7 Appendix

Appendix A: Limiting Normality

The density function of W is given in equation (4). Let $k = q^{-2}$, the density function can be written as

$$
\begin{aligned}
f(w;q) &= \frac{|q|}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \exp\left[q^{-2}(qw - e^{qw})\right] \\
&= \frac{\sqrt{k^{-1}}}{\Gamma(k)}(k)^k \exp\left[q^{-2}(qw - e^{qw})\right] \\
&= \frac{k^k\sqrt{2\pi k}}{\Gamma(k+1)e^k}\frac{(k+1)e^k}{\sqrt{k}\sqrt{2\pi k}} \exp\left[q^{-2}(qw - e^{qw})\right] \\
&= \frac{k^k\sqrt{2\pi k}}{\Gamma(k+1)e^k}\frac{(k+1)}{k\sqrt{2\pi}} \exp\left[\frac{qw - e^{qw} + 1}{q^2}\right]
\end{aligned}
$$

As the shape parameter q approximates zero, or equivalently k approximates infinity, the density function becomes

$$
\begin{aligned}
\lim_{q\to 0} f(x;q) &= \lim_{q\to 0}\frac{(k+1)}{k\sqrt{2\pi}} \exp\left[\frac{qw - e^{qw} + 1}{q^2}\right] \\
&= \frac{1}{\sqrt{2\pi}} \exp\left[\lim_{q\to 0}\frac{w - e^{qw}w}{2q}\right] \\
&= \frac{1}{\sqrt{2\pi}} \exp\left[\lim_{q\to 0}\frac{-e^{qw}w^2}{2}\right] \\
&= \frac{1}{\sqrt{2\pi}} e^{\frac{-w^2}{2}}
\end{aligned}
$$

The first equality holds by *Stirling*'s formula,

$$\lim_{k\to\infty}\frac{k^k\sqrt{2\pi k}}{\Gamma(k+1)e^k} = 1$$

whereas the second and third equalities hold by using L'Hôpital rule.
Hence, as q reasonably close to zero, the log-generalized gamma density becomes the standard normal.

Appendix B.1: Distribution Function

By integrating equation (4), one yields the cumulative density function of W

$$F(w;q) = \int_{-\infty}^{w} \frac{|q|}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \exp\left[q^{-2}(qw - e^{qw})\right] dw$$

Since $f(w;q)$ becomes standard normal density function as q approximates zero, one considers only two cases, $q < 0$ and $q > 0$

Case 1. $q > 0$

$$F(w;q) = \frac{|q|}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \int_{-\infty}^{w} \exp\left[q^{-2}(qw - e^{qw})\right] dw$$

Let $e^{qw} = y$

$$\begin{aligned}
F(w;q) &= \frac{|q|}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \int_{0}^{e^{qw}} \exp\left[q^{-2}(q\frac{\ln y}{q} - y)\right] \frac{1}{qy}dy \\
&= \frac{|q|}{q\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \int_{0}^{e^{qw}} y^{q^{-2}-1}e^{q^{-2}y}dy \\
&= \frac{1}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}}(q^2)^{q^{-2}+1-1} \int_{0}^{q^{-2}e^{qw}} t^{q^{-2}-1}e^{-t}dt \\
&= \frac{1}{\Gamma(q^{-2})}\gamma(q^{-2}, 0, q^{-2}e^{qw})
\end{aligned}$$

Case 2. $q < 0$

$$F(w;q) = \frac{|q|}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \int_{-\infty}^{w} \exp\left[q^{-2}(qw - e^{qw})\right] dw$$

Let $e^{qw} = y$

$$\begin{aligned}
F(w;q) &= \frac{|q|}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \int_{\infty}^{e^{qw}} \exp\left[q^{-2}(q\frac{\ln y}{q} - y)\right] \frac{1}{qy}dy \\
&= \frac{|q|}{q\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \int_{\infty}^{e^{qw}} y^{q^{-2}-1}e^{q^{-2}y}dy \\
&= \frac{-1}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}}(q^2)^{q^{-2}+1-1}(-1) \int_{q^{-2}e^{qw}}^{\infty} t^{q^{-2}-1}e^{-t}dt \\
&= \frac{1}{\Gamma(q^{-2})}\gamma(q^{-2}, q^{-2}e^{qw}, \infty)
\end{aligned}$$

where γ is generalized incomplete gamma function with following form

$$\gamma(a, z_1, z_2) = \int_{z_1}^{z_2} t^{a-1}e^{-t}dt$$

Appendix B.2: Expectation

Conditional on $G = 3$, the expected WTP can be derived as follows.
Refer to regression equation in (3),

$$\begin{aligned}\ln(\text{WTPP}|G=3) &= \boldsymbol{x'\theta} + \sigma W \\ \text{WTPP}|G=3 &= e^{\boldsymbol{x'\theta}} \cdot e^{\sigma W} \\ E\text{WTPP}|G=3 &= e^{\boldsymbol{x'\theta}} \cdot E(e^{\sigma W})\end{aligned}$$

To investigate the mean WTPP, we need to know how $\exp(\sigma W)$ is distributed. We hence derive the density function of $e^{\sigma W}$ first
Let $Y = \phi(W) = e^{\sigma W}$, $W = \phi^{-1}(Y) = \frac{\ln Y}{\sigma}$ with Jacobian $|J| = \frac{1}{\sigma Y}$, the density function of Y conditional on $q \neq 0$ is then given by

$$\begin{aligned}f_Y(y) &= f_W(\phi^{-1}(y))|J| \\ &= \frac{|q|}{\Gamma(q^{-2})}(q^{-2})^{q^{-2}} \exp\left[\frac{q\frac{\ln y}{\sigma} - e^{q\frac{\ln y}{\sigma}}}{q^2}\right]\frac{1}{\sigma y} \\ &= \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}} \exp\left[\frac{1}{\sigma q}\ln y - \frac{y^{q/\sigma}}{q^2}\right]\frac{1}{y} \\ &= \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}} y^{\frac{1}{\sigma q}-1} \exp\left[-\frac{y^{q/\sigma}}{q^2}\right] \qquad y \in (0, \infty)\end{aligned}$$

It follows that

$$EY = \int_0^\infty y \cdot \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}} y^{\frac{1}{\sigma q}-1} \exp\left[-\frac{y^{q/\sigma}}{q^2}\right] dy$$

Let $\frac{y^{q/\sigma}}{q^2} = x$, $(xq^2)^{\sigma/q} = y$ and $dy = \frac{\sigma}{q}(xq^2)^{\sigma/q-1}q^2 dx$, then for $q > 0$, we have

$$\begin{aligned}EY &= \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}} \int_0^\infty (xq^2)^{\frac{\sigma}{q}\frac{1}{\sigma q}} e^{-x}\frac{\sigma}{q}(xq^2)^{\frac{\sigma}{q}-1} q^2 dx \\ &= \frac{|q|}{\Gamma(q^{-2})q}(q^{-2})^{q^{-2}}(q^2)^{\frac{1}{q^2}+\frac{\sigma}{q}-1+1} \int_0^\infty x^{\frac{1}{q^2}+\frac{\sigma}{q}-1} e^{-x} dx \\ &= \frac{1}{\Gamma(q^{-2})}(q^2)^{\frac{\sigma}{q}}\Gamma(q^{-2} + \frac{\sigma}{q})\end{aligned}$$

Similarly, for $\frac{-1}{\sigma} < q < 0$, we can also show that

$$EY = \frac{1}{\Gamma(q^{-2})}(q^2)^{\frac{\sigma}{q}}\Gamma(q^{-2} + \frac{\sigma}{q})$$

Hence, the expectation of WTPP|G=3 is

$$E\text{WTPP}|G=3 = e^{\boldsymbol{x'\theta}} \cdot \frac{1}{\Gamma(q^{-2})}(q^2)^{\frac{\sigma}{q}}\Gamma(q^{-2} + \frac{\sigma}{q})$$

if $q > \frac{-1}{\sigma}$

If the shape parameter, q, were reasonably close to zero, the density function could be approximated by the standard normal density. Under this situation, the expectation can be found as follow

$$\begin{aligned} \ln(\text{WTPP}|G=3) &= \boldsymbol{x}'\boldsymbol{\theta} + \sigma W \\ \text{WTPP}|G=3 &= e^{\boldsymbol{x}'\boldsymbol{\theta}} \cdot e^{\sigma W} \\ E\text{WTPP}|G=3 &= e^{\boldsymbol{x}'\boldsymbol{\theta}} \cdot E(e^{\sigma W}) \end{aligned}$$

Since W is distributed as a standard normal, it follows immediately that $e^{\sigma W}$ is distributed as log-normal variable with parameters 0 and σ^2. The density function of log-normal variate is

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} x^{-1} \exp\left(\frac{-(\log x - \mu)^2}{2\sigma^2}\right) I_{(0,\infty)}(x)$$

Hence, the expectation of $e^{\sigma W}$ and $\text{WTPP}|G=3$ are given by

$$\begin{aligned} E(e^{\sigma W}) &= e^{\frac{1}{2}\sigma^2} \\ E\text{WTPP}|G=3 &= e^{\boldsymbol{x}'\boldsymbol{\theta}} \cdot e^{\frac{1}{2}\sigma^2} \end{aligned}$$

Appendix B.3: Median

To compute the median of WTPP conditional on $G = 3$, one has to determine how $\exp(\boldsymbol{x}'\boldsymbol{\theta} + \sigma W)$ distribute. Refer to regression equation in (3)

$$\begin{aligned}
\ln(WTPP|G=3) &= \boldsymbol{x}'\boldsymbol{\theta} + \sigma W \\
(WTPP|G=3) &= e^{\boldsymbol{x}'\boldsymbol{\theta}} \cdot e^{\sigma W} \\
Med(WTPP|G=3) &= Med\left(e^{\boldsymbol{x}'\boldsymbol{\theta}} \cdot e^{\sigma W}\right)
\end{aligned}$$

In order to find the median of $\exp(\boldsymbol{x}'\boldsymbol{\theta} + \sigma W)$, we need to find the distribution function of $\exp(\boldsymbol{x}'\boldsymbol{\theta} + \sigma W)$ first.

Denote $k = \exp(\boldsymbol{x}'\boldsymbol{\theta})$ and let $Z = \phi(Y) = kY$ where $Y \sim e^{\sigma W}$

$$\begin{aligned}
f_Z(z) &= f_Y(\phi^{-1}(z))|J| \\
&= \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}}\left(\frac{z}{k}\right)^{\frac{1}{q\sigma}-1}\exp\left[-\frac{(z/k)^{q/\sigma}}{q^2}\right]\frac{1}{k}
\end{aligned}$$

with support $Z \in (0, \infty)$

$$F_Z(c) = \int_0^c \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}}\left(\frac{z}{k}\right)^{\frac{1}{q\sigma}-1}\exp\left[-\frac{(z/k)^{q/\sigma}}{q^2}\right]\frac{1}{k}dz$$

Let $\frac{(z/k)^{q/\sigma}}{q^2} = t$, $dz = k\frac{\sigma}{q}(tq^2)^{\frac{\sigma}{q}-1}q^2dt$
Case 1: $q > 0$

$$\begin{aligned}
F_Z(c) &= \int_0^{\frac{(c/k)^{q/\sigma}}{q^2}} \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}}\left[(tq^2)^{\sigma/q}\right]^{\frac{1}{q\sigma}-1} e^{-t}\frac{\sigma}{q}(tq^2)^{\frac{\sigma}{q}-1}q^2dt \\
&= \frac{|q|}{\Gamma(q^{-2})q}(q^{-2})^{q^{-2}}(q^2)^{q^{-2}}\int_0^{\frac{(c/k)^{q/\sigma}}{q^2}} t^{q^{-2}-1}e^{-t}dt \\
&= \frac{1}{\Gamma(q^{-2})}\gamma(q^{-2}, 0, \frac{(c/k)^{q/\sigma}}{q^2})
\end{aligned}$$

Case 2: $q < 0$

$$\begin{aligned}
F_Z(c) &= \int_\infty^{\frac{(c/k)^{q/\sigma}}{q^2}} \frac{|q|}{\Gamma(q^{-2})\sigma}(q^{-2})^{q^{-2}}\left[(tq^2)^{\sigma/q}\right]^{\frac{1}{q\sigma}-1} e^{-t}\frac{\sigma}{q}(tq^2)^{\frac{\sigma}{q}-1}q^2dt \\
&= \frac{|q|}{\Gamma(q^{-2})q}(q^{-2})^{q^{-2}}(q^2)^{q^{-2}}(-1)\int_{\frac{(c/k)^{q/\sigma}}{q^2}}^{\infty} t^{q^{-2}-1}e^{-t}dt \\
&= \frac{1}{\Gamma(q^{-2})}\gamma(q^{-2}, \frac{(c/k)^{q/\sigma}}{q^2}, \infty)
\end{aligned}$$

In sum, the distribution function of $Z = \exp(\boldsymbol{x}'\boldsymbol{\theta} + \sigma W)$ is

$$F_Z(z) = \begin{cases} \frac{1}{\Gamma(q^{-2})}\gamma(q^{-2}, 0, \frac{(z/k)^{q/\sigma}}{q^2}) & \text{if } q > 0 \\ \frac{1}{\Gamma(q^{-2})}\gamma(q^{-2}, \frac{(z/k)^{q/\sigma}}{q^2}, \infty) & \text{if } q < 0 \\ \mathbb{F}(z; \boldsymbol{x}'\boldsymbol{\theta}, \sigma^2) & \text{if } q = 0 \end{cases}$$

where $\mathbb{F}(\cdot;\mu,\sigma^2)$ is the cumulative density function of log-normal variable.

The case $q = 0$ holds because as q is reasonably close to zero, $\exp(\sigma W)$ will be log-normal distributed. It can be proven without difficulty that scale change on log-normal variate will only alter the location parameter. Hence $\exp(\boldsymbol{x}'\boldsymbol{\theta} + \sigma W)$ will be log-normal distributed with parameters $\boldsymbol{x}'\boldsymbol{\theta}$ and σ^2.

With the above distribution function, one may, theoretically, yield any desired percentile accordingly. Unfortunately, for any percentile z_p such that $F_Z(z_p) = p,\ p \in (0,1)$, we can't find the percentile by the inverse function, $F_Z^{-1} : (0,1) \to (0,\infty)$, since the lack of closed form expression of F_Z^{-1}. Therefore, median or other percentiles can only be found by numerical method.

Appendix C: Asymptotic Variance of Estimated Mean of WTPP

Under suitable regularity condition, the maximum likelihood estimators of the likelihood function in equation (7), will be asymptotically normally distributed. That is

$$\sqrt{n}(\hat{\boldsymbol{\theta}}^a - \boldsymbol{\theta}^a) \sim \mathbb{N}_k(\mathbf{0}, I(\boldsymbol{\theta}^a))$$

Suppose a function $g : \mathcal{R}^k \mapsto \mathcal{R}$ with continuous first-order derivatives with respect to each parameters, the asymptotic distribution of $g(\hat{\boldsymbol{\theta}}^a)$, by multivariate Delta method, is given by

$$\sqrt{n}\left(g(\hat{\boldsymbol{\theta}}^a) - g(\boldsymbol{\theta}^a)\right) \sim \mathbb{N}(\mathbf{0}, g'_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}^a) I(\boldsymbol{\theta}^a) g_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}^a))$$

where $g_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}^a)$ is a $k \times 1$ column vector containing first-order derivatives of g with respect to each parameters evaluated at $\boldsymbol{\theta}^a = \hat{\boldsymbol{\theta}}^a$. In this case,

$$g(\boldsymbol{\theta}, \sigma, q) = \exp(\boldsymbol{x}'\boldsymbol{\theta}) \frac{\Gamma(q^{-2} + \sigma/q)}{\Gamma(q^{-2})} (q^2)^{\sigma/q}$$

and

$$g_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}^a) = \left[\frac{\partial}{\partial \boldsymbol{\theta}} g(\boldsymbol{\theta}, \sigma, q) \quad \frac{\partial}{\partial \sigma} g(\boldsymbol{\theta}, \sigma, q) \quad \frac{\partial}{\partial q} g(\boldsymbol{\theta}, \sigma, q)\right]$$

The partial derivatives are enumerated as follow

$$\begin{aligned}
\frac{\partial g}{\partial \boldsymbol{\theta}} &= \exp(\boldsymbol{x}'\boldsymbol{\theta}) \frac{\Gamma(q^{-2} + \sigma/q)}{\Gamma(q^{-2})} (q^2)^{\sigma/q} \cdot \boldsymbol{x} \\
\frac{\partial g}{\partial \sigma} &= \exp(\boldsymbol{x}'\boldsymbol{\theta}) \frac{\Gamma(q^{-2} + \sigma/q)}{\Gamma(q^{-2})} \left[\psi(q^{-2} + \frac{\sigma}{q}) \frac{1}{q} (q^2)^{\sigma/q} + (q^2)^{\sigma/q} \ln(q^2) \frac{1}{q}\right] \\
&= \exp(\boldsymbol{x}'\boldsymbol{\theta}) \frac{\Gamma(q^{-2} + \sigma/q)}{\Gamma(q^{-2})} (q^2)^{\sigma/q} \cdot \frac{1}{q} \left[\psi(q^{-2} + \frac{\sigma}{q}) + \ln(q^2)\right] \\
\frac{\partial g}{\partial q} &= \exp(\boldsymbol{x}'\boldsymbol{\theta}) \frac{\left(\frac{\partial}{\partial q}\Gamma(q^{-2} + \sigma/q)(q^2)^{\sigma/q}\right)\Gamma(q^{-2}) - \Gamma(q^{-2} + \sigma/q)(q^2)^{\sigma/q}\Gamma(q^{-2})\psi(q^{-2})(-2)q^{-3}}{(\Gamma(q^{-2}))^2} \\
&= \exp(\boldsymbol{x}'\boldsymbol{\theta}) \frac{\Gamma(q^{-2} + \sigma/q)}{\Gamma(q^{-2})} (q^2)^{\sigma/q} \cdot \left[-\psi(q^{-2} + \frac{\sigma}{q})(2q^{-3} + \frac{\sigma}{q}) + q^{-2}(2\sigma - \sigma \ln(q^2)) + \psi(q^{-2} 2q^{-3})\right]
\end{aligned}$$

where $\psi(\cdot)$ is the derivative of logarithm of gamma function.

Appendix D: Linear versus Exponential WTPP function

In this section, a simulation result will be demonstrated to corroborate the claim that we made about that the exponential WTPP function has an inclination to underestimate with respect to linear function.

Simulation Scenario:
Step 1: Two different mechanisms will be employed to generate random numbers of WTPP.
   1.(i) Linear WTPP= $\theta^* + \sigma^* W$, $W \sim LGG(w, q^*)$
   1.(ii) Exponential WTPP= $\exp(\theta^* + \sigma^* W)$, $W \sim LGG(w, q^*)$
Step 2: Fit the following 2 models
   2.(i) WTPP= $\theta + \sigma W$
   2.(ii)lnWTPP= $\theta + \sigma W$
Step 3: Calculate expectation of WTPP

In order to prove our assertion, we will concentrate only on the estimate of expectation of WTPP while the estimate of $\hat{\theta}$ or $\hat{\sigma}$ are relatively minor. 60(200) samples are generated in each simulation. Total 1000 simulations results are summarized as follow.

Result I: WTPP= $\theta^* + \sigma^* W$

		θ^*							
$q^* = -1$		5		10		15		20	
		6.1544		*11.1544*		*16.1544*		*21.1544*	
	2	6.161	6.134	11.159	11.148	16.187	16.160	21.147	21.140
		(6.142)	(6.132)	(11.152)	(11.124)	(16.125)	(16.158)	(21.138)	(21.158)
		7.8861		*12.8861*		*17.8861*		*22.8861*	
	5	7.898	7.601	12.910	12.858	17.874	17.866	22.804	22.871
		(7.902)	(7.797)	(12.887)	(12.857)	(17.865)	(17.854)	(22.887)	(22.884)
σ^*		*10.7721*		*15.7721*		*20.7721*		*25.7721*	
	10	10.761	10.305	15.772	15.243	20.723	20.376	25.771	25.695
		(10.715)	(10.602)	(15.781)	(15.625)	(20.812)	(20.550)	(25.787)	(25.776)
		13.6582		*18.6582*		*23.6582*		*28.6582*	
	15	13.717	12.904	18.659	17.703	23.604	22.768	28.639	27.792
		(13.709)	(13.477)	(18.661)	(18.474)	(23.671)	(23.344)	(28.669)	(28.325)
$q^* = -3$		5		10		15		20	
		9.8071		*14.8071*		*19.8071*		*24.8071*	
	2	9.784	9.783	14.801	14.976	19.789	19.783	24.886	24.705
		(9.746)	(9.745)	(14.792)	(14.762)	(19.742)	(19.796)	(24.886)	(24.913)
		17.0178		*22.0178*		*27.0178*		*32.0178*	
	5	17.069	17.053	21.876	21.944	27.122	27.046	31.994	32.092
		(17.092)	(16.998)	(21.976)	(22.060)	(27.762)	(26.971)	(31.978)	(32.676)
σ^*		*29.0357*		*34.0357*		*39.0357*		*44.0357*	
	10	29.175	29.070	33.757	34.080	38.992	38.696	44.298	44.201
		(29.056)	(29.044)	(33.949)	(34.319)	(38.899)	(38.648)	(44.051)	(44.054)
		41.0359		*46.0359*		*51.0359*		*56.0359*	
	15	41.010	40.716	45.626	45.251	50.799	51.262	55.928	55.771
		(41.122)	(41.057)	(46.206)	(46.126)	(51.097)	(52.036)	(56.383)	(56.206)

[a] The numbers in *italic* case are true expectation.
[b] The numbers in the parentheses are estimated with 200 samples.
[c] For each cell, left and right column represents the result estimated by linear and exponential function, respectively.

Result II: WTPP$=\exp(\theta^* + \sigma^* W)$

		θ^*							
$q^* = 1$		1		2		3		4	
		2.4090		*6.5484*		*17.8003*		*48.3863*	
	0.5	2.400	2.402	6.551	6.541	17.768	17.672	48.358	48.196
		(2.410)	(2.391)	(6.553)	(6.511)	(17.835)	(17.789)	(48.385)	(48.384)
		2.7183		*7.3890*		*20.0855*		*54.5981*	
	1	2.726	2.709	7.396	7.344	20.008	19.953	54.579	54.610
		(2.732)	(2.698)	(7.401)	(7.335)	(20.051)	(19.383)	(54.744)	(54.433)
σ^*		*3.6135*		*9.8225*		*26.7005*		*72.5795*	
	1.5	3.612	3.607	9.796	9.827	26.891	26.584	72.401	72.680
		(3.618)	(3.594)	(9.816)	(9.787)	(26.612)	(26.529)	(72.551)	(71.998)
		5.4365		*14.7781*		*40.1711*		*109.1963*	
	2	5.467	5.513	14.646	14.628	40.977	40.451	109.362	108.856
		(5.449)	(5.478)	(14.800)	(14.805)	(40.556)	(40.121)	(109.116)	(108.042)
$q^* = 2$		1		2		3		4	
		1.8793		*5.1085*		*13.8864*		*37.7473*	
	0.5	1.881	1.871	5.078	5.110	13.870	13.934	37.973	37.774
		(1.879)	(1.882)	(5.118)	(5.099)	(13.912)	(13.902)	(37.718)	(37.751)
		1.8374		*4.9948*		*13.5774*		*36.9071*	
	1	1.833	1.835	5.021	4.990	13.501	13.569	37.205	37.097
		(1.840)	(1.839)	(4.978)	(4.982)	(13.563)	(13.624)	(36.882)	(36.881)
σ^*		*2.1206*		*5.7644*		*15.6692*		*42.5933*	
	1.5	2.117	2.131	5.811	5.726	15.517	15.650	42.887	42.190
		(2.128)	(2.123)	(5.750)	(5.774)	(15.590)	(15.619)	(42.291)	(42.465)
		2.7182		*7.3890*		*20.0855*		*54.5981*	
	2	2.695	2.715	7.387	7.514	20.010	20.136	54.499	55.070
		(2.716)	(2.720)	(7.322)	(7.389)	(20.108)	(20.081)	(54.641)	(54.677)

Summary:

Among $2 \cdot 4^3$ cells, there are 83 cells in which the linear WTPP function has greater estimated expectation of WTPP than that of exponential function. Therefore, one may aver that the exponential WTPP function tends to underestimate WTPP than linear function. Such claim can be analogous to the comparison between functional oriented method and utility based one.

Appendix E: Conditional Expectation of AFT Model Residuals

For a given set of covariates $\boldsymbol{x}_i$ (including the intercept term), the standardized residual for the i^{th} observation is

$$\hat{\varepsilon}_i = \frac{\ln(\text{WTPP}_i) - \boldsymbol{x}_i'\hat{\boldsymbol{\theta}}}{\hat{\sigma}}$$

If the observation is right censored or interval censored, the value of y_i is replaced by its lower bound. Under the circumstance, the observed residual $\hat{\varepsilon}_i$ is also right censored or the lower bound of a interval censored value. Denote δ_i as an indicator with value 0 if $\hat{\varepsilon}_i$ is right censored and 1 if interval censored. Then we collect the pair $(\hat{\varepsilon}_i, \delta_i)$ from parametric model.

Our objective is to use a mechanism by which the incomplete residual could be transformed into complete data. The mechanism is quiet intuitive because we just take its conditional expectation.

When $\delta_i = 1$, for instance, the censored residual will be replaced by the conditional expectation $E(\varepsilon_i|\varepsilon_i > \hat{\varepsilon_i})$. Since we have assumed the error term is log-generalized gamma distributed, the conditional expectation is given by

$$E(\varepsilon_i|\varepsilon_i > \hat{\varepsilon_i}) = \int_{\hat{\varepsilon_i}}^{\infty} w \cdot \frac{f(w;\hat{q})}{1 - F_W(\hat{\varepsilon}_i;\hat{q})} dw$$

It takes one more step to process the case $\delta_i = 0$: compute the upper bound for the i^{th} residual $\tilde{\varepsilon}$. Once the upper bound is known, one can compute the following conditional expectation

$$E[\varepsilon_i|\varepsilon_i \in (\hat{\varepsilon}, \tilde{\varepsilon})] = \int_{\hat{\varepsilon_i}}^{\tilde{\varepsilon}} w \cdot \frac{f(w;\hat{q})}{F_W(\tilde{\varepsilon};\hat{q}) - F_W(\hat{\varepsilon}_i;\hat{q})} dw$$

The above two conditional expectations can't be written more explicitly as the absence of closed form solution. Therefore, we have to count on numerical integration to find the conditional expectation.