4 Numerical illustration

In this numerical illustration, we use the numerical simulation to demonstrate both the dynamic behaviors of the optimal portfolio strategy and the optimal separation portfolio strategy, which were derived in the previous section. Table 1 reports the set of parameters representing the financial market and background risks. Note that some parameters are consistent with the numerical analysis presented by Battocchio and Menoncin (2004).

Table 1. Parameter values used in the numerical analysis

Parameter Description	Notation	Values	Parameter Description	Notation	Values
Interest rate			Fix-maturity bond		
Mean reversion	a	0.2	Maturity	K	10
Mean rate	b	0.05	Market price of risk	λ_r	0.15
Volatility factor	σ_r	0.02	Defined Contribution Process		
Initial rate	r_0	0.03	Labor income growth	μ_L	0.046
Stock			Volatility factor	$\sigma_{L,r}$	0.014
Market price of risk	λ_r	0.15	Volatility factor	$\sigma_{L,m}$	0.153
Market price of risk	λ_m	0.31	Volatility factor	σ_L	0.01
Volatility factor	$\sigma_{S,r}$	0.06	Initial labor income	L_0	100
Volatility factor	$\sigma_{S,m}$	0.17	Contribution rate	γ	0.06
Inflation process			Time horizon	T	10
Mean rate	μ_π	0.015	expense ratio	e	0.01
Volatility factor	$\sigma_{\pi,r}$	0.018	Utility parameter	β_2	-20
Volatility factor	$\sigma_{\pi,m}$	0.136			
Volatility factor	σ_π	0.015			

To provide the calculations of labor income and CPI effects on the optimal choice, the salary increase rates and rate of inflations are simulated in Figure 5. Figure 6 plots the simulated market values of the cash, the stock index and the bond funds over the investment horizon.

Figure 7 plots the optimal portfolio holdings of cash, stock and nominal bonds as a function of investment of horizon, i.e., 120 months, for an investment. The real fund wealth is also plotted for comparison.

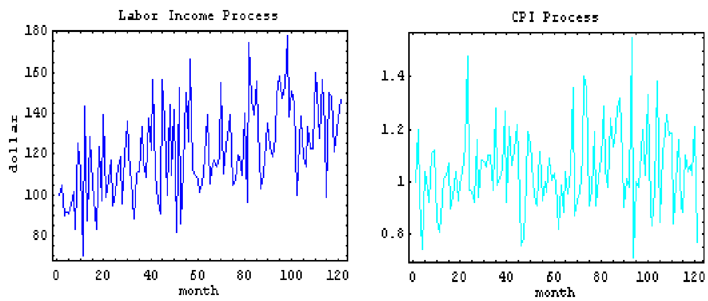

Figure 1: Processes of labor income and CPI

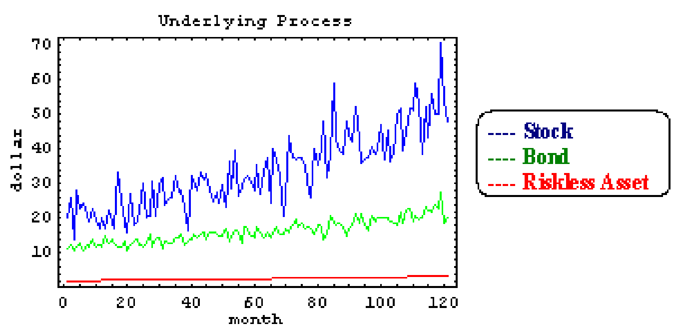

Figure 2: Dynamic process of the underlying assets

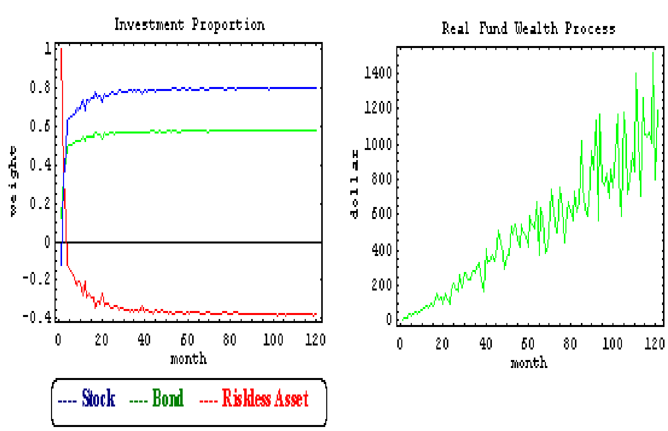

Figure 3: Proportion composition of optimal portfolio and real wealth

Figure 8 confirms the separation of four fund effects in the optimal portfolio selection and their behaviors in each component over time. The market portfolio has shown a decreasing trend for stock index and bond fund holdings due to the utility maximization principle. In contrast the state variable hedge portfolio shows a steady pattern for the optimal weight for bond fund and stock index holdings. To hedge the risk from the state variables, the investment strategy needs to hold a fixed proportion of bond fund and also reduce the holding of the stock index. In the inflation hedge portfolio, the investors are required to hold a high proportion of the stock index, up to 80%, to hedge the inflation risks, while only a small proportion of bond fund is sufficient in the hedge portfolio. However, in the labor income hedge portfolio, the investor should short sell his stock index and the bond portfolio in order to preserve the salary uncertainty over his investment horizon.

In Figure 9, the weights of the stock index in the entire optimal portfolio and the weights for the separated mutual funds are shown as illustration. The results indicate that the inflation hedge portfolio constitute the overwhelming proportion (75%) of the optimal portfolios. While, the state variable hedge portfolio, the market myopic portfolio and labor income hedge portfolio play only minor parts in the optimal portfolio selection problem.

The weights of the bond fund in the entire optimal portfolio and the weights for the separated mutual funds are shown in Figure 10. These results indicate that the inflation hedge portfolio (around 35%) and the state variable hedge portfolio (around 20%) constitute the largest proportions of all long-term financial portfolios. However, the market myopic portfolio and labor income hedge portfolio play only minor parts in the optimal portfolio selection problem. Further work is required to asses more precisely the dynamics of the optimal portfolio under several plausible scenarios.

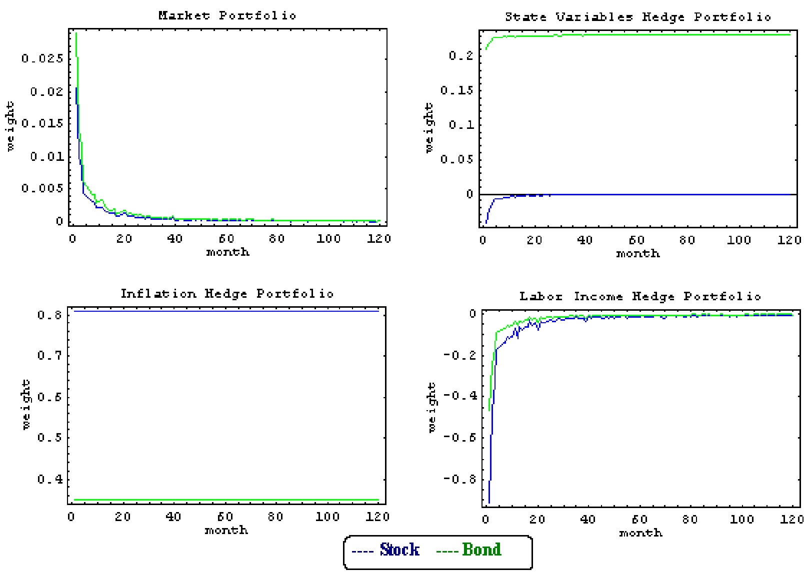

Figure 4: Percentage composition of optimal separated portfolios

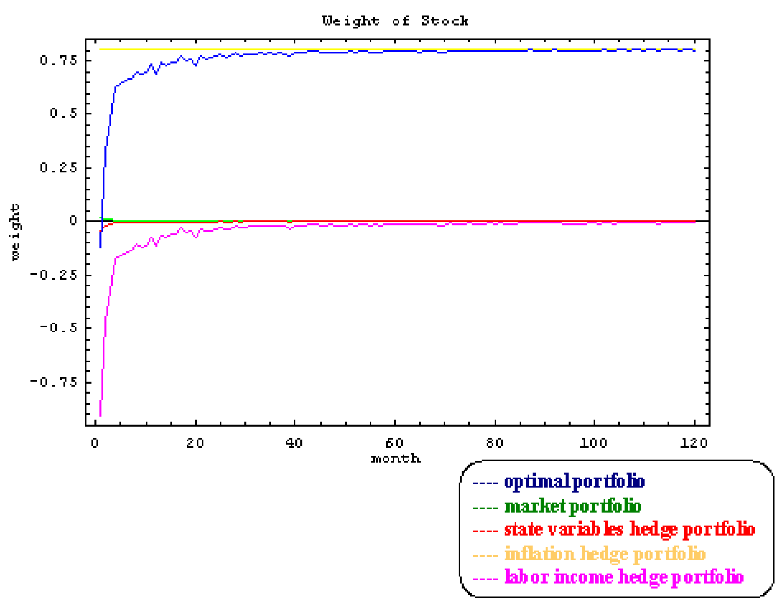

Figure 5: Proportion of stock in the optimal separted portfolio

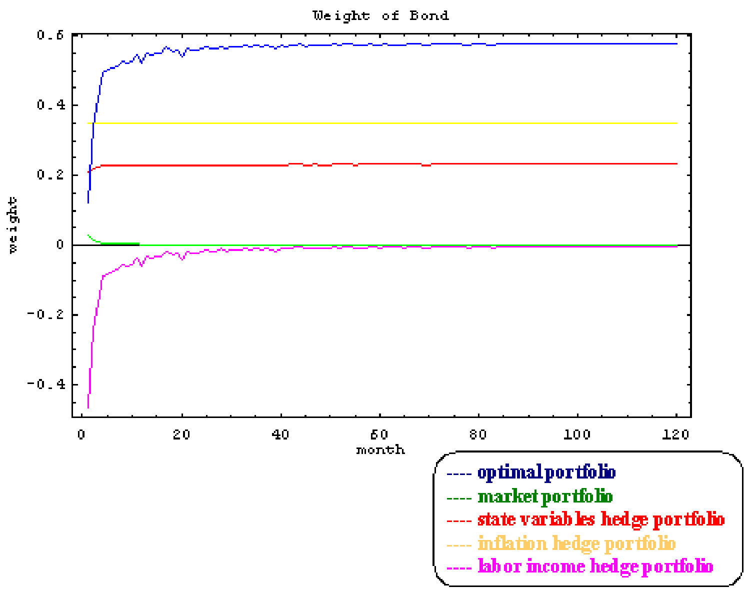

Figure 6: Proportion of bond in the optimal separted portfolio