Surplus Management Under a Stochastic Process

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Abstract

To immunize an insurance company's surplus against interest-rate fluctuations, asset-liability managers commonly adopt the so-called classical immunization strategy to set the duration of the surplus equal to zero. Unfortunately, this strategy is derived on the basis of the flat-term structure. This article examines the immunization strategy with a stochastic process, which can generate a mean-reverting term structure. By means of the stochastic process, the authors provide a measurement for evaluating the assets and liabilities of the insurance company. The authors also show that the immunization strategy suggested in the article is a general model and includes the classical immunization strategy as a special case. Furthermore, if a firm's objective is to maximize its convexity of the surplus subject to zero surplus duration and its budget constraint, the authors demonstrate that linear programming can implement the optimal immunization strategy in this case. Moreover, the results of this simulation show that the cost of failing to recognize stochastic interest-rate changes can be extremely high.

Introduction

Traditionally, to immunize an insurance company's surplus against interest-rate fluctuation, asset-liability managers need to carefully arrange both the cash inflows and outflows of the firm. A classical immunization strategy serves to maintain the duration of assets equal to the asset/liability ratio times the duration of liabilities (Bierwag, 1987; Grove, 1974; Reitano, 1992). Although this strategy is easy to apply in practice, it is derived without recognizing the existence of the stochastic behavior of asset returns and liability returns.

Interest rates are actually mean-reverting and should be guided by a stochastic process (Vasicek, 1977; Dothan, 1978; Cox, Ingersoll, and Ross, 1979; Dothan and Feldman,

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1986; Ho and Lee, 1986; Chen et al., 1992; Heath, Jarrow, and Morton, 1992). Many researchers (e.g., Bierwag, Corrado, and Kaufman, 1992; Bierwag, Fooladi, and Roberts, 1993; Bierwag, Kaufman, and Toevs, 1993; Gagnon and Johnson, 1994; Vetzal, 1994; Reitano, 1996; Zenios et al., 1998) have adopted alternative stochastic processes to analyze the immunization strategies of income investments. However, relatively few (e.g., Briys and Varenne, 1997) have employed a stochastic process to investigate the immunization strategy of an insurance company. This article intends to fill this gap. The authors introduce a stochastic process suggested by Vasicek[1] (1977) to examine an insurance company's immunization strategy. Under the stochastic process, the authors use the measurement suggested by Cox, Ingersoll, and Ross (1979) to evaluate the assets and liabilities of the company. The authors also calculate the firm's surplus, the duration of the surplus, and the convexity of the surplus, and they show that the immunization strategy suggested in this article is a general model and includes the classical immunization strategy as a special case.

Some have suggested that convexity is a valuable supplement to duration in the case of interest rate-dependent portfolio investment. Douglas (1990) alleged that maximizing the convexity of the portfolio would maximize the convexity gain because of random changes in the interest rate. Christensen and Sorensen (1994) also demonstrated that if an investor expects interest-rate volatility to be greater than what appears from the term-structure, then maximizing convexity would induce the convexity gain to exceed the time-value effect. Thus, assuming that the firm's objective is to maximize its convexity of the surplus subject to zero surplus duration and its budget constraint, the authors demonstrate that linear programming can implement the optimal immunization strategy in this case. Moreover, the results of this simulation show that the cost of failing to recognize the stochastic structure of interest-rate changes can be extremely high.

Model

Consider an insurance company with assets A and liabilities L. Let $A(i)$ and $L(i)$ denote the cash inflows and cash outflows of the insurance company at period i. Following Cox, Ingersoll, and Ross (1979), assume that the present value of future cash flows of n periods is equal to the amount of cash flows times the value of a one-dollar zero-coupon bond. To cope with the spread of asset returns and liability returns, as suggested by Barney (1997), let $P_A(i)$ and $P_L(i)$ denote the value of a one-dollar zero-coupon bond discounted by the current rate of return on assets and liabilities, r_A and, r_L respectively. Under insurance practices, r_A and r_L represent the investment return of assets and the interest-rate change of the liability reserve. They usually differ both from the average interest rate/average rate of inflation (such as CPI) and from each other. One of the reasons for the spread between asset and liability returns is that changes in the inflation rate may have different effects on investment and liability claims for an insurance company.

[1] Vasicek (1977) recognized the mean-reverting of the interest-rate change. Cox, Ingersoll, and Ross (1979) further alleged that interest-rate volatility is proportional to the level of the interest rate. For simplicity, the authors have adopted Vasicek's model in this article. However, many models—such as those of Cox, Ingersoll, and Ross (1979); Dothan and Feldman (1986); Ho and Lee (1986); Chen et al. (1992); and Heath, Jarrow, and Morton (1992)—can also be chosen by insurance companies for their own management purposes.

Assume that

$$A = \sum_{i=0}^{n} A(i) P_A(i)$$

and

$$L = \sum_{i=0}^{n} L(i) P_L(i)\,. \tag{1}$$

Equation (1) is a general model for measuring the value of a firm's assets and liabilities.

Traditionally, many researchers usually assume that interest rate r is given, that $r_A = r_L = r$, and that they are the same at each period. Under these assumptions, a firm's assets and liabilities can be valued as the net present value of the future cash inflows and cash outflows, respectively. This traditional method is, indeed, a special case; i.e., when $P_A(i) = P_L(i) = \frac{1}{(1+r)^i}$ of Equation (1).

However, if the interest-rate follows a stochastic process rather than being a constant, the traditional method cannot evaluate a firm's assets and liabilities because of a lack of future interest-rate information. Intuitively, $\frac{1}{(1+r)^i}$ represents the value of a one-dollar zero-coupon bond evaluated by the net present value method. Thus, instead of using the net present value approach to evaluate the firm's value, the authors propose that the assets and liabilities of an insurance company under a stochastic process can be measured by Equation (1).

Assume that the returns of assets and liabilities follow the stochastic process suggested by Vasicek[2] (1977) and can be expressed as

$$dr_A = a_A(b_A - r_A)dt + \sigma_A dz\,, \tag{2}$$

where r_A is the spot rate of assets, and a_A, b_A, and σ_A are constants, and

$$dr_L = a_L(b_L - r_L)dt + \sigma_L dz\,, \tag{3}$$

where r_L is the spot rate of liabilities, and a_L, b_L, and σ_A are constants.

In the above stochastic process, dz follows a standard Brownian motion. The drift rates of the return on assets and liabilities are $a_A(b_A - r_A)$ and $a_L(b_L - r_L)$. The standard deviations of asset and liability returns are σ_A and σ_L, respectively. To estimate the drift rates and the standard deviations of assets and liabilities, asset-liability

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[2] Other authors have modeled the returns of assets and liabilities by many other stochastic processes, such as Cox, Ingersoll, and Ross (1979); Ho and Lee (1986); and Heath, Jarrow, and Morton (1992), among others.

managers can regress the difference of the returns on the returns.[3] Vasicek (1977) solved Equations (2) and (3) and showed that

$$P_A(i) = \alpha_A(i)\exp(-\beta_A(i)r_A), \tag{4}$$

where
$$\beta_A(i) = \frac{1-\exp(-a_A i)}{a_A} \tag{5}$$

and
$$\alpha_A(i) = \exp\left(\frac{(\beta_A(i)-i)(a_A^2 b_A - 0.5\sigma_A^2)}{a_A^2} - \frac{\sigma_A^2\beta_A^2(i)}{4a_A}\right). \tag{6}$$

$$P_L(i) = \alpha_L(i)\exp(-\beta_L(i)r_L), \tag{7}$$

where
$$\beta_L(i) = \frac{1-\exp(-a_L i)}{a_L} \tag{8}$$

and
$$\alpha_L(i) = \exp\left(\frac{(\beta_L(i)-i)(a_L^2 b_L - 0.5\sigma_L^2)}{a_L^2} - \frac{\sigma_L^2\beta_L^2(i)}{4a_L}\right). \tag{9}$$

If $a_A = a_L = 0$ and $\sigma_A = \sigma_L = 0$, then $\beta_A(i) = \beta_L(i) = i$ and $\alpha_A(i) = \alpha_L(i) = 1$. The term structures of Equations (2) and (3) approach flat-term structures. Moreover, $P_A(i)$ and $P_L(i)$ convert to $\exp(-ir_A)$ and $\exp(-ir_L)$, which are continuous discount factors of assets and liabilities at period *i*. Thus, we can conclude that this model is a general one and includes the traditional models as a special case.

From an accounting point of view, the surplus of firm *S* is equal to the difference between its assets and liabilities. Thus

$$S = \sum_{i=0}^{n} A(i)P_A(i) - \sum_{i=0}^{n} L(i)P_L(i). \tag{10}$$

Equation (10) provides a new way for the insurance company to discount its cash outflows and inflows in each period and further measure the value of the firm. Now the issue is what immunization problem may concern the managers if the surplus of the insurance company is valued by Equation (10). The firm's surplus is immunized locally by the change in interest rates if $\frac{dS}{dr} = 0$.[4] Assume that a change in interest rate

[3] By using the Generalized Method of Moments, Chen et al. (1992) provided a valuable methodology to estimate a variety of models of the short-term interest rate.

[4] $\frac{dS}{dr} = 0$ is well defined for a nonstochastic interest rate change but may be undefined under the stochastic process. Since $P_A(i)$ and $P_L(i)$ can be determined explicitly by Equations (4) and (7), it is reasonable to assume that $\frac{dS}{dr}$ can be derived directly.

r has different effects on the spot rates of assets and liabilities; i.e., $\frac{dr_A}{dr} = C_A$ and $\frac{dr_L}{dr} = C_L$, where C_A and C_L are constants. Differentiating the surplus with respect to the interest rate gives us

$$\frac{dS}{dr} = \sum_{i=0}^{n} A(i) C_A \frac{dP_A(i)}{dr_A} - \sum_{i=0}^{n} L(i) C_L \frac{dP_L(i)}{dr_L} = 0 \tag{11}$$

The duration of assets D_A and the duration of liabilities D_L are defined as follows:

$$D_A = -\frac{\sum_{i=0}^{n} A(i) C_A \frac{dP_A(i)}{dr_A}}{\sum_{i=0}^{n} A(i) P_A(i)}$$

and

$$D_L = -\frac{\sum_{i=0}^{n} L(i) C_L \frac{dP_L(i)}{dr_L}}{\sum_{i=0}^{n} L(i) P_L(i)}$$

Thus, substituting D_A and D_L into Equation (11) and rearranging terms, we obtain

$$D_A = \left(\frac{L}{A}\right) D_L . \tag{12}$$

Like classical immunization strategy, Equation (12) suggests that the firm's surplus is immunized against interest-rate risk if the duration of the firm's assets is set equal to the debt ratio times the duration of the firm's liabilities. Moreover, the measurement of duration in Equation (12) is a general model that can be applied to both flat and nonflat term-structures. Equation (12) can generate the same results as Bierwag (1987) and Barney (1997). If $C_A = C_L$, $a_A = a_L = 0$, and $\sigma_A = \sigma_L = 0$, and the future cash flows are discounted discretely rather than continuously—i.e., $\exp(-ir_A)$, and $\exp(-ir_L)$ are replaced by $\frac{1}{(1+r_A)^i}$ and $\frac{1}{(1+r_L)^i}$—Equation (12) corresponds to Barney (1997). If r_A is even equal to r_L, then Equation (12) is precisely the classical immunization strategy.

As Douglas (1990) and Christensen and Sorensen (1994) suggested, if an asset-liability manager expects the volatility of interest rates to be greater than what appears in the term-structure, then the firm's objective would be to maximize its convexity of the surplus subject to the zero surplus duration and its budget constraints. In general, $\frac{dS}{dr}$ is a good measure for the interest-rate risk only if the movement of the interest rate is small. From Taylor's expansion series, we know $\Delta S = \sum_{k=1}^{m} \frac{d^k S}{dr^k} \frac{(\Delta r)^k}{k!}$. If the shift in the interest rate is small, then the change in the surplus can be approximated by Taylor's expansion with $m = 1$; that is, $\Delta S \approx \frac{dS}{dr}\Delta r$. It is obvious that the surplus of the firm is immunized by the change in the interest rate if $\frac{dS}{dr} = 0$, which is also the rationale of Equation (12). However, if the shift in the interest rate is not small, then Taylor's expansion with $m = 2$ will provide a more accurate approximation, thus, $\Delta S \approx \frac{dS}{dr}\Delta r + \frac{1}{2}\frac{d^2S}{dr^2}(\Delta r)^2$. Note that the second term of ΔS, $\frac{1}{2}\frac{d^2S}{dr^2}(\Delta r)^2$, will always be positive if $\frac{d^2S}{dr^2} > 0$. In other words, if $\frac{d^2S}{dr^2} > 0$, then the surplus of the firm increases whether the interest rate increases or decreases. Therefore, if the shift in the interest rate is not small, maximizing $\frac{d^2S}{dr^2}$ is the best strategy for immunization when $\frac{dS}{dr} = 0$. In this case, the firm's optimal immunization strategy can be expressed as

$$\operatorname*{Max}_{A(i)} \quad \frac{d^2S}{dr^2}$$

subject to

$$S = A - L,$$

and

$$\frac{dS}{dr} = 0. \tag{13}$$

From Equation (10), it is obvious that the convexity of surplus can be calculated as

$$\frac{d^2S}{dr^2} = \sum_{i=0}^{n} A(i)\frac{d^2P_A(i)}{dr^2} - \sum_{i=0}^{n} -(i)\frac{d^2P_L(i)}{dr^2}. \tag{14}$$

Thus, substituting Equations (10), (12), and (14) into Equation (13), we obtain

$$\operatorname*{Max}_{A(i)} \quad \frac{d^2S}{dr^2} = \sum_{i=0}^{n} A(i)\frac{d^2P_A(i)}{dr^2} - \sum_{i=0}^{n} L(i)\frac{d^2P_L(i)}{dr^2}$$

subject to

$$S = \sum_{i=0}^{n} A(i)P_A(i) - \sum_{i=0}^{n} L(i)P_L(i),$$

and

$$D_A = \left(\frac{L}{A}\right)D_L \tag{15}$$

In Equation (15), *A(i)* is the insurer's investment decision for asset-liability management in each period. It is important to recognize that when the firm's surplus *S* and both the liability schedule[5] $L(i)$ and the stochastic processes of asset returns and liability returns are given as parameters, Equations (10), (12), and (14) are all linear functions with respect to $A(i)$. Therefore, linear programming can solve Equation (15).

Numerical Examples and Comparisons

The previous section provides theoretical models for an insurance company's surplus management. To demonstrate the application of this model, the authors will construct a hypothetical insurance company. In real practice, an insurance company may need to fulfill certain solvency constraints, such as a risk-based capital adequacy requirement or other minimum solvency margins required by rating agencies. Thus minimum solvency margins and non-negative constraints are further considered to make this simulation more realistic.

The results of the simulation show that linear programming can implement the optimal immunization strategy of this model. The authors also find that the cost of failing to recognize the stochastic term-structure can be extremely high.

For simulation purposes, the authors have created a balance sheet for the hypothetical insurance company at period 0 (see Exhibit 1).

Exhibit 1
Balance Sheet of a Hypothetical Insurance Company

Assets	Liabilities	Surplus
$3,337,782	$2,837,782	$500,000

Without losing any generality, the authors assume the liabilities are to be paid out in five years, as shown in Exhibit 2.

Exhibit 2
Liability Schedule of the Hypothetical Insurance Company

Periods	Liabilities
1	$591,500
2	$633,700
3	$677,400
4	$723,500
5	$775,800

[5] Under real insurance practices, the firm's liability schedule is sometimes hard to predict. Recent research findings of "effective duration of insurance liabilities"—e.g., Babbel, Merril, and Planning (1997) and Briys and Varenne (1997)—can help to make more accurate predictions.

Assume that the current interest rate r = 5 percent. Also assume that $r_A = r$ and $r_L = 0.01 + 1.2r$; i.e., $C_A = 1$, and $C_L = 1.2$. Further assume that $a_A = 0.1$, $a_L = 0.2$, $b_A = 0.05$, $b_L = 0.08$, $\sigma_A = 0.03$, and $\sigma_L = 0.1$. By means of Equation (15), the firm's immunization strategy can be modeled as:

$$\max_{A(i)} \quad \sum_{i=0}^{n} [C_A^2 \alpha_A(i) \beta_A^2(i) \exp(-\beta_A(i) r_A)] A(i)$$

subject to

$$\sum_{i=0}^{n} [C_A \alpha_A(i) \beta_A(i) \exp(-\beta_A(i) r_A)] A(i) = 3{,}337.78,$$

and

$$\sum_{i=0}^{n} [\alpha_A(i) \exp(-\beta_A(i) r_A)] A(i) = 7{,}374.23. \tag{16}$$

Assume that the insurance company can reinvest its net cash flows at each period in the same investment portfolio. The minimum solvency margin, K = \$100,000, and the non-negative constraints of the firm are further required. These constraints can be expressed as

$$\sum_{i=0}^{j} (A(i) - L(i)) \frac{P_A(i)}{P_A(j)} \geq K, j = 1,2,3,4,5$$

and

$$A(i) \geq 0, i = 0,1,2,3,4,5. \tag{17}$$

Equation (16), combined with the constraints of Equation (17), obviously can be solved by linear programming. The results of the linear programming are as shown in Exhibit 3.

EXHIBIT 3
Optimal Asset Allocation

Periods	Assets
0	\$1,102,823
1	\$0
2	\$136,780
3	\$672,806
4	\$717,866
5	\$1,198,887

To demonstrate the cost of failing to recognize the existence of the stochastic processes of asset returns and liability returns, the authors generate a counter-example in which the asset-liability manager fails to recognize the existence of these stochastic processes. Therefore, he or she assumes that $a_A = 0$, $a_L = 0$, $\sigma_A = 0$, and $\sigma_L = 0$.

Other parameters in the counter-example are the same as those in the stochastic process case (optimal case). Thus the asset-liability manager allocates the firm's assets as shown in Exhibit 4.

EXHIBIT 4
Asset Allocation of the Counter-Example

Periods	Assets
0	$273,068
1	$404,506
2	$628,655
3	$672,300
4	$718,498
5	$1,092,471

To compare the effects of immunization results in the optimal case and in the counter-example, the authors assume that the interest rate shifts from 5 percent to 3 percent, 4 percent, 6 percent, or 7 percent.[6] The surpluses of the optimal case and the counter-example are as follows:

EXHIBIT 5
The Cost of Mismatch

(1)	(2)	(3)	(4)=(3)–(2) ((4)/$500,000)
Interest Rate	Optimal Case	Counter-Example	Differences
5%	$500,000	$416,000	–$84,000 (–16.77%)
6%	$500,000	$408,000	–$92,000 (–18.42%)
4%	$500,000	$425,000	–$75,000 (–15.03%)
7%	$500,000	$400,000	–$100,000 (–19.99%)
3%	$500,000	$433,000	–$67,000 (–13.22%)

Exhibit 5 illustrates that the cost of failing to recognize the existence of the stochastic process is extremely high, ranging from –13 percent to nearly –20 percent. However, in all the situations, the firm's surplus in the stochastic process case is nearly immunized against the interest-rate risk.

[6] In this simulation, the authors assume the shift in the interest rate is nonstochastic for simplicity of demonstration, although our model can be applied for both stochastic change and nonstochastic change of interest rates. From a practical point of view, the managers of the insurance company may notice the existence of the stochastic structure of interest rates in the long run but may be concerned with the nonstochastic change in the interest rate in the short run. Thus to evaluate a more precise value of the firm, the managers of the firm choose $P_A(i)$ and $P_L(i)$ to discount the cash flows of the firm rather than $\frac{1}{(1+r)^i}$. Since $P_A(i)$ and $P_L(i)$ can be determined explicitly by spot rates, the firm's managers treat the interest rate risk of the firm only as a shift of the turn structure caused by a change in the spot rate, after determining the discount factors for each period by Equations (4) and (7).

Conclusions

In this article, the authors have examined the immunization strategy of surplus management under a stochastic process for an insurance company. This immunization strategy is a general model that one can apply to both flat and nonflat term structures. The authors have shown that this model can produce the results of the classical immunization strategy suggested by Bierwag (1987) and Barney (1997). Furthermore, if an asset-liability manager expects the interest-rate volatility to be greater than what appears in the term structure, then the firm's objective is to maximize its convexity of the surplus subject to a zero surplus duration and its budget constraint. The authors have demonstrated that in this case, the firm's optimal immunization strategy can be implemented by linear programming. Moreover, the results of this simulation have shown that the cost of failing to recognize the stochastic interest changes can be extremely high. As a future extension of this article, the authors suggest exploring a more general term-structure model related to different interest-rate-dependent portfolio situations. In addition, they also suggest dynamic programming for further research on this issue, since maximizing the convexity of the surplus does not always result in superior surplus management results.[7]

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[7] Gagnon and Johnson (1994) and Barber and Copper (1997) have demonstrated that matching the convexities of asset and liability does not always improve the immunization results.

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