I. Introduction

The term asset allocation means different things to different people in different contexts.

In practice, asset allocation is divided into three categories: strategic asset allocation (SAA), tactical asset allocation (TAA) and dynamic asset allocation.

The strategic asset allocation can be characterized as a long-term asset allocation decision. Strategic asset allocation is the process of spreading money across various asset classes in proportions matched with investor’s risk tolerance and long-term financial objectives.

When we revert to tactical asset allocation, we make periodic short-term adjustments in asset weights to take advantage of perceived market opportunities. Tactical asset allocation in founded on the premise that asset returns are on average driven by the economic fundamentals. When asset classes and markets diverge frequently from their long-term equilibrium levels, there's often an opportunity to add value through TAA. However, the chances to add or subtract value tend to come irregularly.

Dynamic asset allocation strategies are designed to reshape the distribution pattern of the portfolio return. The best well known of these strategies is the portfolio insurance strategy, which is designed to set the floor for the value of the portfolio. Through dynamic trading strategies an investor is able to create a new asset (one that may not exist in a pure form in financial markets). For example, some hedge funds employ dynamic trading strategies to create a new asset class that will outperform simple buy and hold strategies when there is increased volatility in markets, while it will perform relatively poorly in less volatile markets.

Modern portfolio theory is often thought to stem from mean-variance analysis of Markowitz(1952). Markowitz showed how investors should allocate their wealth on risky assets if they only care about not only the mean but also the variance of return over a single time period. However, this static setup prevents the construction of dynamic portfolios that properly address the progressive uncertainty. The striking conclusion of the Mean-Variance is that all investors will hold the same risky portfolio, called the tangency portfolio, and they will not alter the relative proportions of risky assets in the tangency portfolio, this observation is known as the mutual fund separation theorem.

In real life, the patterns in financial planning advices contradict such conclusion, which is so called the asset allocation puzzle. A breakthrough in dealing with mutiperiod investment problem is due to Merton’s contribution (1971,1973). Merton

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1 Canner, Mankiw, and Weil (1997) call this asset allocation puzzle.
solved the optimal portfolio problem based on dynamic programming under a continuous time framework. Unfortunately, Merton’s dynamic programming technique, except in the Black-Scholes assumptions, also proves cumbersome, and doesn’t yield a closed-form formula for the optimal policies. Complexity of the results and their dependence on utility function parameters are the main reasons that Merton’s model has not been popular among fund managers, and even less acceptable than the Markowitz’s model.

Another approach is to rely on the martingale property of asset prices with respect to an adequate numeraire. Harisson and Kreps(1979) and Harisson and Pliska(1981) introduce change in probability that has successfully been applied to price the contingent claim, and also enable us to derive the optimal strategies under either complete market(Cox and Huang,1989) or incomplete market (He and Pearson,1991). The new equivalent probability yielding martingale prices, called martingale measure, depends on the specified numeraire chosen to express the relative price of assets and portfolios. This is the approach most closely related with this paper. In this paper, we introduce a special numeraire to translate the dynamic programming problem into a static problem. This special numeraire, known as numeraire portfolio or growth optimal portfolio, yields the martingale price under the historical probability measure and is the solution to the optimal portfolio problem of investors with a logarithm utility.

The dynamic mean-variance strategies are first characterized by Richardson(1990) and Bajeux-Besnainu and Portait(1998) in. Yet mean-variance strategies may become over-leveraged in some cases and can yield negative wealth. In that case, for an individual allocating his/her retirement savings or for institutional investors investing on behave of themselves or clients, mean-variance strategies expose to investors potential downside risk. In order to prevent such undesirable feature, we impose a principal-guaranteed restriction on the dynamic mean-variance program. We show that for any given horizon T, the constraint DMVE (dynamic mean-variance efficient) strategies are static combination of two funds: zero coupon bond with maturity T and principal-guaranteed put option on minimum norm portfolio.