4 Model Specifications

To begin with, it would be helpful to define notations and enumerate model assumptions

Notation	Description
G	Subjective Preference Groups
	$G = 1$: Purchase *Non-GM* only.
	$G = 2$: Purchase *GM* only.
	$G = 3$: Either one will do.
Γ_1	$\{i$: respondent i belongs to first preference group $\}$
Γ_2	$\{i$: respondent i belongs to second preference group $\}$
Γ_3	$\{i$: respondent i belongs to third preference group $\}$
WTPP_i	i^{th} respondent's WTP for premium.
I_i	(L_i, U_i) : Observed censoring interval for i^{th} respondent's WTPP.

Assumptions:

1. We observe random samples $\text{WTPP}_1, \text{WTPP}_2, ..., \text{WTPP}_n$, where $\text{WTPP}_i \sim \mathcal{P}_{\theta^a}$, $\boldsymbol{\theta}^a \in \boldsymbol{\Theta} \subset \mathbb{R}^k$ independently.

2. The parameter space $\boldsymbol{\Theta}$ contains an open set in $\mathbb{R}^k$ and the true parameter value $\boldsymbol{\theta}_0^a$ is an interior point.

3. The distribution of WTPP_i is positively skewed.

4. The stochastic component of $\ln \text{WTPP}_i$ is distributed as log-generalized gamma family.

Preference Analysis

In order to identify the preference group a respondent belongs, a multinominal logistic model is utilized. Let

$$\begin{aligned}\ln \frac{P(G=1|\boldsymbol{s})}{P(G=3|\boldsymbol{s})} &= \boldsymbol{\alpha}^{\mathrm{T}}\boldsymbol{s} \\ \ln \frac{P(G=2|\boldsymbol{s})}{P(G=3|\boldsymbol{s})} &= \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{s} \end{aligned} \tag{1}$$

where $\boldsymbol{s}$ denotes the set of observable personal attributes, such as gender, age, or other socioeconomic variables, for a given respondent.
With some arrangements, the probability of observing $G = g$ conditional on observable attributes become

$$\begin{aligned} P(G=1|\boldsymbol{s}) &= \frac{e^{\boldsymbol{\alpha}^T\boldsymbol{s}}}{1+e^{\boldsymbol{\alpha}^T\boldsymbol{s}}+e^{\boldsymbol{\beta}^T\boldsymbol{s}}} \\ P(G=2|\boldsymbol{s}) &= \frac{e^{\boldsymbol{\beta}^T\boldsymbol{s}}}{1+e^{\boldsymbol{\alpha}^T\boldsymbol{s}}+e^{\boldsymbol{\beta}^T\boldsymbol{s}}} \\ P(G=3|\boldsymbol{s}) &= \frac{1}{1+e^{\boldsymbol{\alpha}^T\boldsymbol{s}}+e^{\boldsymbol{\beta}^T\boldsymbol{s}}} \end{aligned} \tag{2}$$

The reason why we treat the third group as the baseline or reference group is to take into account the number of observations in each group, there are 58, 8, and 268 observations in each group. In order to improve estimation accuracy, a general guideline is to use the group with the most observations as reference.

WTP for Premium

Regression Form:

Since the distribution of WTPP is positively skewed, we attempt to model the logarithm of the WTPPP measurements. Specially, conditional on the third preference group, we assume that these respondents' WTP for premium can be modeled as

$$\ln \quad (\text{WTPP}_i | G = 3) = \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{\theta} + \sigma W_i \tag{3}$$

where $\boldsymbol{x}_i$ and W_i are observable traits for a given respondent and stochastic component, respectively. Moreover, we assume that W_i are *i.i.d.* log-generalized gamma random variables with shape parameter q. The density function and distribution function[4] of log-generalized gamma variable are

$$f(w; q) = \frac{|q|}{\Gamma(q^{-2})} (q^{-2})^{q^{-2}} \exp\left[q^{-2}(qw - e^{qw})\right] \quad , \ w \in \mathbb{R} \tag{4}$$

$$F(w; q) = \begin{cases} \frac{1}{\Gamma(q^{-2})} \gamma(q^{-2}, 0, q^{-2} e^{qw}) & \text{if } q > 0 \\ \frac{1}{\Gamma(q^{-2})} \gamma(q^{-2}, q^{-2} e^{qw}, \infty) & \text{if } q < 0 \\ \Phi(w) & \text{if } q \approx 0 \end{cases} \tag{5}$$

where $\gamma(a, z_1, z_2)$ is generalized incomplete gamma function with the form

$$\int_{z_1}^{z_2} t^{a-1} e^{-t} dt$$

and $\Phi(\cdot)$ is the distribution function of standard normal variate.

The regression form that we use in equation (3) is often called *accelerated failure-time*(AFT) model in survival analysis. The term "accelerated failure-time" means that the median time to event with covariate $\boldsymbol{x}$ is the baseline median time to event divided by the acceleration factor $\exp(\boldsymbol{x}'\boldsymbol{\theta})$. "Time to event" is analogous to the amount which the respondent will pay in our content. In the following, the regression form that we use to evaluate WTPP will be called AFT model.

A noteworthy point is that we will not fit the AFT model for preference group 1 and 2 because we assume their responses shows nothing but their resolute preference. Actually, a preliminary analysis also reveals that their WTPP are unreasonably high. Rarely will we expect consumer pay such a great amount of premium just to buy a salmon. Therefore, our attention is fully paid to group 3.

Estimation of Mean and Median WTPP

Based on regression function, the expected value of WTPP[5] is given by

$$E(\text{WTPP}|G = 3) = \begin{cases} e^{\boldsymbol{x}'\boldsymbol{\theta}} \cdot \frac{1}{\Gamma(q^{-2})} (q^2)^{\frac{\sigma}{q}} \Gamma(q^{-2} + \frac{\sigma}{q}) & \text{if } \ q > -\frac{1}{\sigma}, q \neq 0 \\ e^{\boldsymbol{x}'\boldsymbol{\theta}} \cdot e^{\frac{1}{2}\sigma^2} & \text{if } \ q = 0 \end{cases} \tag{6}$$

Unfortunately, unlike mean WTPP, median WTPP doesn't have a analytical form[6]. Therefore, the estimation of median must be performed by numerical method. It is also the absence of closed form that makes the estimation of standard error of median more arduous.

[4]See Appendix B.1 for derivation

[5]Refer to Appendix B.2 for derivation

[6]Refer to Appendix B.3 for derivation.

Likelihood Function

According to the survey data, we observe each respondent's subjective preference and his/her censoring range for willingness to pay for premium. Therefore, each respondent contributes the following quantity to the likelihood function

$$P(G = g|\boldsymbol{s}_i)\,[P(\mathrm{WTPP}_i \in I_i)]^{I_{\{3\}}(G)}$$

where $I_{\{3\}}(G)$ is an indicator, 1 if i^{th} respondent belongs to $G = 3$ and 0 otherwise.

It follows that the likelihood function is given by the product of all the likelihood contributions

$$\begin{aligned}
\mathcal{L} &= \prod_{i=1}^{n} P(G = g|\boldsymbol{s}_i)\,[P(\mathrm{WTPP}_i \in I_i)]^{I_{\{3\}}(G)} \\
&= \prod_{i\in\Gamma_1} P(G_i = 1|\boldsymbol{s}_i) \prod_{i\in\Gamma_2} P(G_i = 2|\boldsymbol{s}_i) \prod_{i\in\Gamma_3} P(\mathrm{WTPP}_i \in I_i)P(G_i = 3|\boldsymbol{s}_i)
\end{aligned}$$

the second equality holds since the indicator $I_{\{g=3\}}$ becomes 0 for those respondent classified as $G = 2$ or $G = 3$.

Plugging in equations (2) and (3), the likelihood function can be written more explicitly

$$\begin{aligned}
\mathcal{L} &= \prod_{i\in\Gamma_1} \left(\frac{e^{\boldsymbol{\alpha}'\boldsymbol{s}_i}}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right) \prod_{i\in\Gamma_2} \left(\frac{e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right) \\
&\quad \prod_{i\in\Gamma_3} P\left(\frac{\ln(L_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma} < W_i < \frac{\ln(U_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma}\right)\left(\frac{1}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right)
\end{aligned}$$

Hence, the log-likelihood function is given by

$$\begin{aligned}
\ln\mathcal{L} &= \sum_{i\in\Gamma_1} \ln\left(\frac{e^{\boldsymbol{\alpha}'\boldsymbol{s}_i}}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right) + \sum_{i\in\Gamma_2} \ln\left(\frac{e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right) + \\
&\quad \sum_{i\in\Gamma_3}\left[\ln P\left(\frac{\ln(L_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma} < W_i < \frac{\ln(U_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma}\right) + \ln\left(\frac{1}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right)\right] \\
&= \sum_{i\in\Gamma_1} \ln\left(\frac{e^{\boldsymbol{\alpha}'\boldsymbol{s}_i}}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right) + \sum_{i\in\Gamma_2} \ln\left(\frac{e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right) + \sum_{i\in\Gamma_3} \ln\left(\frac{1}{1 + e^{\boldsymbol{\alpha}'\boldsymbol{s}_i} + e^{\boldsymbol{\beta}'\boldsymbol{s}_i}}\right) + \\
&\quad \sum_{i\in\Gamma_3} \ln P\left(\frac{\ln(L_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma} < W_i <_i \frac{\ln(U_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma}\right) \\
&= \ln\mathcal{L}_1 + \ln\mathcal{L}_2 \qquad (7)
\end{aligned}$$

Note that the log-likelihood function is composed of two sub-log-likelihood functions which respectively represent multinominal logistic model and AFT model. This special structure allows us to maximize each log-likelihood function separately and the log-likelihood function will be maximized if and only if each of the sub-log-likelihood function attains its maximum.

A computational issue that needs to be mentioned is that the likelihood contribution involves the calculation of

$$P(\mathrm{WTPP}_i \in I_i) = P\left(\frac{\ln(L_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma} < W_i < \frac{\ln(U_i) - \boldsymbol{x}_i'\boldsymbol{\theta}}{\sigma}\right)$$

For some subjects whose censoring interval I_i is negative, the logarithm can't be undefined. To solve this problem, we shift all the observed censoring interval to the right by C such that the censoring is positive for all subjects.

The value of C is set to be 150 in this study, this amount is also the threshold price. It may be argued that the arbitrary choice of C is so crude that the consequence of the study is highly dependent on the value of C. A sensitivity check has been performed to see how serious the effect of C is, and it turns out to be minor. In sum, though the choice of C is up to the investigator's arbitrary decision, this artificial device has negligible effect to our conclusion.

Maximum Likelihood Estimators

Theoretically, the maximum likelihood estimators (MLE) can be yielded analytically by solving the simultaneous equations

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\alpha}} \ln \mathcal{L} &= \mathbf{0} \\ \frac{\partial}{\partial \boldsymbol{\beta}} \ln \mathcal{L} &= \mathbf{0} \\ &\vdots \end{aligned}$$

provided that the log likelihood function is differentiable on their parameter space. Nonetheless, the solution to the simultaneous equations doesn't have a closed form. Therefore, numerical method, such as Newton-Raphson algorithm, must be utilized to solve the maximization problem in this case.

In this study, other numerical method, namely Nelder-Mead algorithm, will be adopted to find the maximum likelihood estimators for the likelihood function in equation (7). Relative slow convergence as it is, this algorithm is preferred for its robustness. Apart from searching for the MLE, observed information matrix is also obtained accordingly.