I. Conclusion

Although Markowitz’s mean-variance approach has been accepted and appreciated by practitioners and academics for half century, the main drawback of this approach is the static nature of the problem. By allowing the possibility of trading at time instant, an investor can react immediately if the situation dictates this. Merton's breakthrough provides closed form solution in continuous time framework. However, his assumption of CRRA investors and constant opportunity set is the crucial reason that Merton’s results are not popular and accessible for practitioners. In this paper, we use martingale approach, choose an adequate numeraire such that the relative prices are martingale, to derive the optimal mean-variance strategies that guarantee the principle. We characterize the strategies as a static combination of put option on minimum norm portfolio and zero coupon bond. We also provide closed form expression for optimal weights so that the model is easy to implement. Finally, a numerical example is provided to illustrate the behavior of the optimal policies.

Some practical assumption of our results would imply a nontrivial problem of construction the numeraire portfolio. Several authors discuss the representative advantages of proxies for numeraire portfolio (Long (1990), Roll (1973)). Several extensions of the model are worthwhile. For instance, since the transaction cost has a direct impact on investment performance, it should be taken into consideration in the dynamic rebalancing model. On the other hands, there are several empirical evidences showing that the portfolio returns are not normally distributed. For that reason, adding higher moment into mean-variance model would close match with the market, even though it may make the optimization problem more sophisticated to solve.