

1 Introduction

Ever since 2003, tight spreads in the credit markets have forced investors to look for innovative structures that offer enhanced yields. One such structure is the synthetic CDO of CDO tranches, commonly known as a CDO-squared, or CDO². The synthetic nature of a CDO-squared is characterized by a pool of underlying tranches of synthetic CDOs, otherwise known as the “inner CDOs”, which are often created only conceptually for the sole purpose of being referenced. The portfolio of synthetic CDO tranches are then tranced into so-called “super tranches”.

The inner CDOs in turn reference on individual pools of underlying entities, such as credit default swaps. The overall pool often consists of up-to 250-350 entities. Not only can the number of referenced entities in a CDO-squared be large, they can at the same time be allowed to overlap. Because of the size of the underlying credits being referenced in the transaction, a CDO-squared in fact offers a risk-return profile that are different from that of an ordinary synthetic CDO tranche.

An additional subordination layer provides extra flexibility in the capital structure of a CDO-squared: Different combinations of subordination levels and widths of the super tranches can be offered; The composition of the underlying pool of individual CDO tranches can be tailor-made; And the risk-reward profile of the tranches can be fine-tuned by varying the degree of overlapping entities in the individual pools. However, such flexibility is coupled with difficult pricing issues. A CDO-squared is particularly sensitive to the following factors: First, the number of defaults; Second, the location of defaults among inner CDOs; Third, the degree of overlapping between inner CDOs; Finally, the correlation of defaults between the entities underlying the inner CDOs.

The number of defaults is defined as the sum of defaulted underlying names in each inner CDOs. It should be noted that the number of defaulted names is irrespective of the overlapping structure, for instance, if an underlying name being referenced by 2 inner CDOs defaults, each of the 2 inner CDOs counts one default, and this contributes a sum of 2 to the number of defaults.

The meaning of the location of defaults is twofold. First, the location of default specifies which inner CDOs are affected when an underlying name defaults. Second, it also specifies which tranches of the particular inner CDOs are consumed. The location of default is closely-related to the overlapping structure of CDO-squared. For example, in one particular case a defaulted underlying name X, being referenced by inner CDO A and B, consumes the notional amount of the mezzanine tranche of inner CDO A and that of the equity tranche of inner CDO B.

The degree of overlapping refers to the extent of repetition of the underlying names. If a large portion of underlying names are being referenced repeatedly by the of inner CDOs, we say there is a high degree of overlapping and also a high correlation of defaults among the inner CDOs.

We have found out that the previous studies of CDO² failed to characterize the relationship between the number of defaults and the location of defaults.

And these are the main issues and problems to which we are going to provide a solution. Furthermore, we have also discovered the presence of overlapping risk which is generated by the overlapping structure of CDO-squared, and how a higher premium is demanded as a compensation for the overlapping risk.

In this paper we address the pricing issues of CDO of CDOs. Underlying the conditional independence assumption we use the factor copula approach to characterize the correlation of defaults events. We provide an efficient recursive algorithm that constructs the loss distribution. Our algorithm accounts for the number of defaults, the location of defaults among inner CDOs, and in addition the degree of overlapping between inner CDOs. Our algorithm is a natural extension of the probability bucketing method of Hull and White (2004). We analyze the sensitivity of different parameters on the tranche spreads of a CDO-squared, and in order to characterize the risk-reward profiles of CDO-squared tranches, we introduces appropriate risk measures that quantify the degree of overlapping among the inner CDOs.

Hull and White (2004) presents a recursive scheme known as probability bucketing approach to construct conditional loss distribution of CDO. However, this approach is insufficient to capture the complexities of CDO².

In the case of the modeling of CDO, we are concerned for the probabilities of different number of defaults upon a time horizon t , e.g., the probabilities of 3 defaults happened within a year. With the mentioned probabilities, we can then calculate the expected loss within the time horizon, which enables us to figure out the spreads of CDO.

However, in the modeling of CDO², an appropriate valuation should be able to overcome two more difficulties:

(1) The overlapping structure of the underlying CDOs

In practice, a CDO² usually consists of a portfolio (of tranches) of the inner CDOs, in which there are overlapping in the fundamental layer—the underlying names being referenced by the inner CDOs. This contributes to the correlation of the inner CDOs, resulting simultaneous jumps whenever the defaults of overlapping credits happened. In another words, the overlapped fundamental layer determines the dependency among the inner CDOs, and in turn complicated the solution to tackle the correlation structure in the valuation of CDO².

(2) The location where defaults happened

As mentioned above, in the modeling of CDO we only need to pay attention the number of defaults in the underlying pool before the maturity; assuming equally weighted composition of principals and constant recovery, which particular set of defaults happened is insignificant. However, for CDO² we also have to consider the location of the defaults; i.e. to specify the payoff of CDO², we need to know both the number of defaults and how these default events are distributed in the inner CDOs.

Subsequent parts of this paper are structured as follows: Section 2 reviews the literature on credit risk modeling and the pricing and hedging issues of CDO-squared; Section 3 develops a recursive algorithm which extends the probability bucketing method of Hull and White (2004). We use the algorithm to construct the loss distribution a CDO of CDOs. Section 4 introduces appropriate risk

measures that quantify the effect of overlapping among inner CDOS. We present the numerical results in Section 5; We conduct sensitivity analysis for the pricing results and discuss relevant implications. Section 6 concludes this paper.