

# 1 Introduction

Although the defined contribution (hereafter DC) pension plans have been the primary engine of growth in the U.S. private pension market over the last two decades (see Lachance et al., 2003), the pension systems in most of Asian countries have traditionally been more tied to defined benefit (hereafter DB) pension plans. The benefits in DB plans are fixed in advance and the plan sponsor adjusts the contributions annually, whereas the benefits in DC plans are determined by the performance of the invested portfolio and the contributions are fixed. But recently the legislatures in many Asian countries have introduced DC pension schemes into their pension system. To do so, they either reform their pension to DC schemes or plan to restructure their retirement program to a mixture of DC and DB pension schemes. A DC plan offers participants flexibility, portability, and investment portfolio choice, all of which can improve an employer's ability to attract and retain workers. But the investment risk that had been assumed by the plan sponsor under the DB promise is transferred to the worker in a DC plan (Bodie, 1990).

Recently, some states in the U.S. have introduced a guarantee mechanism to help protect DC plan participants. One such guarantee takes the form of an option permitting DC plan participants to buy back their DB benefit for a price (see Lachance et al., 2003; Milevsky and Promislow, 2004). Lachance et al. (2003) found that if employees were to exercise the buy-back option optimally, the market value of this option could represent up to 100 percent of the DC contributions over their work life. Thus the potential costs involved with the provision of such a guarantee are easily misunderstood and can be quite large. In this study, a theoretical framework is developed to analyze the optimal portfolio strategy and illustrate how the portfolio characteristics influence the behaviors of the wealth accumulation. Asset allocation is a decision-making process in which the investment funds of an individual or a group of individuals are allocated to investment categories rather than to individual assets. Studies by Brinson, et al. (1986, 1990) have shown convincingly that allocation of investment funds to asset categories is far more important than the selection of individual securities within each asset category (also see Smith, 1998).

Since a major part of the wealth of the plan participant comes from the labor income and may suffer from the inflation risk due to the long time before his retirement, three issues listed in the following are considered and investigated in detail.

1. How can the dynamic hedge demands for future inflation and labor income uncertainties be incorporated in making the long-term financial planning for retirement ?
2. How can the incentive fees or the floor protections provided in the fund management be reasonably controlled through the capital market, based on the mutual fund separation theorem ?
3. How can the risks and rewards from the financial market be reasonably

considered with different risk factors, investment time horizons and attitudes toward risks ?

The investment performance of pension funds has become a crucial theme in more recent years, especially due to the aging society worldwide. Pension contributions can be regarded as a form of mandatory savings for the plan participants before their retirement. According to the Labor Worker Standard Law enacted by Taiwan government in 1984, an employer is required to contribute 2% to 15% of each employee's pensionable payroll to a government-managed trust fund with minimum returns guaranteed by the government. The mandatory pension plan is a defined benefit scheme since the participant's retirement benefits are calculated according to the length of employed time and the final salary upon retirement. With over seven million employees, the labor worker retirement system is one of the major pension plans in Taiwan. Although the pension plan has minimum guaranteed returns from the government, the plan is subject to insolvency risk because the insufficient contributions coupled with low investment returns may not be able to match the benefit payments.

After lengthy debates over several decades, the Legislation Yuan in Taiwan finally reformed its Labor Standard Law in June 2004. The legislature has adopted a new system that provide two kinds of retirement options to the labor workers through the personal retirement account and the annuity insurance program. This new system will be enacted in July 2005. Under article 6, employers shall contribute on a monthly basis to the Labor Pension Fund by contributing into the individual pension fund accounts at the Bureau of Labor Insurance for those employees who are subject to this Act. Under the personal retirement account, the retirement provisions for the new labor workers belong to the defined contribution schemes, and the rate of contribution by an employer to the Labor Pension Fund per month shall not be less than 6% of the employee's monthly wages. An employee may voluntarily contribute per month, up to 6% of his/her monthly wages to his/her pension fund account. The full amount of the voluntary pension contribution made by an employee may be deducted from the employee's taxable income in the year concerned. Under article 24, an employee who is 60 years old or above and whose seniority is more than 15 years, is entitled to monthly pension payments. An employee whose seniority is less than 15 years shall be entitled to a lump sum pension payment .

The investment performance of the pension fund is guaranteed by the government and in accordance with this Act shall not be less than that of the interest paid for a two-year term time deposit by local banks. In the event of any deficiency, the Treasury shall make up the shortfall. The labor workers can also select the annuity insurance program run by the private insurance companies if the company has more than two hundreds employees and half of the employees agree to this arrangement. The labor workers are eligible to draw their pension at the age of sixty. In addition, this new system also provides the option for active workers to choose between the old system and the new one.

The change from the DB plans to the DC plans appears to be significant for plan participants, since the accumulation process of fund wealth is totally

different from the previous systems. In the new DC scheme, the investment risk that was previously borne by the sponsors in DB plans has now devolved to the plan participants. In the DC pension plan, the fund manager plays a crucial role in providing the retirement benefits for the plan participants against the cost of living adjustment (*COLA*) since the final wealth at his retirement date depends on the fund performance. Therefore, a comprehensive study is required to clarify how the fund manager handles the pension assets to achieve the required performance under this new pension system.

### **1.1 The retirement system**

A DB pension plan is a pension scheme where the benefits are defined in advance by the sponsor. Contributions are set and subsequently adjusted so as to ensure that the scheme remains in balance. Conversely, a DC pension plan is a pension scheme where only contributions are fixed and benefits therefore depend solely on the return of the assets. Thus, DC plans allow the contributors to trace the value of their retirement accounts. The only aspect that a plan participant can determine is to make reasonable assumptions on the replacement ratio, i.e. the percentage of her income which will be replaced by the pension. Currently, most of the proposed pension plans are based on DC schemes involving a considerable transfer of risks to employees. As we have already highlighted, the employee's contributions are determined in advance, while the final retirement account depends on the administrative expenses and investment performance of the fund managers. Consequently, an efficient financial management is essential to gain contributors' trust.

### **1.2 The asset allocation problem**

The goal of the fund manager is to invest the accumulated wealth to maximize the expected utility of the terminal fund wealth. Most conventional pension management is constructed under one-period and constant interest rate assumptions, and uses the Markowitz (1952) approach, i.e. mean-variance method. Sharpe (1991) describes the mean-variance approach as a highly parsimonious characterization of investor goals, employing a myopic view and focusing on only two aspects of the probability distribution of possible returns over that period. The main drawback in the method which discussed in Sharpe (1991) is that the aggregation of single-period optimal decisions across periods might not be optimal for multiple periods as a whole. Similarly, the pension fund holders are usually long-term investors, generally from 20 to 40 years, the assumption of constant interest rates is not suitable for our purpose. Thus, pension plan management should be considered within multi-period framework. Methods of optimal control solve the long-term financial planning problems through global optimization across periods instead of local optimization within a period.

### 1.3 The developments of the multi-period problems

The control theory has been developed in engineering fields since the 1930s and the applications to economics emerged in 1950s. Samuelson (1969), Merton (1969, 1970, 1990), Brennan and Schwartz (1982), Karatzas et al. (1986), Karatzas and Shreve (1991), Duffie (1996), Kim and Omberg (1996), Brennan, Schwartz and Lagnado (1997), Boyle and Yang (1997), Brennan and Schwartz (1998), Sorensen (1999), Wachter (2002), and Campbell and Vicerira (1999, 2001) studied the optimal consumption and investment problems under different setting using the control theory.

The application of control theory to pension plan management began with O'Brien (1986, 1987). Subsequent papers along this line include Haberman and Sung (1994), Runggaldier (1998), Schäl (1998), Chang (1999, 2002), Chang et al. (2003) and related works. Cairn (2000) further introduces the asset allocation into the controlled process, studying the optimal investment strategies as well as optimal funding policies to minimize certain quadratic loss functions. Josa-Fombellidal and Rinc-Zapatero (2001) minimize the contribution rate risk and the solvency risk with  $n$  risky assets and a risk-free security in the presence of short selling constraints. Chang et al. (2002) study the dynamic funding policy and investment strategy for the defined benefit pension schemes. They formulate the optimal decisions of pension plans as a stochastic control problem and solve the problem through dynamics programming. Chang et al. (2003) show that failing to recognize the under-funding risk and the over-contribution risk will lead to a significant difference in optimal funding schedule. The weighting factors and the returns of investment also play critical roles in obtaining the optimal strategy.

### 1.4 Background risks

Since the background risks proposed in Menoncin (2002) are not fully investigated in the previous pension studies, in addition to the state variables (i.e., the risks from the financial market), we consider a set of stochastic processes to describe the inflation and labor income uncertainties, called the background variables in this study. These factors significantly affect the growth of investor wealth and by means of incorporating these variables into the models we are capable of considering several crucial factors for financial planning.

First, we consider the uncertainty of labor income since this has a profound effect on the plan contributions. According to Campbell and Viceira (2002), the investors own tradable financial assets as part of their total wealth portfolio, but they also own a valuable asset that is not readily tradable. This asset is their human wealth, the expected discounted value of their future labor income. Figure 1 shows that the personal average wages have changed every year from 1988 to 2004 in Taiwan manufacturing industries. In addition, Figure 2 shows the range of the nominal increment rates over the same time period. Imrohorglu, Imrohorglu and Joines (1995, 1999a) investigate the role of social security in a general equilibrium setting with salary uncertainty but non-stochastic rates of

return and no annuities. Huang, Imrohoroglu and Sargent (1997) focus on the intergenerational impact of various social security systems on transition paths. They allow for stochastic salary uncertainty but there is no uncertainty on rates of return. Campbell, Cocco, Gomes and Maenhout (2001) consider the long-run pattern of lifetime savings and portfolio allocation in the presence of income and rate of return uncertainty and with various pension arrangements. They do not consider the impact of varying degrees of imperfection in annuity markets but do consider fixed costs of entering the equity market. Since we consider the DC plan in which employees have to contribute a fixed rate of their income to their personal accounts. These can be regarded as a personal pensions. Personal pensions mean that labor income risk from working years, which will have an impact on the contributions to a personal pension fund, has lasting effects upon pension income.

Campbell and Viceira (2002) discuss the effects due to the labor income risk. First, the existence of other income prospects tends to substitute for bonds in the investor portfolio. Hence a relatively young investor with extensive future earnings prospects will tend to have a higher proportion of stocks than an investor at a later stage of his working life. However, this effect is reduced if the income prospects are uncertain. In line with the literature on background risk, in effect the investor becomes more risk-averse to the market risks and, hence, buys less stocks. Viceira (2001) optimizes the intertemporal investment-consumption policy of an investor who has uncertain salary. In his model, labor income follows a geometric process and any savings out of labor income are invested in the portfolio. The single risky asset also follows a possibly correlated geometric process. Viceira finds that the ratio of portfolio wealth to labor income is stationary, and using a log-linear approximation he derives an optimal portfolio policy which has a constant stock proportion. He also finds that when salary risk is independent of the asset return risk, employed investors hold a larger fraction of their savings in the risky asset than retired investors. Koo (1998) and Heaton and Lucas (1997) also derive optimal consumption and portfolio policies with stochastic wage. Koo uses a continuous time model and shows that the optimal level of risk taking is lower in the presence of an uninsurable labor income risk. Heaton and Lucas, in an infinite horizon model, do not find any significant effect of labor income risk on portfolio composition.

Second, we take the inflation risk into account. Actually, when a long period of time is considered, this risk becomes significant. In Figure 3, we draw the annual increasing rates of Consumption Price Index (*CPI*) from 1988 to 2004 of the major countries worldwide. It can be seen that apart from Japan, all countries increase every year. The CPI data in Taiwan is selected and is shown in Figure 4, which shows that the CPI has increased almost every year, so inflation risk exists. Since the pension fund is a long-term plan, the managers should consider this inflation risk and determine the optimal strategy to resist the inflation uncertainties. Modigliani and Cohn (1979), Madsen (2002) and Ritter and Warr (2002) have shown that stock market investors suffer from inflation illusion. Menoncin (2002) considers both the salary uncertainty and also the inflation risk to analyze the portfolio problem of an investor maximiz-

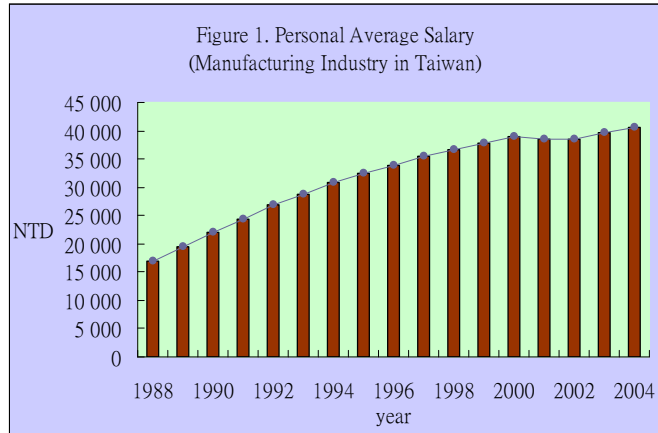


Figure 1: Personal Average Salary in Manufacturing Industry (Data from Directorate General of Budget, Accounting and Statistics, Executive Yuan, R.O.C.)

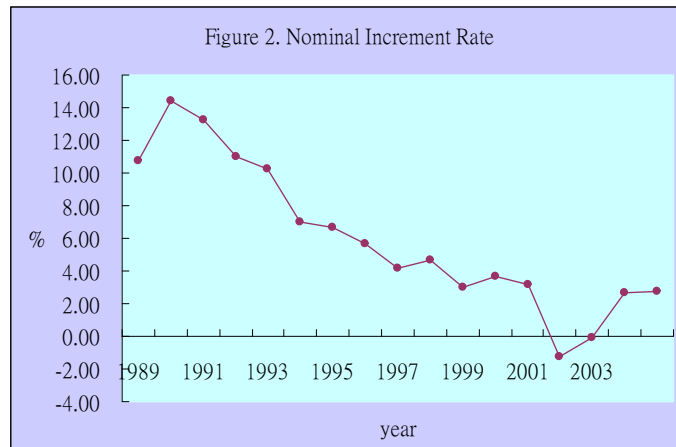


Figure 2: Nominal Increment Rate in Taiwan (Data from Directorate General of Budget, Accounting and Statistics, Executive Yuan, R.O.C.)

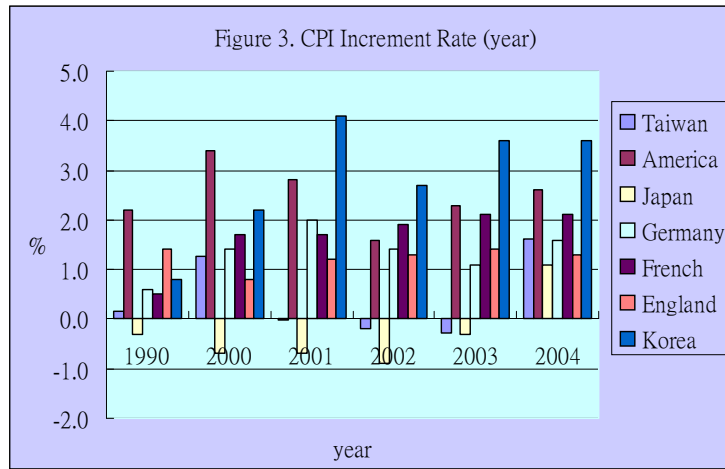


Figure 3: CPI Increment Rate per year (Data from Directorate General of Budget, Accounting and Statistics, Executive Yuan, R.O.C.)

ing the expected exponential utility of his terminal real wealth. In his model, the investor must cope with both a set of stochastic investment opportunities and a set of background risks. If the market is complete this model can find an exact solution. If the market is incomplete, they suggest an approximated general solution. Contrary to other exact solutions obtained in the literature, all their results are obtained considering a stochastic inflation risk and without specifying any particular functional form for the stochastic variables involved in the problem.

### 1.5 The main approach in this article

In this article, we employ the dynamic programming approach presented in Battocchio and Menoncin (2004), in analyzing the optimal portfolio problem for a DC pension fund with labor income risk and inflation risk. We note that in Battocchio and Menoncin (2004), the constraint that the sum of the total invested assets should equal the fund wealth is not clearly verified. At the same time, since the fund performance is also significantly related to the fund management fee and the incentive bonus. Hence a detailed constraint for the assets and the management expenses are incorporated into our framework. The proposed model is summarized as follows.

1. The interest rates vary in the multi-period setting, so the stochastic model proposed in Vasicek (1977) is employed to describe its dynamics.
2. The financial market consists of three assets: a riskless asset (cash), stock index and bond fund, which can be bought and sold without incurring any

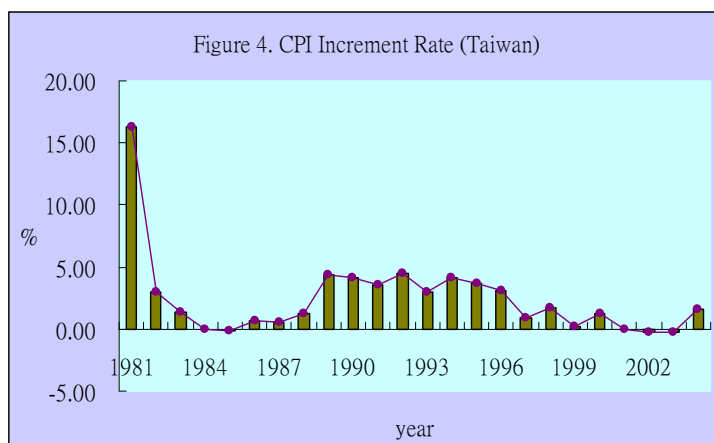


Figure 4: CPI Increment Rate in Taiwan (Data from Directorate General of Budget, Accounting and Statistics, Executive Yuan, R.O.C.)

transaction costs or restriction on short sales.

3. Two stochastic processes are introduced to describe the random behaviors of the labor incomes and the uncertainties of the consumption price index.
4. The management incentives for the fund manager that the pension board must consider are incorporated in evaluating the financial decisions.

In order to fully characterize the characteristic of the DC pension plans, we consider a plan participant who, at each period, contributes a constant proportion of his/her salary income to a personal pension account. At the date of retirement, the accumulated pension fund will be converted into a life annuity.

Similar models have been recently presented by Blake et al. (2000), Boulier et al. (2001) and Deelstra et al. (2003). Especially, Blake et al. (2000) assume a stochastic process for salary including a non-hedgable risk component and focus on the replacement ratio as the central measure for determining the pension flow. Boulier et al. (2001) assume a deterministic process for salary and consider a guarantee on the benefits. Accordingly, they strongly support the real need for a downside protection of contributors who are more directly exposed to the financial risk borne by the pension fund. Also Deelstra et al. (2003) allow for a minimum guarantee in order to minimize the randomness of the retirement account; although they describe the contribution flow through a non-negative, progressive measurable, and square-integrable process. A recent model for a DC pension scheme in discrete time is proposed by Haberman and Vigna (2001). In particular, they study both the investment risk, that is the risk of incurring a poor investment performance during the accumulation phase of the fund, and



the annuity risk, that is the risk of purchasing an annuity at retirement in a particular recessionary economic scenario involving a low conversion rate.

The methodological approach employed in solving the optimal asset allocation problem is dynamic programming. Alternative approaches (see for example Deelstra et al. (2003); Lioui and Poncet (2001)) are based on the Cox-Huang (1991) methodology (the so-called martingale approach), where the resulting partial differential equation is often simpler to solve than the Hamilton-Jacobi-Bellman equation from the dynamic programming. In this article, closed form solutions are constructed to understand the dynamic hedging demands through mutual fund separation theorem originally proposed in Merton (1971) for cases where different risk factors from the financial market, inflation and labor income are involved in the pension economy. An important result called five-fund separation is constructed, which means that investment choices are reduced to the allocation of assets between five specific investment funds (portfolio). Finally, the numerical simulations are presented to investigate the dynamics behavior of optimal portfolio strategy through several separated components.

The work is organized as follows. In Section 2, we introduce the general framework and reveal the financial market structure, the dynamics processes describing the behavior of asset values, the background risks of salary risk and inflation risk, and the fund's wealth. In Section 3, we present the stochastic optimal control problem and use the dynamic programming algorithm to compute the problem and derive an explicit solution. In Section 4, we present a numerical simulation. Finally, Section 5 concludes and summarizes the paper.