Chapter 1

Introduction

To expand the range of life insurance products, life insurance companies have recently begun offering complex contracts with embedded options: participating policies with interest rate guarantees and surrender options; and equity-linked contracts with a guaranteed payoff. Each option contributes to firm liabilities. According to Briys and de Varenne (1997), numerous life insurance companies neglected the significance of these options, thus exposing themselves to the risk of insolvency. Consequently, fair valuation of life insurance liabilities recently has aroused considerable attention.

Fair value is an accounting term which has similar definitions in the U.S. accounting standards, such as the Statement of Financial Accounting Standard (SFAS) 87 with respect to pension assets, and SFAS 107 (1992) with respect to financial instruments. The general concept of fair valuation in these accounting standards is that the insurance liabilities are to be valued as if they were traded among well-informed investors (or willing and able to transact) in liquid marketplace. The application of fair values in life insurance companies' balance sheets means that assets and liabilities will have to be marked-to-market.

FASB(2004) introduces a fair values hierarchy. Fair value estimates are classified at three quality levels:

(1) Use market value when available.
(2) When no market value is available for the exact same instrument, use the market value of similar instruments, adjusted for the differences between the instrument to be valued and the similar instruments.
(3) If no market value is available and no suitable similar instruments are available, use a present value estimate of future cash flows. The present value should include an adjustment for risk.

According to Jørgensen (2004), the market prices for life insurance policies are rarely available, and not possible to find traded securities with a sufficiently close similarity to the obligation of life insurance policies. Therefore, we must step down in the fair value hierarchy and aim for level 3 which inevitably involves some financial modeling techniques.

Owing to the pioneering work by Brennan and Schwartz (1976, 1979a, 1979b) and Boyle and Schwartz (1977), the valuation of options in life insurance products recently has advanced markedly.

Participating policies enable policyholders to receive dividends (bonuses) in addition to the promised payments implied by the guaranteed interest rate. Because of the existence of the interest rate guarantee in the participating policies, the participating mechanism resembles that of European call options. This specific feature has been studied by Briys and de Varenne (1994, 1997a, 1997b), Grosen and Jørgensen (2000, 2002), Jensen, Jørgensen, and Grosen (2001), Miltersen and Persson (2003), and Bacinello (2001).

Briys and de Varenne (1994, 1997a, 1997b) evaluated the asset and the liability of a single-period participating policy under the stochastic interest rate model. Their model did not take the mortality risk into account and assumed that the default could occur only at maturity. Their framework was of European type, and they can therefore obtain closed-form formulae.

Grosen and Jørgensen (2000) and Jensen, Jørgensen, and Grosen (2001) analyzed a participating policy with a minimum interest rate guarantee and the dynamics of asset and liability related to the policy. The European bonus option was associated with the percentage of the positive performance of the company's asset portfolio and the minimum interest rate guarantee. The value of bonus option can be priced by Monte Carlo simulation.

Grosen and Jørgensen (2002) considered a participating policy with a minimum interest rate guarantee. They constructed the initial asset which was invested by two agents: the policyholders and the equityholders. At maturity, the policyholders had prior claim on the companies’ assets, whereas the equityholders had the limited liabilities. The policyholders also were entitled to receive a bonus if the market value of assets evolved sufficiently favorable. The maturity payoff of policyholders can be decomposed into three terms: a promised amount based on guaranteed interest rates, a bonus option and a shorted put option associated with a default risk. The maturity payoff of equityholders can be decomposed into two terms: a long call option and a shorted call option. Under the Black and Scholes (1973) and Merton (1973) framework, they worked with a constant interest rate and derived the closed-form formulae for the underlying options. Finally, they considered the rebate given to the policyholders in case of default prior to maturity.

Miltersen and Persson (2003) studied the Guaranteed Investment Contracts, which are like a deposit account with guaranteed rate of return. They calculated the fair value of the contracts by all the possible combinations of the guaranteed rate of return and the fraction of the positive excess rate of return. They proposed a multi-period extension and also provided closed-form formulae under the Heath, Jarrow, and Morton (1992) framework.

Bacinello (2001) analyzed a life insurance endowment participating policies with a minimum guaranteed interest rate, in which the benefits were annually adjusted according to the performance of a reference portfolio. Under the Black and Scholes (1973) and Merton (1973) framework, the closed-form formulae of such policies were derived in terms of one-year call options. The pricing was achieved with the assumption of the independence between mortality risk and financial risk. Bacinello (2001) further analyzed the sensitivity of contractual parameters, such as the risk-free interest rate (discount rate), the technical rate (minimum guaranteed interest rate), the participation level and the volatility of the reference portfolio. Bacinello (2001) pointed out that an American-type put option (surrender option), which authorized the policyholder to sell back the policy to the insurer at the surrender cash value, had not been evaluated.

The presence of a surrender option in a contract implies that the policyholders can sell the contract back to the issuer before maturity. The problem of valuing surrender options embedded in life insurance contracts has been studied within the framework of constant risk-free interest rate.

Grosen and Jørgensen (1997) analyzed the valuation of American-type contracts with interest rate guarantee using the optional sampling theorem. Grosen and Jørgensen (2000) priced surrender options embedded in bonus policies via a binomial tree approach. Meanwhile, Jensen, Jørgensen, and Grosen (2001) assessed such surrender options using a finite difference approach. Bacinello (2003a) also used the model of Cox, Ross and Rubinstein (1979) to determine the fair value of the contract. Moreover, Bacinello (2003a) performed sensitivity analysis for the contractual parameters. Additionally, Bacinello (2003b) finished calculating the periodic premiums for this policy. Tanskanen and Lukkarinen (2003) numerically solved the partial differential equation for the fair value of participating contracts.

Life insurance policyholders may surrender their contracts to exploit the higher yields available in financial markets. Surrender option is a concern for life insurance companies, particularly during high interest rate volatility. If the guaranteed return is not sufficiently high compared to other forms of investment, mainly when interest rates rise, policyholders may terminate their existing policies early and chase higher yields offered in capital markets. Therefore, this work considers the stochastic interest rates.

The surrender option complicates the valuation owing to the problems of the path dependency and the optimal stopping time, especially in situations involving stochastic interest rates. The problem of valuing the surrender option embedded in life insurance contracts under stochastic interest rate model has been tackled, with different assumptions and by various methodologies, such as those by Longstaff and Schwartz (2001), Bernard et. al. (2005).

Longstaff and Schwartz (2001) developed the least squares Monte Carlo (LSM) approach for valuing and optimally exercising American-type options. Moreover, Bernard et. al. (2005) studied the valuation of life insurance participating contracts as introduced by Grosen and Jørgensen (2002) in a stochastic interest rate environment. Meanwhile, Bernard et. al. (2005) considered early firm default and approximated default time distribution and derived the semi-closed formulae for the policy.

The main aim of this study is to price the participating policies introduced by Bacinello (2001) embedded with surrender options under a stochastic interest rate model. This work proposes a two-dimensional Cox-Ross-Rubinstein (CRR) model capable of efficiently calculating the embedded surrender option in the policy. Two-dimensional CRR approaches are applied to analyze the importance and sensitivity of a stochastic interest rate model for the policy.

The rest of this study is organized as follows. Chapter 2 discusses the structure of the participating policy and a stochastic interest rate model. Two-dimensional CRR models are then developed to assess the contract in Chapter 3. Next, Chapter 4 analyzes the accuracy and convergence of the two-dimensional CRR model, and analyzes the sensitivity of the fair value to the volatility parameters. Finally, conclusions and further research directions are outlined in Chapter 5.