Chapter 7.
CONCLUSION AND FUTURE RESEARCHES

7.1 Conclusions

In this dissertation, we propose a new reliability model, DwACM (Dichotomous-data with Auxiliary Continuous Measurements) which contains both dichotomous and continuous observations. By regarding the lifetime as a latent variable, these two kinds of data can be linked. Integrating out the lifetimes, the likelihood can be obtained. However, most of the cases they can not be integrated out analytically. We use EM-algorithm and MCMC to carry out the estimates under the frequentist and Bayesian frameworks, respectively.

The regular EM-algorithm can not be applied directly because of the complexity of the likelihood. We use a MCEM with an importance sampling technique to overcome the difficulty. An modification whose idea is borrowed from Laplace method is also applied to moderate some computational problems. For the Bayesian inferences, we consider the Gibbs sampler to approximate the posterior distribution.

We also use this model to select a suitable degradation measurement. And the "correct classification probability (CCP)" is chosen as the measurement selection criterion. For the random-slope linear degradation model, the measurement with the highest $r = |\beta_1|/\sigma$ can be proved to be the one with the highest CCP, which means it is the most suitable degradation measurement.

In the end, the whole procedure is illustrated by simulation studies, and the results seems to be very promising.

7.2 Future Researches

We present a simple model to illustrate the main issues in the dissertation and it is easy to extend the model in several directions. First, the lifetime distribution in the model can be varied, for example, to be Weibull or lognormal. Next we can replace $\exp(\alpha_1 W_i)$ in (3.2) by $\sum_{k=1}^{K} \exp(\alpha_k W_{ki})$, a model with K covariates. The basic idea of treating the extended model is the same as the simple one, although some computational difficulties may be raised. Here we point out two of them, and give some suggestions for these problems as future research directions.

First, the new lifetime distribution may cause the latent parts in the complete log-likelihood are then not linear. Thus, the maximizer in the M-step can not be obtained easily by replacing the latent parts with their conditional expectations. The maximization must be done by maximizing the sample average approximation

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(p)}) \approx \sum_{i=1}^{N} \frac{\sum_{s=1}^{B} \log L_i^c(\boldsymbol{\theta}; T_i^s, \boldsymbol{Y}_i) \prod_{j=1}^{n_i} \phi(\frac{Y_{ij} - \beta_0^{(p)} - \beta_1^{(p)} \frac{t_{ij}}{T_i^s}}{\sigma^{(p)}})}{\sum_{s=1}^{B} \prod_{j=1}^{n_i} \phi(\frac{Y_{ij} - \beta_0^{(p)} - \beta_1^{(p)} \frac{t_{ij}}{T_i^s}}{\sigma^{(p)}})},$$

where $T_i^1, \cdots, T_i^B$ are random samples from the conditional distribution of T_i given Z_i.

Second, the increasing covariates will cause the problem of high dimensional maximization. If the proportional hazards structure is maintained, the maximization respect to α_k given the rests can be down as that in Chapter 5. Therefore, the ECM algorithm provided by Meng and Rubin (1993) gives a possible approach to this problem.

We can also consider to release the assumption of the random-slope-linear degradation. When the intercept is also a random variable, one needs to choose the joint distribution of the random slope and intercept so that the derived lifetime distribution satisfies the assumption. In traditional degradation data analysis, the model of degradation process can contain more than one random parameters. Usually, we assume that those random parameters follow a multivariate normal distribution. Then the lifetime distribution can be obtained by either numerical integrations or Monte Carlo Methods (cf. Chapter 13 of Meeker and Escobar, 1998.) However, if the lifetime distribution is given, there are infinite many ways to choose the joint distribution of these two random components. Hence, how to choose a suitable distribution, which can model the real situation well and will not cause many computational difficulties, is another future research direction.

Also, instead of the soft failures, if the hard failures are taken into consideration, one will face the same difficulty in seeking for a suitable joint distribution of random parameters and the random threshold so that the assumed lifetime distribution can be derived.