

[4] International Organization for Standardization. *Open Systems Interconnection - Basic Reference Model*, ISO 7498, 1984.

[5] J. W. Wong, "Throughput of DQDB networks under heavy load," in *Proceedings of EFOC/LAN Conference*, Amsterdam, The Netherlands, June 14-16, 1989, pp. 146-151.

[6] H. R. Van As, J. W. Wong, and P. Zafiropulo, "Fairness, Priority and Predicability of the DQDB MAC protocol under heavy load," *International Zürich Seminar*, Switzerland, March 5-8, 1990.

[7] H. R. Van As, J. W. Wong, and P. Azfiropulo, "QA DQDB analysis: fairness, predicability, and priority," *IEEE September 802.6-89/49 Contribution*, Montreal, Cannada, September 25-29, 1989.

[8] E. L. Hahne, N. F. Maxemchuk, and A. K. Choudhury, "Improving DQDB fairness," *IEEE September 802.6-89/49 Contribution*, Montreal, Canada, September 25-29, 1989.

[9] Mark W. Garrett and San-Qi Li, "a study of slot reuse in dual bus multiple access networks," in *Proceedings of INFOCOM'90*, San Francisco, June 1990, pp. 617-629.

- [10] Chatschik Bisdikian, "Waiting time analysis in a single buffer DQDB (802.6) network," in *Proceedings of INFOCOM'90*, San Francisco, June 1990, pp. 610-616.
- [11] M. Conti, E. Gregori, and L. Lenzini, "DQDB/FBS: a fair MAC protocol stemming from DQDB fairness analysis," in *Proc. 2nd Workshop Future Trends Distribut. Comput. Syst. 1990's*, pp. 66-74.
- [12] M. Conti, E. Gregori, and L. Lenzini, "DQDB under heavy load: performance evaluation and fairness analysis," in *Proceedings of INFOCOM'90*, San Francisco, June 1990, pp. 133-145.
- [13] M. Conti, E. Gregori, and L. Lenzini, "A methodological approach to an extensive analysis of DQDB performance and fairness," *IEEE Journal on Selected Areas in Communications*, Vol. 9, No. 1, January 1991, pp. 76-87.
- [14] Eugenia Y. Huang and L. F. Merakos, "On the access fairness of the DQDB MAN protocol," in *IPCCC'90 Proceedings*, pp. 325-329, March 1990.
- [15] Eugenia Y. Huang and L. F. Merakos, "Delay analysis of a unidirectional dual bus with one erasure station," in *Proceedings of the 1990 Conference on Information Science and Systems*, pp. 730-733, Princeton University, March 1990.
- [16] Eugenia Y. Huang and L. F. Merakos, "Mean packet delay analysis of a dual bus with one erasure station," *International Journal of Performance Evaluation*, Vol. 16, No. 1-3, pp. 177-184.
- [17] Special Issue of *IEEE Network - Focus on ATM LANs*, Vol. 7, No. 2, March 1993.
- [18] Hai Xie and L.F. Merakos, "Rate based head end controlled bandwidth balancing allocation in unidirectional bus metropolitan area networks," in *Proceedings of the 1990 Conference on Information Science and Systems*, The Johns Hopkins University, Baltimore, Maryland, March 1991.
- [19] Hai Xie and L.F. Merakos, "Rate based head end controlled bandwidth allocation in unidirectional bus metropolitan area networks," *IEEE J-SAC*, Vol. 11, No. 8, pp. 1259-1267, October 1993.
- [20] E. L. Hahne and N. F. Maxemchuk, "Fair access of multiple-priority traffic to DQDB networks," in *Proceedings of INFOCOM'91*.
- [21] H. R. Van As, "Performance evaluation of bandwidth balancing in the DQDB MAC protocol," in *Proceedings of EFOC/LAN Conference* Munich, Germany, June 27-29, 1990.
- [22] M. M. Nassehii, "CRMA: an access scheme for high-speed LANs and MANs," in *Proc. Supercom/ICC*, Atlanta, GA, April 1990.
- [23] Hans R. Muller, *et. al.*, "DQMA and CRMA: new access schemes for Gbit/s LANs and MANs," in *Proceedings of INFORCOM '90*, San Francisco, June 1990, pp. 185-191.
- [24] M. De Sanctis, *et. al.*, "Performance issues in CRMA networks for integrated broadband communications," in *Proceedings of INFOCOM '92*, Florence, Italy, May 1992, pp. 811-16.

- [25] L. Proiette, *et. al.*, "A CRMA based Gbit/s LAN-ATM based broadband ISDN gateway," in *GLOBECOM '92*, Orlando, FL, Dec 1992, pp. 1207-13.
- [26] Samuel M. Selby. *Standard Mathematical Tables*. The Chemical Rubber Co., 7th edition,
- [27] Athanasios Papoulis. *Probability, Random Variables, and Stochastic Processes*. John Wiley & Sons, second edition, 1975. Chapter 12. 1975.
- [28] Leonard Kleinrock. *Queueing Systems*. John Wiley & Sons, 1975.
- [29] D. V. Lindley, "The theory of queues with a single server," *Proc. Cambridge Philosophical Society* Vol. 48, (1952) pp. 277-289.
- [30] Phillip Wolfe, "Method for simultaneous nonlinear equations," *Communations of the ACM*, Vol. 2, (1959).
- [31] Werner Greub. *Linear Algebra*. Springer-Verlag, 4th edition, 1975. Chapter IV.
- [32] Steven J. Leon. *Linear Algebra with Applications*. Macmillan Publishing Company, 3rd edition, 1990. Chapter 7 and Appendix.