附錄一

利用 (3.5) 式的 BGGD 衍生版本的聯合機率密度函數，可分別推導出 (X,Y) 的邊際分配機率密度函數，推導的過程如下：

$$f(x)=\int_0^{\infty} f(x,y)dy$$

$$
\begin{aligned}
&=\int_0^{\infty}\frac{r_1 r_2 x^{\frac{r_1}{2}-1} y^{\frac{r_2}{2}-1}}{8\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\{\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2}-2\rho\frac{(x^{\frac{r_1}{2}}-m)(y^{\frac{r_2}{2}}-n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}-n)^2}{\sigma_2^2})] \\
&\qquad +\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2}-2\rho\frac{(x^{\frac{r_1}{2}}+m)(y^{\frac{r_2}{2}}+n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}+n)^2}{\sigma_2^2})] \\
&\qquad +\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2}+2\rho\frac{(x^{\frac{r_1}{2}}+m)(y^{\frac{r_2}{2}}-n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}-n)^2}{\sigma_2^2})] \\
&\qquad +\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2}+2\rho\frac{(x^{\frac{r_1}{2}}-m)(y^{\frac{r_2}{2}}+n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}+n)^2}{\sigma_2^2})]\}dy
\end{aligned}
$$

$$=\frac{r_1 r_2 x^{\frac{r_1}{2}-1}}{8\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}[(A)+(B)]$$

其中

$$
\begin{aligned}
(A)=\int_0^{\infty} y^{\frac{r_2}{2}-1}&\{\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2}-2\rho\frac{(x^{\frac{r_1}{2}}-m)(y^{\frac{r_2}{2}}-n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}-n)^2}{\sigma_2^2})] \\
&+\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2}+2\rho\frac{(x^{\frac{r_1}{2}}-m)(y^{\frac{r_2}{2}}+n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}+n)^2}{\sigma_2^2})]\}dy
\end{aligned}
$$

$$
\begin{aligned}
(B)=\int_0^{\infty} y^{\frac{r_2}{2}-1}&\{\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2}-2\rho\frac{(x^{\frac{r_1}{2}}+m)(y^{\frac{r_2}{2}}+n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}+n)^2}{\sigma_2^2})] \\
&+\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2}+2\rho\frac{(x^{\frac{r_1}{2}}+m)(y^{\frac{r_2}{2}}-n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}-n)^2}{\sigma_2^2})]\}dy
\end{aligned}
$$

我們先對 (A) 與 (B) 作推導

(A)

$$=\int_0^\infty y^{\frac{r_2}{2}-1}\{\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2}-2\rho\frac{(x^{\frac{r_1}{2}}-m)(y^{\frac{r_2}{2}}-n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}-n)^2}{\sigma_2^2})]$$

$$+\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2}+2\rho\frac{(x^{\frac{r_1}{2}}-m)(y^{\frac{r_2}{2}}+n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}+n)^2}{\sigma_2^2})]\}dy$$

令 $w=y^{\frac{r_2}{2}}$，可得 $dy=\frac{2}{r_2}w^{\frac{2}{r_2}-1}dw$，代入上式

$$=\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2})]\int_0^\infty\frac{2}{r_2}\{\exp[-\frac{1}{2(1-\rho^2)}(\frac{(w-n)^2}{\sigma_2^2}-2\rho\frac{(w-n)(x^{\frac{r_1}{2}}-m)}{\sigma_1\sigma_2})]$$

$$+\exp[-\frac{1}{2(1-\rho^2)}(\frac{(w+n)^2}{\sigma_2^2}+2\rho\frac{(w+n)(x^{\frac{r_1}{2}}-m)}{\sigma_1\sigma_2})]\}dw$$

$$=\frac{2}{r_2}\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2})]\exp[-\frac{1}{2(1-\rho^2)}(\frac{n^2}{\sigma_2^2}+2\rho\frac{n(x^{\frac{r_1}{2}}-m)}{\sigma_1\sigma_2})]\times$$

$$\{\int_0^\infty\exp[-\frac{1}{2(1-\rho^2)}(\frac{w^2}{\sigma_2^2}-2(\frac{n}{\sigma_2}+\rho\frac{(x^{\frac{r_1}{2}}-m)}{\sigma_1})\frac{w}{\sigma_2})]+\exp[-\frac{1}{2(1-\rho^2)}(\frac{w^2}{\sigma_2^2}+2(\frac{n}{\sigma_2}+\rho\frac{(x^{\frac{r_1}{2}}-m)}{\sigma_1})\frac{w}{\sigma_2})]dw\}$$

$$=\frac{2}{r_2}\exp[-\frac{1}{2(1-\rho^2)}(1-\rho^2)(\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2})]\times$$

$$\{\int_0^\infty\exp[-\frac{1}{2(1-\rho^2)}(\frac{w}{\sigma_2}-(\frac{n}{\sigma_2}+\rho\frac{(x^{\frac{r_1}{2}}-m)}{\sigma_1}))^2]+\exp[-\frac{1}{2(1-\rho^2)}(\frac{w}{\sigma_2}+(\frac{n}{\sigma_2}+\rho\frac{(x^{\frac{r_1}{2}}-m)}{\sigma_1}))^2]dw\}$$

令 $\frac{w}{\sigma_2}=t$，可得 $dw=\sigma_2 dt$，代入上式

$$=\frac{2}{r_2}\exp[-\frac{1}{2}\frac{(x^{\frac{r_1}{2}}-m)^2}{\sigma_1^2}]\times$$

$$\sigma_2\{\int_0^\infty \exp[-\frac{1}{2(1-\rho^2)}(t-(\frac{n}{\sigma_2}+\rho\frac{(x^{\frac{r_1}{2}}-m)}{\sigma_1}))^2]dt+\int_0^\infty \exp[-\frac{1}{2(1-\rho^2)}(t+(\frac{n}{\sigma_2}+\rho\frac{(x^{\frac{r_1}{2}}-m)}{\sigma_1}))^2]dt\}$$

$$=\frac{2\sigma_2}{r_2}\exp[-\frac{(x^{\frac{r_1}{2}}-m)^2}{2\sigma_1^2}]\sqrt{2\pi}\sqrt{1-\rho^2}$$

(B)

$$=\int_0^\infty y^{\frac{r_2}{2}-1}\{\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2}-2\rho\frac{(x^{\frac{r_1}{2}}+m)(y^{\frac{r_2}{2}}+n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}+n)^2}{\sigma_2^2})]$$
$$+\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2}+2\rho\frac{(x^{\frac{r_1}{2}}+m)(y^{\frac{r_2}{2}}-n)}{\sigma_1\sigma_2}+\frac{(y^{\frac{r_2}{2}}-n)^2}{\sigma_2^2})]\}dy$$

令 $w=y^{\frac{r_2}{2}}$ ，可得 $dy=\frac{2}{r_2}w^{\frac{2}{r_2}-1}dw$ ，代入上式

$$=\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2})]\int_0^\infty \frac{2}{r_2}\{\exp[-\frac{1}{2(1-\rho^2)}(\frac{(w+n)^2}{\sigma_2^2}-2\rho\frac{(w+n)(x^{\frac{r_1}{2}}+m)}{\sigma_1\sigma_2})]$$
$$+\exp[-\frac{1}{2(1-\rho^2)}(\frac{(w-n)^2}{\sigma_2^2}+2\rho\frac{(w-n)(x^{\frac{r_1}{2}}+m)}{\sigma_1\sigma_2})]\}dw$$

$$=\frac{2}{r_2}\exp[-\frac{1}{2(1-\rho^2)}(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2})]\exp[-\frac{1}{2(1-\rho^2)}(\frac{n^2}{\sigma_2^2}-2\rho\frac{n(x^{\frac{r_1}{2}}+m)}{\sigma_1\sigma_2})]\times$$
$$\{\int_0^\infty \exp[-\frac{1}{2(1-\rho^2)}(\frac{w^2}{\sigma_2^2}+2(\frac{n}{\sigma_2}-\rho\frac{(x^{\frac{r_1}{2}}+m)}{\sigma_1})\frac{w}{\sigma_2})]+\exp[-\frac{1}{2(1-\rho^2)}(\frac{w^2}{\sigma_2^2}-2(\frac{n}{\sigma_2}-\rho\frac{(x^{\frac{r_1}{2}}+m)}{\sigma_1})\frac{w}{\sigma_2})]dw\}$$

$$=\frac{2}{r_2}\exp[-\frac{1}{2(1-\rho^2)}(1-\rho^2)(\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2})]\times$$

$$\{\int_0^\infty \exp[-\frac{1}{2(1-\rho^2)}(\frac{w}{\sigma_2}+(\frac{n}{\sigma_2}-\rho\frac{(x^{\frac{r_1}{2}}+m)}{\sigma_1}))^2]+\exp[-\frac{1}{2(1-\rho^2)}(\frac{w}{\sigma_2}-(\frac{n}{\sigma_2}-\rho\frac{(x^{\frac{r_1}{2}}+m)}{\sigma_1}))^2]dw\}$$

令 $\frac{w}{\sigma_2}=t$ ，可得 $dw=\sigma_2 dt$ ，代入上式

$$=\frac{2}{r_2}\exp[-\frac{1}{2}\frac{(x^{\frac{r_1}{2}}+m)^2}{\sigma_1^2}]\times$$

$$\sigma_2\{\int_0^\infty \exp[-\frac{1}{2(1-\rho^2)}(t+(\frac{n}{\sigma_2}-\rho\frac{(x^{\frac{r_1}{2}}+m)}{\sigma_1}))^2]dt+\int_0^\infty \exp[-\frac{1}{2(1-\rho^2)}(t-(\frac{n}{\sigma_2}-\rho\frac{(x^{\frac{r_1}{2}}+m)}{\sigma_1}))^2]dt\}$$

$$=\frac{2\sigma_2}{r_2}\exp[-\frac{(x^{\frac{r_1}{2}}+m)^2}{2\sigma_1^2}]\sqrt{2\pi}\sqrt{1-\rho^2}$$

綜合 (A) 與 (B) ， $f(x)=\frac{r_1 r_2 x^{\frac{r_1}{2}-1}}{8\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}[(A)+(B)]$

$$=\frac{r_1 x^{\frac{r_1}{2}-1}}{2\sqrt{2\pi}\sigma_1}\{\exp[-\frac{(x^{\frac{r_1}{2}}-m)^2}{2\sigma_1^2}]+\exp[-\frac{(x^{\frac{r_1}{2}}+m)^2}{2\sigma_1^2}]\}$$

$0<x<\infty$

相同的步驟可以導出 $f(y)=\int_0^\infty f(x,y)dx$

$$=\frac{r_2 y^{\frac{r_2}{2}-1}}{2\sqrt{2\pi}\sigma_2}\{\exp[-\frac{(y^{\frac{r_2}{2}}-n)^2}{2\sigma_2^2}]+\exp[-\frac{(y^{\frac{r_2}{2}}+n)^2}{2\sigma_2^2}]\}$$

$0<y<\infty$

附錄二

一、機率值通式

在 (3.4) 模型下並利用 BGGD 與二元常態分配之間可以相互轉換的特性，我們可以得到各種發生可能機率值的通式如下，其中假設受訪者針對「甲療程」的願付價值介於 A 與 B 之間，而對「乙療程」之願付價值則於 C 與 D 之間。

$$\pi_{mnpq} = p(A \le WTP_{1i} \le B \text{ , } C \le WTP_{2i} \le D)$$

$$= p(A \le (x_{1i}\beta_1 + e_{1i})^{\frac{2}{r_1}} \le B \text{ , } C \le (x_{2i}\beta_2 + e_{2i})^{\frac{2}{r_2}} \le D)$$

$$= p(A^{r_1} \le (x_{1i}\beta_1 + e_{1i})^2 \le B^{r_1} \text{ , } C^{r_2} \le (x_{2i}\beta_2 + e_{2i})^2 \le D^{r_2})$$

$$= p(-B^{\frac{r_1}{2}} \le (x_{1i}\beta_1 + e_{1i}) \le -A^{\frac{r_1}{2}} \text{ , } A^{\frac{r_1}{2}} \le (x_{1i}\beta_1 + e_{1i}) \le B^{\frac{r_1}{2}} \text{ , } -D^{\frac{r_2}{2}} \le (x_{2i}\beta_2 + e_{2i}) \le -C^{\frac{r_2}{2}} \text{ , } C^{\frac{r_2}{2}} \le (x_{2i}\beta_2 + e_{2i}) \le D^{\frac{r_2}{2}})$$

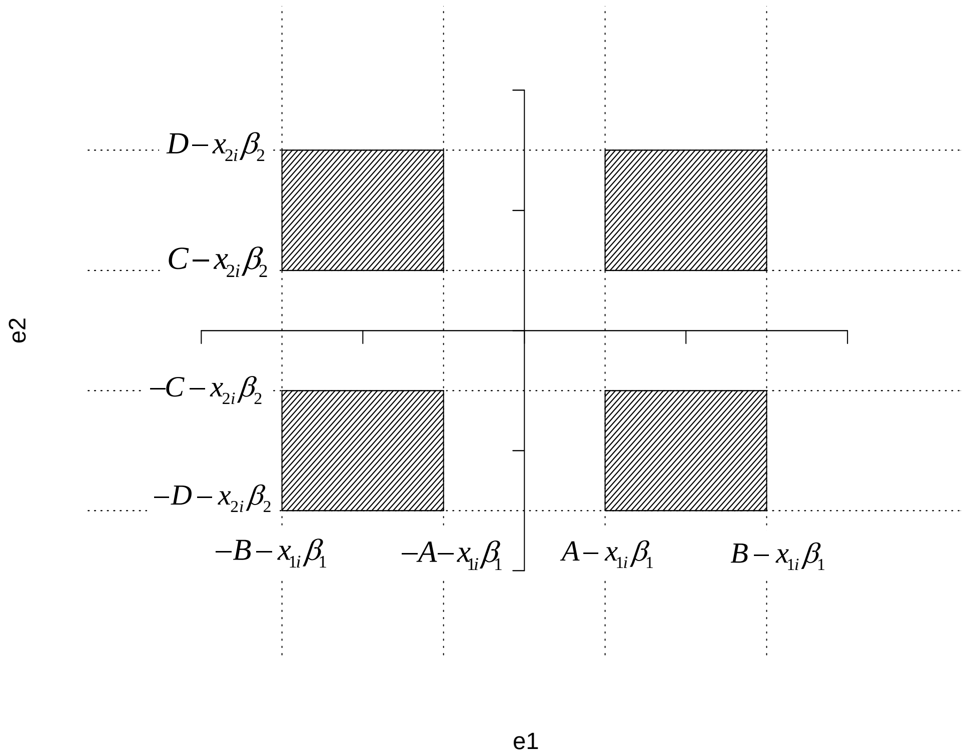
$$= p(-B^{\frac{r_1}{2}} - x_{1i}\beta_1 \le e_{1i} \le -A^{\frac{r_1}{2}} - x_{1i}\beta_1 \text{ , } A^{\frac{r_1}{2}} - x_{1i}\beta_1 \le e_{1i} \le B^{\frac{r_1}{2}} - x_{1i}\beta_1 \text{ , } -D^{\frac{r_2}{2}} - x_{2i}\beta_2 \le e_{2i} \le -C^{\frac{r_2}{2}} - x_{2i}\beta_2 \text{ , } C^{\frac{r_2}{2}} - x_{2i}\beta_2 \le e_{2i} \le D^{\frac{r_2}{2}} - x_{2i}\beta_2)$$

$$\begin{aligned} &= p(A^{\frac{r_1}{2}} - x_{1i}\beta_{11} \le e_{1i} \le B^{\frac{r_1}{2}} - x_{1i}\beta_1 \text{ , } C^{\frac{r_2}{2}} - x_{2i}\beta_2 \le e_{2i} \le D^{\frac{r_2}{2}} - x_{2i}\beta_2) \\ &+ p(-B^{\frac{r_1}{2}} - x_{1i}\beta_1 \le e_{1i} \le -A^{\frac{r_1}{2}} - x_{1i}\beta_1 \text{ , } C^{\frac{r_2}{2}} - x_{2i}\beta_2 \le e_{2i} \le D^{\frac{r_2}{2}} - x_{2i}\beta_2) \\ &+ p(-B^{\frac{r_1}{2}} - x_{1i}\beta_1 \le e_{1i} \le -A^{\frac{r_1}{2}} - x_{1i}\beta_1 \text{ , } -D^{\frac{r_2}{2}} - x_{2i}\beta_2 \le e_{2i} \le -C^{\frac{r_2}{2}} - x_{2i}\beta_2) \\ &+ p(A^{\frac{r_1}{2}} - x_{1i}\beta_1 \le e_{1i} \le B^{\frac{r_1}{2}} - x_{1i}\beta_1 \text{ , } -D^{\frac{r_2}{2}} - x_{2i}\beta_2 \le e_{2i} \le -C^{\frac{r_2}{2}} - x_{2i}\beta_2) \end{aligned}$$

(根據附錄圖一所推導出)

$$
\begin{aligned}
&= p\left(\frac{A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \;,\; \frac{C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right) \\
&+ p\left(\frac{-B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{-A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \;,\; \frac{C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right) \\
&+ p\left(\frac{-B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{-A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \;,\; \frac{-D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{-C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right) \\
&+ p\left(\frac{A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \;,\; \frac{-D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{-C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right)
\end{aligned}
$$

其中 ($z_{e_{1i}}, z_{e_{2i}}$) 服從標準二元常態分配$(0,0,1,1,\rho)$，其相關係數等於ρ，且 $0 \le A \le B$、$0 \le C \le D$。



附錄圖一

二、16 種發生可能之機率值

將各種發生可能代入 (3.8) 式，即可求出各種可能之機率值分別如下。其中 $a_i init$ 、 $a_i up$ 和 $a_i low$ 分別為第 i 個受訪者回答甲療程願付價值時隨機分配到的起始金額、起始金額的 2 倍及起始金額的 1/2 倍；而 $b_i init$ 、 $b_i up$ 和 $b_i low$ 則分別為第 i 個受訪者回答乙療程願付價值時隨機分配到的起始金額、起始金額的 2 倍及起始金額的 1/2 倍。

(1)「願意、願意、願意、願意」：

$$\pi_{1111} = p(a_i up \le WTP_{1i} \text{ , } b_i up \le WTP_{2i})$$

$$= p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \text{ , } \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \text{ , } z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(2)「願意、願意、願意、不願意」:

$$\pi_{1110} = p(a_i up \le WTP_{1i}\ ,\ b_i init \le WTP_{2i} < b_i up)$$

$$
\begin{aligned}
&= p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}}\ ,\ \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}}\ ,\ \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})
\end{aligned}
$$

(3)「願意、願意、不願意、願意」:

$$\pi_{1101} = p(a_i up \le WTP_{1i}\ ,\ b_i low \le WTP_{2i} < b_i init)$$

$$
\begin{aligned}
&= p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}}\ ,\ \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}}\ ,\ \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})
\end{aligned}
$$

(4)「願意、願意、不願意、不願意」：

$$\pi_{1100} = p(a_i up \le WTP_{1i}\ ，0 \le WTP_{2i} < b_i low)$$

$$= p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}}\ ，\frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ，\frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(z_{e_{1i}} \le \frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ，\frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}}\ ，\frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

(5)「願意、不願意、願意、願意」：

$$\pi_{1011} = p(a_i init \le WTP_{1i} < a_i up\ ，b_i up \le WTP_{2i})$$

$$= p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ，\frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ，\frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ，z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ，z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(6)「願意、不願意、願意、不願意」：

$$\pi_{1010} = p(a_i init \le WTP_{1i} < a_i up \text{ ， } b_i init \le WTP_{2i} < b_i up)$$

$$= p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(7)「願意、不願意、不願意、願意」：

$$\pi_{1001} = p(a_i init \le WTP_{1i} < a_i up \text{ ， } b_i low \le WTP_{2i} < b_i init)$$

$$= p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(8)「願意、不願意、不願意、不願意」：

$$\pi_{1000} = p(a_i init \le WTP_{1i} < a_i up \text{ , } 0 \le WTP_{2i} < b_i low)$$

$$= p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i up^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

(9)「不願意、願意、願意、願意」：

$$\pi_{0111} = p(a_i low \le WTP_{1i} < a_i init \text{ , } b_i up \le WTP_{2i})$$

$$= p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(10)「不願意、願意、願意、不願意」：

$$\pi_{0110} = p(a_i low \le WTP_{1i} < a_i init\ ,\ b_i init \le WTP_{2i} < b_i up)$$

$$\begin{aligned}
&= p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})
\end{aligned}$$

(11)「不願意、願意、不願意、願意」：

$$\pi_{0101} = p(a_i low \le WTP_{1i} < a_i init\ ,\ b_i low \le WTP_{2i} < b_i init)$$

$$\begin{aligned}
&= p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}) \\
&+ p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})
\end{aligned}$$

(12)「不願意、願意、不願意、不願意」：

$$\pi_{0100} = p(a_i low \le WTP_{1i} < a_i init \text{ ， } 0 \le WTP_{2i} < b_i low)$$

$$= p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i init^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

(13)「不願意、不願意、願意、願意」：

$$\pi_{0011} = p(0 \le WTP_{1i} < a_i low \text{ ， } b_i up \le WTP_{2i})$$

$$= p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1} \text{ ， } \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1} \text{ ， } z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ ， } z_{e_{2i}} \le \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(14)「不願意、不願意、願意、不願意」：

$$\pi_{0010} = p(0 \le WTP_{1i} < a_i low\ ,\ b_i init \le WTP_{2i} < b_i up)$$

$$= p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i up^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(15)「不願意、不願意、不願意、願意」：

$$\pi_{0001} = p(0 \le WTP_{1i} < a_i low\ ,\ b_i low \le WTP_{2i} < b_i init)$$

$$= p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-b_i init^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

(16)「不願意、不願意、不願意、不願意」：

$$\pi_{0000} = p(0 \le WTP_{1i} < a_i low \text{ , } 0 \le WTP_{2i} < b_i low)$$

$$= p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} < \frac{b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} < z_{e_{1i}} \le \frac{-x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

$$+ p(\frac{-x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} < \frac{a_i low^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \text{ , } \frac{-b_i low^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} < z_{e_{2i}} \le \frac{-x_{2i}\beta_2}{\sigma_2})$$

附錄三

一、回顧與假設

當要估計未知參數時，需計算概似函數對各個未知參數的一次偏微，由附錄二我們已知各種發生可能機率值的通式如下：

$$\pi_{mnpq} = p(A \le WTP_{1i} \le B\ ,\ C \le WTP_{2i} \le D)$$

$$= p\left(\frac{A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right) \qquad (a)$$

$$+ p\left(\frac{-B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{-A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right) \qquad (b)$$

$$+ p\left(\frac{-B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{-A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{-C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right) \qquad (c)$$

$$+ p\left(\frac{A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1} \le z_{e_{1i}} \le \frac{B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ \frac{-D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2} \le z_{e_{2i}} \le \frac{-C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\right) \qquad (d)$$

為求簡化，我們先假設（a）式中

$$a_1 = \frac{A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ b_1 = \frac{B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}\ ,\ c_1 = \frac{C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}\ ,\ d_1 = \frac{D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}$$

（b）式中 $a_2 = \frac{-A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}$ ，$b_2 = \frac{-B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}$ ，$c_2 = \frac{C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}$ ，$d_2 = \frac{D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}$

（c）式中 $a_3 = \frac{-A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}$ ，$b_3 = \frac{-B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}$ ，$c_3 = \frac{-C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}$ ，$d_3 = \frac{-D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}$

(d) 式中 $a_4 = \dfrac{A^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}$ ，$b_4 = \dfrac{B^{\frac{r_1}{2}} - x_{1i}\beta_1}{\sigma_1}$ ，$c_4 = \dfrac{-C^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}$ ，$d_4 = \dfrac{-D^{\frac{r_2}{2}} - x_{2i}\beta_2}{\sigma_2}$

於是各種發生可能機率值的通式可簡化如下：

$$\pi_{mnpq} = p(A \le WTP_{1i} \le B\text{ ，} C \le WTP_{2i} \le D)$$

$$= p(a_1 \le z_{e_{1i}} \le b_1\text{ ，} c_1 \le z_{e_{2i}} \le d_1) + p(b_2 \le z_{e_{1i}} \le a_2\text{ ，} c_2 \le z_{e_{2i}} \le d_2)$$
$$+ p(b_3 \le z_{e_{1i}} \le a_3\text{ ，} d_3 \le z_{e_{2i}} \le c_3) + p(a_4 \le z_{e_{1i}} \le b_4\text{ ，} d_4 \le z_{e_{2i}} \le c_4)$$

二、Leibnitz's Rule

在一次微分的推導過程中，將會運用到下列各式：

(I) $$\frac{d}{d\theta}\int_{a(\theta)}^{b(\theta)} f(x,\theta)dx = f(b(\theta),\theta)\frac{d}{d\theta}b(\theta) - f(a(\theta),\theta)\frac{d}{d\theta}a(\theta) + \int_{a(\theta)}^{b(\theta)}\frac{\partial}{\partial\theta}f(x,\theta)dx$$

(II) $$\frac{d}{d\theta}\int_{a(\theta)}^{b(\theta)} f(x)dx = f(b(\theta))\frac{d}{d\theta}b(\theta) - f(a(\theta))\frac{d}{d\theta}a(\theta)$$

(III) $$\frac{d}{d\theta}\int_{a}^{b} f(x,\theta)dx = \int_{a}^{b}\frac{\partial}{\partial\theta}f(x,\theta)dx$$

其中 (II) 式及 (III) 式均為 (I) 式的特例

三、概似函數對各個參數的一次微分

(1)概似函數對 β_1 的一次微分*

$$LogL_{\beta_1}^{(1)} = \sum_{i=1}^{n} \frac{\pi_{\beta_1\ mnpq}^{(1)}}{\pi_{mnpq}}$$

其中

$$\begin{aligned}\pi_{\beta_1\ mnpq}^{(1)} &= b_{1\beta_1}^{(1)}\int_{c_1}^{d_1} f(b_1, y)dy - a_{1\beta_1}^{(1)}\int_{c_1}^{d_1} f(a_1, y)dy + a_{2\beta_1}^{(1)}\int_{c_2}^{d_2} f(a_2, y)dy - b_{2\beta_1}^{(1)}\int_{c_2}^{d_2} f(b_2, y)dy \\ &+ a_{3\beta_1}^{(1)}\int_{d_3}^{c_3} f(a_3, y)dy - b_{3\beta_1}^{(1)}\int_{d_3}^{c_3} f(b_3, y)dy + b_{4\beta_1}^{(1)}\int_{d_4}^{c_4} f(b_4, y)dy - a_{4\beta_1}^{(1)}\int_{d_4}^{c_4} f(a_4, y)dy\end{aligned}$$

$$a_{1\beta_1}^{(1)} = a_{2\beta_1}^{(1)} = a_{3\beta_1}^{(1)} = a_{4\beta_1}^{(1)} = \frac{-x_{1i}}{\sigma_1}$$

$$b_{1\beta_1}^{(1)} = b_{2\beta_1}^{(1)} = b_{3\beta_1}^{(1)} = b_{4\beta_1}^{(1)} = \frac{-x_{1i}}{\sigma_1}$$

(2)概似函數對 β_2 的一次微分為

$$LogL_{\beta_2}^{(1)} = \sum_{i=1}^{n} \frac{\pi_{\beta_2\ mnpq}^{(1)}}{\pi_{mnpq}}$$

其中

$$\begin{aligned}\pi_{\beta_2\ mnpq}^{(1)} &= d_{1\beta_2}^{(1)}\int_{a_1}^{b_1} f(x, d_1)dx - c_{1\beta_2}^{(1)}\int_{a_1}^{b_1} f(x, c_1)dx + d_{2\beta_2}^{(1)}\int_{b_2}^{a_2} f(x, d_2)dx - c_{2\beta_2}^{(1)}\int_{b_2}^{a_2} f(x, c_2)dx \\ &+ c_{3\beta_2}^{(1)}\int_{b_3}^{a_3} f(x, c_3)dx - d_{3\beta_2}^{(1)}\int_{b_3}^{a_3} f(x, d_3)dx + c_{4\beta_2}^{(1)}\int_{a_4}^{b_4} f(x, c_4)dx - d_{4\beta_2}^{(1)}\int_{a_4}^{b_4} f(x, d_4)dx\end{aligned}$$

---

*本文概似函數對 β_1 的一次微分表示為 $LogL_{\beta_1}^{(1)}$

$$c_{1\beta_2}^{(1)} = c_{2\beta_2}^{(1)} = c_{3\beta_2}^{(1)} = c_{4\beta_2}^{(1)} = \frac{-x_{2i}}{\sigma_2}$$

$$d_{1\beta_2}^{(1)} = d_{2\beta_2}^{(1)} = d_{3\beta_2}^{(1)} = d_{4\beta_2}^{(1)} = \frac{-x_{2i}}{\sigma_2}$$

(3)概似函數對σ_1的一次微分為

$$LogL_{\sigma_1}^{(1)} = \sum_{i=1}^{n} \frac{\pi_{\sigma_1\ mnpq}^{(1)}}{\pi_{mnpq}}$$

其中

$$\begin{aligned}\pi_{\sigma_1\ mnpq}^{(1)} &= b_{1\sigma_1}^{(1)} \int_{c_1}^{d_1} f(b_1, y)dy - a_{1\sigma_1}^{(1)} \int_{c_1}^{d_1} f(a_1, y)dy + a_{2\sigma_1}^{(1)} \int_{c_2}^{d_2} f(a_2, y)dy - b_{2\sigma_1}^{(1)} \int_{c_2}^{d_2} f(b_2, y)dy \\ &+ a_{3\sigma_1}^{(1)} \int_{d_3}^{c_3} f(a_3, y)dy - b_{3\sigma_1}^{(1)} \int_{d_3}^{c_3} f(b_3, y)dy + b_{4\sigma_1}^{(1)} \int_{d_4}^{c_4} f(b_4, y)dy - a_{4\sigma_1}^{(1)} \int_{d_4}^{c_4} f(a_4, y)dy\end{aligned}$$

$a_{1\sigma_1}^{(1)} = -\frac{1}{\sigma_1} a_1$ ， $a_{2\sigma_1}^{(1)} = -\frac{1}{\sigma_1} a_2$ ， $a_{3\sigma_1}^{(1)} = -\frac{1}{\sigma_1} a_3$ ， $a_{4\sigma_1}^{(1)} = -\frac{1}{\sigma_1} a_4$

$b_{1\sigma_1}^{(1)} = -\frac{1}{\sigma_1} b_1$ ， $b_{2\sigma_1}^{(1)} = -\frac{1}{\sigma_1} b_2$ ， $b_{3\sigma_1}^{(1)} = -\frac{1}{\sigma_1} b_3$ ， $b_{4\sigma_1}^{(1)} = -\frac{1}{\sigma_1} b_4$

(4)概似函數對σ_2的一次微分為

$$LogL_{\sigma_2}^{(1)} = \sum_{i=1}^{n} \frac{\pi_{\sigma_2\ mnpq}^{(1)}}{\pi_{mnpq}}$$

其中

$$\begin{aligned}\pi_{\sigma_2\ mnpq}^{(1)} &= d_{1\sigma_2}^{(1)} \int_{a_1}^{b_1} f(x, d_1)dx - c_{1\sigma_2}^{(1)} \int_{a_1}^{b_1} f(x, c_1)dx + d_{2\sigma_2}^{(1)} \int_{b_2}^{a_2} f(x, d_2)dx - c_{2\sigma_2}^{(1)} \int_{b_2}^{a_2} f(x, c_2)dx \\ &+ c_{3\sigma_2}^{(1)} \int_{b_3}^{a_3} f(x, c_3)dx - d_{3\sigma_2}^{(1)} \int_{b_3}^{a_3} f(x, d_3)dx + c_{4\sigma_2}^{(1)} \int_{a_4}^{b_4} f(x, c_4)dx - d_{4\sigma_2}^{(1)} \int_{a_4}^{b_4} f(x, d_4)dx\end{aligned}$$

$$c_{1\sigma_2}^{(1)} = -\frac{1}{\sigma_2}c_1 \text{ ， } c_{2\sigma_2}^{(1)} = -\frac{1}{\sigma_2}c_2 \text{ ， } c_{3\sigma_2}^{(1)} = -\frac{1}{\sigma_2}c_3 \text{ ， } c_{4\sigma_2}^{(1)} = -\frac{1}{\sigma_2}c_4$$

$$d_{1\sigma_2}^{(1)} = -\frac{1}{\sigma_2}d_1 \text{ ， } d_{2\sigma_2}^{(1)} = -\frac{1}{\sigma_2}d_2 \text{ ， } d_{3\sigma_2}^{(1)} = -\frac{1}{\sigma_2}d_3 \text{ ， } d_{4\sigma_2}^{(1)} = -\frac{1}{\sigma_2}d_4$$

(5)概似函數對 r_1 的一次微分為

$$LogL_{r_1}^{(1)} = \sum_{i=1}^{n} \frac{\pi_{r_1\ mnpq}^{(1)}}{\pi_{mnpq}}$$

其中

$$\begin{aligned}\pi_{r_1\ mnpq}^{(1)} &= b_{1r_1}^{(1)}\int_{c_1}^{d_1} f(b_1,y)dy - a_{1r_1}^{(1)}\int_{c_1}^{d_1} f(a_1,y)dy + a_{2r_1}^{(1)}\int_{c_2}^{d_2} f(a_2,y)dy - b_{2r_1}^{(1)}\int_{c_2}^{d_2} f(b_2,y)dy \\ &+ a_{3r_1}^{(1)}\int_{d_3}^{c_3} f(a_3,y)dy - b_{3r_1}^{(1)}\int_{d_3}^{c_3} f(b_3,y)dy + b_{4r_1}^{(1)}\int_{d_4}^{c_4} f(b_4,y)dy - a_{4r_1}^{(1)}\int_{d_4}^{c_4} f(a_4,y)dy\end{aligned}$$

$$a_{1r_1}^{(1)} = a_{4r_1}^{(1)} = \frac{1}{2\sigma_1}A^{\frac{r_1}{2}}\ln A \text{ ， } a_{2r_1}^{(1)} = a_{3r_1}^{(1)} = -\frac{1}{2\sigma_1}A^{\frac{r_1}{2}}\ln A$$

$$b_{1r_1}^{(1)} = b_{4r_1}^{(1)} = \frac{1}{2\sigma_1}B^{\frac{r_1}{2}}\ln B \text{ ， } b_{2r_1}^{(1)} = b_{3r_1}^{(1)} = -\frac{1}{2\sigma_1}B^{\frac{r_1}{2}}\ln B$$

(6)概似函數對 r_2 的一次微分為

$$LogL_{r_2}^{(1)} = \sum_{i=1}^{n} \frac{\pi_{r_2\ mnpq}^{(1)}}{\pi_{mnpq}}$$

其中

$$\begin{aligned}\pi_{r_2\ mnpq}^{(1)} &= d_{1r_2}^{(1)}\int_{a_1}^{b_1} f(x,d_1)dx - c_{1r_2}^{(1)}\int_{a_1}^{b_1} f(x,c_1)dx + d_{2r_2}^{(1)}\int_{b_2}^{a_2} f(x,d_2)dx - c_{2r_2}^{(1)}\int_{b_2}^{a_2} f(x,c_2)dx \\ &+ c_{3r_2}^{(1)}\int_{b_3}^{a_3} f(x,c_3)dx - d_{3r_2}^{(1)}\int_{b_3}^{a_3} f(x,d_3)dx + c_{4r_2}^{(1)}\int_{a_4}^{b_4} f(x,c_4)dx - d_{4r_2}^{(1)}\int_{a_4}^{b_4} f(x,d_4)dx\end{aligned}$$

$$c_{1r_2}^{(1)} = c_{2r_2}^{(1)} = \frac{1}{2\sigma_2} C^{\frac{r_2}{2}} \ln C \text{ ，} c_{3r_2}^{(1)} = c_{4r_2}^{(1)} = -\frac{1}{2\sigma_2} C^{\frac{r_2}{2}} \ln C$$

$$d_{1r_2}^{(1)} = d_{2r_2}^{(1)} = \frac{1}{2\sigma_2} D^{\frac{r_2}{2}} \ln D \text{ ，} d_{3r_2}^{(1)} = d_{4r_2}^{(1)} = -\frac{1}{2\sigma_2} D^{\frac{r_2}{2}} \ln D$$

(7)概似函數對ρ的一次微分為

$$LogL_{\rho}^{(1)} = \sum_{i=1}^{n} \frac{\pi_{\rho\ mnpq}^{(1)}}{\pi_{mnpq}}$$

其中

$$\begin{aligned}\pi_{\rho\ mnpq}^{(1)} &= \int_{c_1}^{d_1}\int_{a_1}^{b_1} f_{\rho}^{(1)}(x,y)dxdy + \int_{c_2}^{d_2}\int_{b_2}^{a_2} f_{\rho}^{(1)}(x,y)dxdy \\ &+ \int_{d_3}^{c_3}\int_{b_3}^{a_3} f_{\rho}^{(1)}(x,y)dxdy + \int_{d_4}^{c_4}\int_{a_4}^{b_4} f_{\rho}^{(1)}(x,y)dxdy\end{aligned}$$

$$f_{\rho}^{(1)}(x,y) = \frac{1}{1-\rho^2} f(x,y)[\rho + xy - \frac{\rho}{1-\rho^2}(x^2 - 2\rho xy + y^2)]$$

附錄四

CVDFACTS 第五循環中之「肥胖之願付價格問卷」問卷設計如下：

二、願付價格【問家中有經濟能力之成年人（包括家庭主婦、成年學生）】

肥胖之願付價格

肥胖危險性說明	醫藥界已證實：肥胖和高血壓、高血脂症（即血裏油太多）、痛風、糖尿病密切相關，這些症狀可能會導致腦中風與急性心臟病。此外，肥胖也與乳癌有密切的關係。

您覺得自己需不需要減重？ □(1)需要 □(0)不需要（不需回答問題 A 及 B）

問題 A：

自付額設定：請依<u>*個案編號之第 5 碼*</u>，選取下列 1 組起始值：					
	(0)1 千	(1)1 千 5 百	(2)2 千	(3)3 千	(4)4 千
	(5)5 千	(6)6 千	(7)1 萬	(8)1 萬 5 千	(9)3 萬

假設現在有一種無副作用之新減肥療程，須以「吃藥」及「運動」配合，且可以有效地在 3 個月內減少 5 公斤，但您總共需支付 x 元，請問您是否願意參加？【請注意：您在其它方面之花費將因此少了 x 元。】

（1） □(1)願意【請續答（2)】 □(2)不願意【請跳答（3)】 □(3)不確定 □(4)不知道

（2） 您願意付 2x 元/年？ □(1)願意 □(2)不願意

（3） 您願意付 ½x 元/年？ □(1)願意 □(2)不願意(跳答(4))

（4） 你不願意之原因？ □(1)不願意付費 □(2)根本不願意參加此療程

3. 問題 B：

自付額設定：請依<u>*個案編號之第 5 碼*</u>，選取下列 1 組起始值：					
	(0)1 千	(1)2 千	(2)3 千	(3)4 千	(4)5 千
	(5)6 千	(6)8 千	(7)1 萬	(8)2 萬	(9)5 萬

假設有另一種無副作用之新減肥療程，但僅需吃藥，不需運動配合，也能在 3 個月內減少 5 公斤，但您總共需支付 x 元，您是否願意？【請注意：您在其它方面之花費將因此少了 x 元。】

（1） □(1)願意【請續答（2)】 □(2)不願意【請跳答（3)】 □(3)不確定 □(4)不知道

（2） 您願意付 2x 元/年？ □(1)願意 □(2)不願意

（3） 您願意付 ½x 元/年？ □(1)願意 □(2)不願意 (跳答（4）)

（4） 你不願意之原因？ □(1)不願意付費 □(2)根本不願意參加此療程

附錄五

本文的原始資料下載於中研院生醫所 CVDFACTS 資料庫，資料處理是利用 S-plus 軟體 (資料處理的程式在此省略) ，再將整理好的資料代入 R 軟體中計算。參數估計則是利用 R 中的 optim 指令，詳細的程式如下：

```
#讀資料
A2=matrix(scan("k://thesis//r//17A2.txt"),ncol=33,byrow=T)
#受訪者的16種回應
aans=matrix(scan("k://thesis//r//17aans.txt"),ncol=4,byrow=T)
#解釋變數
rx=cbind(rep(1,length(aans[,1])),matrix(c(scan("k://thesis//r//17rx.txt")),ncol=17,byro
w=T))

#各case的上下界
ID=function(x,i,A2)
{ jx=(x[1]*2^3+x[2]*2^2+x[3]*2^1+x[4]*2^0)
      if (jx==0)      abcd=c(0,A2[i,13],0,A2[i,16])
  else if (jx==1)     abcd=c(0,A2[i,13],A2[i,16],A2[i,14])
  else if (jx==2)     abcd=c(0,A2[i,13],A2[i,14],A2[i,15])
  else if (jx==3)     abcd=c(0,A2[i,13],A2[i,15],Inf)
  else if (jx==4)     abcd=c(A2[i,13],A2[i,11],0,A2[i,16])
  else if (jx==5)     abcd=c(A2[i,13],A2[i,11],A2[i,16],A2[i,14])
  else if (jx==6)     abcd=c(A2[i,13],A2[i,11],A2[i,14],A2[i,15])
  else if (jx==7)     abcd=c(A2[i,13],A2[i,11],A2[i,15],Inf)
  else if (jx==8)     abcd=c(A2[i,11],A2[i,12],0,A2[i,16])
  else if (jx==9)     abcd=c(A2[i,11],A2[i,12],A2[i,16],A2[i,14])
  else if (jx==10)    abcd=c(A2[i,11],A2[i,12],A2[i,14],A2[i,15])
  else if (jx==11)    abcd=c(A2[i,11],A2[i,12],A2[i,15],Inf)
  else if (jx==12)    abcd=c(A2[i,12],Inf,0,A2[i,16])
  else if (jx==13)    abcd=c(A2[i,12],Inf,A2[i,16],A2[i,14])
  else if (jx==14)    abcd=c(A2[i,12],Inf,A2[i,14],A2[i,15])
  else  abcd=c(A2[i,12],Inf,A2[i,15],Inf)
     list(abcd=abcd)}
```

```
#觀測到的上下界
ABCD=NULL
for (ii in 1:length(A2[,1]))
{ xx=aans[ii,]
ABCD=rbind(ABCD,ID(xx,ii,A2)$abcd)}

#Log Likelihood function
LH=function(x){
#beta1個數=截距+解釋變數個數
beta1=c(x[1],x[2],x[3],x[4],x[5],x[6],x[7],x[8], x[9],x[10],x[11])
#beta1個數=截距+解釋變數個數
beta2=c(x[12],x[13],x[14],x[15],x[16],x[17],x[18],x[19],x[20],x[21],x[22])
sigma1=x[23]
sigma2=x[24]
r1=x[25]
r2=x[26]
rho=x[27]

LFF=c(1:length(ABCD[,1]))*0
LF=c(1:length(ABCD[,1]))*0

for(ii in 1:length(ABCD[,1]))
{a1=(ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b1=(ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c1=(ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d1=(ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
a2=(-ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b2=(-ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c2=(ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d2=(ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
a3=(-ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b3=(-ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c3=(-ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d3=(-ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
a4=(ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b4=(ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c4=(-ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d4=(-ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
```

```
if (b1==Inf    &&    d1==Inf )    bd1=1  else bd1=
pmvnorm(lower=c(-Inf,-Inf),upper=c(b1,d1),sigma=matrix(c(1,rho,rho,1),ncol=2))

LFF[ii]=bd1-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a1,d1),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(b1,c1),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a1,c1),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a2,d2),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(b2,d2),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a2,c2),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(b2,c2),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a3,c3),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(b3,c3),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a3,d3),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(b3,d3),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(b4,c4),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a4,c4),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(b4,d4),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a4,d4),sigma=matrix(c(1,rho,rho,1),ncol=2))

if (LFF[ii]<=10^(-323)) LF[ii]=log(10^(-323))
else LF[ii]=log(LFF[ii])}

LL0= sum(LF)
return(LL0)}

# Grad 為 Log Likelihood function 分別對各個要估計的參數所作的一次微分函數
Grad=function(x)
{beta1=c(x[1],x[2],x[3],x[4],x[5],x[6],x[7],x[8], x[9])
beta2=c(x[10],x[11],x[12],x[13],x[14],x[15],x[16],x[17],x[18])
sigma1=x[19]
sigma2=x[20]
r1=x[21]
r2=x[22]
rho=x[23]

LFF=c(1:length(ABCD[,1]))*0
```

```
kkb1=NULL
kkb2= NULL
kks1= c(1:length(ABCD[,1]))*0
kks2= c(1:length(ABCD[,1]))*0
kkr1= c(1:length(ABCD[,1]))*0
kkr2= c(1:length(ABCD[,1]))*0
kkrho= c(1:length(ABCD[,1]))*0

for (ii in 1:length(ABCD[,1]))
{a1=(ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b1=(ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c1=(ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d1=(ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
a2=(-ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b2=(-ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c2=(ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d2=(ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
a3=(-ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b3=(-ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c3=(-ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d3=(-ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
a4=(ABCD[ii,1]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
b4=(ABCD[ii,2]^(r1/2)-rx[ii,]%*%beta1)/sqrt(sigma1^2)
c4=(-ABCD[ii,3]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)
d4=(-ABCD[ii,4]^(r2/2)-rx[ii,]%*%beta2)/sqrt(sigma2^2)

if (b1==Inf    &&    d1==Inf )    bd1=1  else bd1=
pmvnorm(lower=c(-Inf,-Inf),upper=c(b1,d1),sigma=matrix(c(1,rho,rho,1),ncol=2))

LFF[ii]=bd1-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a1,d1),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(b1,c1),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a1,c1),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a2,d2),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(b2,d2),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a2,c2),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(b2,c2),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a3,c3),sigma=matrix(c(1,rho,rho,1),ncol=2))-
```

```
pmvnorm(lower=c(-Inf,-Inf),upper=c(b3,c3),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a3,d3),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(b3,d3),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(b4,c4),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(a4,c4),sigma=matrix(c(1,rho,rho,1),ncol=2))-
pmvnorm(lower=c(-Inf,-Inf),upper=c(b4,d4),sigma=matrix(c(1,rho,rho,1),ncol=2))+
pmvnorm(lower=c(-Inf,-Inf),upper=c(a4,d4),sigma=matrix(c(1,rho,rho,1),ncol=2))

if (LFF[ii]<=10^(-308)) LFF[ii]=10^(-308) else LFF[ii]=LFF[ii]

# Log Likelihood對beta1微分
db1a1=-rx[ii,]/sqrt(sigma1^2)
db1a2=-rx[ii,]/sqrt(sigma1^2)
db1a3=-rx[ii,]/sqrt(sigma1^2)
db1a4=-rx[ii,]/sqrt(sigma1^2)
db1b1=-rx[ii,]/sqrt(sigma1^2)
db1b2=-rx[ii,]/sqrt(sigma1^2)
db1b3=-rx[ii,]/sqrt(sigma1^2)
db1b4=-rx[ii,]/sqrt(sigma1^2)

# Log Likelihood對beta2微分
db2c1=-rx[ii,]/sqrt(sigma2^2)
db2c2=-rx[ii,]/sqrt(sigma2^2)
db2c3=-rx[ii,]/sqrt(sigma2^2)
db2c4=-rx[ii,]/sqrt(sigma2^2)
db2d1=-rx[ii,]/sqrt(sigma2^2)
db2d2=-rx[ii,]/sqrt(sigma2^2)
db2d3=-rx[ii,]/sqrt(sigma2^2)
db2d4=-rx[ii,]/sqrt(sigma2^2)

bnb1y=function(y,b1)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(b1^2-2*rho*y*b1+y^2))}
bna1y=function(y,a1)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(a1^2-2*rho*y*a1+y^2))}
bna2y=function(y,a2)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(a2^2-2*rho*y*a2+y^2))}
bnb2y=function(y,b2)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(b2^2-2*rho*y*b2+y^2))}
```

```
bna3y=function(y,a3)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(a3^2-2*rho*y*a3+y^2))}
bnb3y=function(y,b3)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(b3^2-2*rho*y*b3+y^2))}
bna4y=function(y,a4)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(a4^2-2*rho*y*a4+y^2))}
bnb4y=function(y,b4)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(b4^2-2*rho*y*b4+y^2))}

if (b1==Inf) ic1d1fb1y=0 else
ic1d1fb1y=integrate(bnb1y,b1=b1,lower=c1,upper=d1)$value
ic1d1fa1y=integrate(bna1y,a1=a1,lower=c1,upper=d1)$value
ic2d2fa2y=integrate(bna2y,a2=a2,lower=c2,upper=d2)$value
if (b2==-Inf) ic2d2fb2y=0 else
ic2d2fb2y=integrate(bnb2y,b2=b2,lower=c2,upper=d2)$value
id3c3fa3y=integrate(bna3y,a3=a3,lower=d3,upper=c3)$value
if (b3==-Inf) id3c3fb3y=0 else
id3c3fb3y=integrate(bnb3y,b3=b3,lower=d3,upper=c3)$value
if (b4==Inf) id4c4fb4y=0 else
id4c4fb4y=integrate(bnb4y,b4=b4,lower=d4,upper=c4)$value
id4c4fa4y=integrate(bna4y,a4=a4,lower=d4,upper=c4)$value

bnxd1=function(x,d1)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*d1+d1^2))}
bnxc1=function(x,c1)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*c1+c1^2))}
bnxd2=function(x,d2)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*d2+d2^2))}
bnxc2=function(x,c2)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*c2+c2^2))}
bnxd3=function(x,d3)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*d3+d3^2))}
bnxc3=function(x,c3)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*c3+c3^2))}
bnxd4=function(x,d4)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*d4+d4^2))}
bnxc4=function(x,c4)
{solve(2*pi*(1-rho^2)^0.5)*exp(-solve(2*(1-rho^2))*(x^2-2*rho*x*c4+c4^2))}
```

```
if (d1==Inf) ia1b1fxd1=0 else
ia1b1fxd1=integrate(bnxd1,d1=d1,lower=a1,upper=b1)$value
ia1b1fxc1=integrate(bnxc1,c1=c1,lower=a1,upper=b1)$value
if (d2==Inf) ib2a2fxd2=0 else
ib2a2fxd2=integrate(bnxd2,d2=d2,lower=b2,upper=a2)$value
ib2a2fxc2=integrate(bnxc2,c2=c2,lower=b2,upper=a2)$value
ib3a3fxc3=integrate(bnxc3,c3=c3,lower=b3,upper=a3)$value
if (d3==-Inf) ib3a3fxd3=0 else
ib3a3fxd3=integrate(bnxd3,d3=d3,lower=b3,upper=a3)$value
ia4b4fxc4=integrate(bnxc4,c4=c4,lower=a4,upper=b4)$value
if (d4==-Inf) ia4b4fxd4=0 else
ia4b4fxd4=integrate(bnxd4,d4=d4,lower=a4,upper=b4)$value

# Log Likelihood對sigma1微分
ds1a1=-a1/sqrt(sigma1^2)
ds1a2=-a2/sqrt(sigma1^2)
ds1a3=-a3/sqrt(sigma1^2)
ds1a4=-a4/sqrt(sigma1^2)
if (b1==Inf) ds1b1=10^308 else ds1b1=-b1/sqrt(sigma1^2)
if (b2==-Inf) ds1b2=10^(-308) else ds1b2=-b2/sqrt(sigma1^2)
if (b3==-Inf) ds1b3=10^(-308) else ds1b3=-b3/sqrt(sigma1^2)
if (b4==Inf) ds1b4=10^308 else ds1b4=-b4/sqrt(sigma1^2)

# Log Likelihood對sigma2微分
ds2c1=-c1/sqrt(sigma2^2)
ds2c2=-c2/sqrt(sigma2^2)
ds2c3=-c3/sqrt(sigma2^2)
ds2c4=-c4/sqrt(sigma2^2)
if (d1==Inf) ds2d1=10^308 else ds2d1=-d1/sqrt(sigma2^2)
if (d2==Inf) ds2d2=10^308 else ds2d2=-d2/sqrt(sigma2^2)
if (d3==-Inf) ds2d3=10^(-308) else ds2d3=-d3/sqrt(sigma2^2)
if (d4==-Inf) ds2d4=10^(-308) else ds2d4=-d4/sqrt(sigma2^2)

# Log Likelihood對r1微分
if (ABCD[ii,1]==0) dr1a1=0 else
dr1a1=ABCD[ii,1]^(r1/2)*log(ABCD[ii,1])/(2*sqrt(sigma1^2))
if (ABCD[ii,1]==0) dr1a2=0 else
```

```
dr1a2=-ABCD[ii,1]^(r1/2)*log(ABCD[ii,1])/(2*sqrt(sigma1^2))
if (ABCD[ii,1]==0) dr1a3=0 else
dr1a3=-ABCD[ii,1]^(r1/2)*log(ABCD[ii,1])/(2*sqrt(sigma1^2))
if (ABCD[ii,1]==0) dr1a4=0 else
dr1a4=ABCD[ii,1]^(r1/2)*log(ABCD[ii,1])/(2*sqrt(sigma1^2))

if (ABCD[ii,2]==Inf) dr1b1=0 else
dr1b1=ABCD[ii,2]^(r1/2)*log(ABCD[ii,2])/(2*sqrt(sigma1^2))
if (ABCD[ii,2]==Inf) dr1b2=0 else
dr1b2=-ABCD[ii,2]^(r1/2)*log(ABCD[ii,2])/(2*sqrt(sigma1^2))
if (ABCD[ii,2]==Inf) dr1b3=0 else
dr1b3=-ABCD[ii,2]^(r1/2)*log(ABCD[ii,2])/(2*sqrt(sigma1^2))
if (ABCD[ii,2]==Inf) dr1b4=0 else
dr1b4=ABCD[ii,2]^(r1/2)*log(ABCD[ii,2])/(2*sqrt(sigma1^2))

# Log Likelihood對r2微分
if (ABCD[ii,3]==0) dr2c1=0 else
dr2c1=ABCD[ii,3]^(r2/2)*log(ABCD[ii,3])/(2*sqrt(sigma2^2))
if (ABCD[ii,3]==0) dr2c2=0 else
dr2c2=ABCD[ii,3]^(r2/2)*log(ABCD[ii,3])/(2*sqrt(sigma2^2))
if (ABCD[ii,3]==0) dr2c3=0 else
dr2c3=-ABCD[ii,3]^(r2/2)*log(ABCD[ii,3])/(2*sqrt(sigma2^2))
if (ABCD[ii,3]==0) dr2c4=0 else
dr2c4=-ABCD[ii,3]^(r2/2)*log(ABCD[ii,3])/(2*sqrt(sigma2^2))

if (ABCD[ii,4]==Inf) dr2d1=0 else
dr2d1=ABCD[ii,4]^(r2/2)*log(ABCD[ii,4])/(2*sqrt(sigma2^2))
if (ABCD[ii,4]==Inf) dr2d2=0 else
dr2d2=ABCD[ii,4]^(r2/2)*log(ABCD[ii,4])/(2*sqrt(sigma2^2))
if (ABCD[ii,4]==Inf) dr2d3=0 else
dr2d3=-ABCD[ii,4]^(r2/2)*log(ABCD[ii,4])/(2*sqrt(sigma2^2))
if (ABCD[ii,4]==Inf) dr2d4=0 else
dr2d4=-ABCD[ii,4]^(r2/2)*log(ABCD[ii,4])/(2*sqrt(sigma2^2))

# Log Likelihood對rho的微分
ia1b1c1d1bnfxy=function(y,rho,a1,b1){integrand=function(x,y,rho)
{1/(2*pi*(1-rho^2)^(3/2))*exp(-(1/(2*(1-rho^2)))*(x^2-2*rho*x*y+y^2))*(rho+x*y-
(rho/(1-rho^2))*(x^2-2*rho*x*y+y^2))}unlist(lapply(y,function(y,integrand,rho,a1,b1
```

```
){integrate(integrand,lower=a1,upper=b1,y=y,rho=rho)$value},integrand,rho,a1,b1))}
drhoa1b1c1d1=integrate(ia1b1c1d1bnfxy,a1=a1,b1=b1,lower=c1,upper=d1,rho=rho,
rel.tol=10^1)$value

ib2a2c2d2bnfxy=function(y,rho,a2,b2){integrand=function(x,y,rho)
{1/(2*pi*(1-rho^2)^(3/2))*exp(-(1/(2*(1-rho^2)))*(x^2-2*rho*x*y+y^2))*(rho+x*y-
(rho/(1-rho^2))*(x^2-2*rho*x*y+y^2))}unlist(lapply(y,function(y,integrand,rho,a2,b2
){integrate(integrand,lower=b2,upper=a2,y=y,rho=rho)$value},integrand,rho,a2,b2))}
drhob2a2c2d2=integrate(ib2a2c2d2bnfxy,a2=a2,b2=b2,lower=c2,upper=d2,rho=rho,
rel.tol=10^1)$value

ib3a3d3c3bnfxy=function(y,rho,a3,b3){integrand=function(x,y,rho)
{1/(2*pi*(1-rho^2)^(3/2))*exp(-(1/(2*(1-rho^2)))*(x^2-2*rho*x*y+y^2))*(rho+x*y-
(rho/(1-rho^2))*(x^2-2*rho*x*y+y^2))}unlist(lapply(y,function(y,integrand,rho,a3,b3
){integrate(integrand,lower=b3,upper=a3,y=y,rho=rho)$value},integrand,rho,a3,b3))}
drhob3a3d3c3=integrate(ib3a3d3c3bnfxy,a3=a3,b3=b3,lower=d3,upper=c3,rho=rho,
rel.tol=10^1)$value

ia4b4d4c4bnfxy=function(y,rho,a4,b4){integrand=function(x,y,rho)
{1/(2*pi*(1-rho^2)^(3/2))*exp(-(1/(2*(1-rho^2)))*(x^2-2*rho*x*y+y^2))*(rho+x*y-
(rho/(1-rho^2))*(x^2-2*rho*x*y+y^2))}unlist(lapply(y,function(y,integrand,rho,a4,b4
){integrate(integrand,lower=a4,upper=b4,y=y,rho=rho)$value},integrand,rho,a4,b4))}
drhoa4b4d4c4=integrate(ia4b4d4c4bnfxy,a4=a4,b4=b4,lower=d4,upper=c4,rho=rho,
rel.tol=10^1)$value

kk1=db1b1*ic1d1fb1y-db1a1*ic1d1fa1y+db1a2*ic2d2fa2y-db1b2*ic2d2fb2y+db1a3
*id3c3fa3y-db1b3*id3c3fb3y+db1b4*id4c4fb4y-db1a4*id4c4fa4y
kkb1=rbind(kkb1,kk1)

kk2=db2d1*ia1b1fxd1-db2c1*ia1b1fxc1+db2d2*ib2a2fxd2-db2c2*ib2a2fxc2+db2c3
*ib3a3fxc3-db2d3*ib3a3fxd3+db2c4*ia4b4fxc4-db2d4*ia4b4fxd4
kkb2=rbind(kkb2,kk2)

kks1[ii]=ds1b1*ic1d1fb1y-ds1a1*ic1d1fa1y+ds1a2*ic2d2fa2y-ds1b2*ic2d2fb2y+
ds1a3*id3c3fa3y-ds1b3*id3c3fb3y+ds1b4*id4c4fb4y-ds1a4*id4c4fa4y

kks2[ii]=ds2d1*ia1b1fxd1-ds2c1*ia1b1fxc1+ds2d2*ib2a2fxd2-ds2c2*ib2a2fxc2+
ds2c3*ib3a3fxc3-ds2d3*ib3a3fxd3+ds2c4*ia4b4fxc4-ds2d4*ia4b4fxd4
```

```
kkr1[ii]=dr1b1*ic1d1fb1y-dr1a1*ic1d1fa1y+dr1a2*ic2d2fa2y-dr1b2*ic2d2fb2y+
dr1a3*id3c3fa3y-dr1b3*id3c3fb3y+dr1b4*id4c4fb4y-dr1a4*id4c4fa4y

kkr2[ii]=dr2d1*ia1b1fxd1-dr2c1*ia1b1fxc1+dr2d2*ib2a2fxd2-dr2c2*ib2a2fxc2+
dr2c3*ib3a3fxc3-dr2d3*ib3a3fxd3+dr2c4*ia4b4fxc4-dr2d4*ia4b4fxd4

kkrho[ii]=drhoa1b1c1d1+drhob2a2c2d2+drhob3a3d3c3+drhoa4b4d4c4}

db1LF=kkb1/LFF
db2LF=kkb2/LFF
ds1LF=kks1/LFF
ds2LF=kks2/LFF
dr1LF=kkr1/LFF
dr2LF=kkr2/LFF
drhoLF=kkrho/LFF

# Log Likelihood對參數的微分
grad=c(apply(db1LF,2,sum),apply(db2LF,2,sum),sum(ds1LF),sum(ds2LF),
sum(dr1LF),sum(dr2LF),sum(drhoLF))
return(grad)}

#利用 optim 求估計值
real10finala1=optim(x,fn=LH, gr =Grad,method ="BFGS",control =
list(trace=10,fnscale=-1,maxit=200,REPORT=1,reltol=1e-16), hessian = TRUE)
```