Appendix

Appendix A1: The equation of the stock price.

The solution begins with Eq. (2.15) from the text, which we repeat here for convenience as Eq. (A.1):

\[
\left(\frac{-1}{1-\tau}\right) \cdot \frac{\omega}{b} p_t + \frac{1}{1-\tau} [E_t p_{t+1} - E_t p_t + p_{t-1}] = -X_t, \tag{A.1}
\]

where \(X_t = -\frac{c}{b} - \frac{1}{b} u_t + \frac{\alpha}{b} \varepsilon_t - \frac{\Delta_t}{(1-\tau)^2}, \quad \alpha + \gamma = \omega, \quad \text{and} \quad c = -\alpha\gamma + AR.

Take expectations of Eq. (A.1) as of time \(t - 1\), we obtain:

\[
\left(\frac{-2}{1-\tau}\right) \cdot \frac{\omega}{b} E_{t-1} p_t + \frac{1}{1-\tau} (E_{t-1} p_{t+1} + E_{t-1} p_{t-1}) = -E_{t-1} X_t, \tag{A.2}
\]

This expression is a difference equation in the expectation terms. Use of the lag operator \(L\) allows the substitute:

\[
E_{t-1} p_{t-1} = LE_{t-1} p_t, \quad E_{t-1} p_{t+1} = L^2 E_{t-1} p_t. \tag{A.3}
\]

We therefore obtain:

\[
\left\{ L^2 - \frac{\omega(1-\tau)}{b} L + 1 \right\} E_{t-1} p_t = 0 \tag{A.4}
\]

Therefore, either we require that \(E_{t-1} p_t = 0\) or we require that the quadratic expression in \(L\) be identically equal to zero. As a result, a nontrivial solution requires that:

\[
L^2 - \frac{\omega(1-\tau)}{b} L + 1 = 0. \tag{A.5}
\]

Denote the roots of this quadratic equation by \(\lambda_i, \quad i = 1, 2\), then:
\[ \lambda_1 \lambda_2 = 1 \quad \text{and} \quad \lambda_1 + \lambda_2 = 2 + \frac{\omega(1-\tau)}{b} > 2. \] (A.6)

From Eq. (A.6), we can assure that one root is greater than 1 and the other root is less than 1. Furthermore, the general solution to Eq. (A.1) is given by:

\[ E_{t-1} P_{t+j} = c \lambda_1^j + d \lambda_2^j, \] (A.7)

with \( c \) and \( d \) determined by an initial-value condition. In the special case of \( E_{t-1} P_{t-1} = P_{t-1} \), we can therefore substitute \( j = -1 \) into Eq. (A.7) and obtain:

\[ P_{t-1} = c \lambda_1^{-1} + d \lambda_2^{-1}. \] (A.8)

The constant (\( c \) or \( d \)) corresponding to the greater root is assumed to be equal to zero to ensure a convergent system. Now define \( \lambda = \lambda_1 < \lambda_2 \), where \( \lambda_1 \) is selected as the smaller of the two roots. In this case, \( d \) is supposed to be zero so that Eq. (A.8) is express as:

\[ E_{t-1} P_{t-1} = P_{t-1} = c \lambda^{-1}. \] (A.9)

We can re-express it as: \( c = \lambda P_{t-1} \). Therefore, the rational price expectation process is given by:

\[ E_{t-1} P_{t+j} = \lambda^{j+1} P_{t-1}, \] (A.10)

and so

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1 See Sargent (1987, Ch.9).

2 In calculation, we adopt the "backward" method to obtain \( 0 < \lambda_1 < 1 \) and \( \lambda_2 > 1 \), and then select \( \lambda_1 = \lambda \) to ensure a convergent system. However, if the "forward" method is adopted, we may obtain another two roots: \( 0 < \frac{1}{\lambda_2} < 1 \) and \( \frac{1}{\lambda_1} > 1 \), and then choose \( \frac{1}{\lambda_2} = \lambda \) to ensure a convergent system. In our following analysis, we only use the property of \( 0 < \lambda < 1 \), rather than the value of \( \lambda \). Therefore, either backward solution or forward solution will lead to the same conclusions.
\[ E_{t-1}P_t = \lambda P_{t-1} \quad \text{and} \quad E_tP_{t+1} = \lambda P_t. \]  

(A.11)

Substitute Eq. (A.11) into Eq. (A.1), we obtain:

\[ p_t = \frac{1}{(1-\tau)^{-1}(1-\lambda)} p_{t-1} + \frac{1}{(1-\tau)^{-1}(1-\lambda) + \frac{\omega}{b}} X_t. \]  

(A.12)

**Appendix A2: The dynamic equation of the stock price.**

To obtain the stock price’s dynamic equation, we first have to compute the long-run equilibrium \( \bar{p} \). In the long run, all unexpected shocks will not exist anymore and the stock price in each period will be equal to the long-run equilibrium stock price. Judging from the above, we therefore set \( u_t = \varepsilon_t = \Delta_t = 0 \), and replace \( p_t \) and \( p_{t-1} \) by \( \bar{p} \) in Eq.(A.1) to obtain:

\[ \bar{p} = \frac{-1}{\omega} (-\alpha y_f + AR) = \frac{-c}{\omega}, \]  

(A.13)

where \( c = -\alpha y_f + AR \).

In addition, we can rewrite the RE equation as follows:

\[ \frac{(1-\lambda)^2}{1-\tau} = \frac{\lambda \omega}{b}. \]  

(A.14)

Substitute Eq.(A.13) and Eq.(A.14) into Eq.(A.1), we obtain:

\[ p_t = \lambda p_{t-1} + \frac{\lambda(1-\tau)}{1-\lambda} \left[ -\frac{c}{b} u_t - \frac{1}{b} \varepsilon_t - \frac{\Delta_t}{(1-\tau)^2} \right] \]

\[ = \lambda p_{t-1} + (1-\lambda)\bar{q} - \frac{(1-\lambda)}{\omega} u_t + \frac{\alpha(1-\lambda)}{\omega} \varepsilon_t - \frac{\lambda}{(1-\lambda)(1-\tau)} \Delta_t. \]  

(A.15)
Appendix A3: Derivation of $\frac{d\lambda}{d\tau}$

From Eq.(3.3) and Eq.(3.4), we have:

$$b(1-\tau)^{-1}(1-\lambda)^2 = \lambda \omega,$$  \hspace{1cm} (RE)  \hspace{1cm} (A.16)

$$b(1-\tau)^{-1}(1-\lambda)^2 = \frac{(1-\tau)^{-1}(1-\lambda)^2}{\theta \text{Var}(p_{z,t})} = f.$$  \hspace{1cm} (SB)  \hspace{1cm} (A.17)

Differentiating the above two expressions with respect to $b$, $\lambda$, $\tau$, and then expressing them in matrix notation, we obtain:

$$\begin{bmatrix} (1-\tau)^{-1}(1-\lambda)^2 & -2b(1-\tau)^{-1}(1-\lambda) - \omega \end{bmatrix} \begin{bmatrix} db \end{bmatrix} = \begin{bmatrix} -b(1-\tau)^{-2}(1-\lambda)^2 d\tau \\ -b(1-\tau)^{-2}(1-\lambda)^2 + f \end{bmatrix} d\tau,$$

(A.18)

where $f$ denotes the SB equation:

$$f = \frac{(1-\lambda)^2}{\theta(1-\tau) \left( \frac{(1-\lambda)^2}{\omega^2} \cdot \text{Var}(u) + \frac{\alpha^2 (1-\lambda)^2}{\omega^2} \cdot \text{Var}(\epsilon) + \frac{\lambda^2}{(1-\lambda)^2 (1-\tau)^2} \cdot \text{Var}(\Delta) \right)},$$

(A.19)

$f_{\lambda}$ denotes the partial derivative of $f$ with respect to $\lambda$:

$$f_{\lambda} = -\frac{1}{\theta(1-\tau) \left[ \frac{1}{\omega^2} \cdot \text{Var}(u) + \frac{\alpha^2}{\omega^2} \cdot \text{Var}(\epsilon) + \frac{\lambda^2}{(1-\lambda)^2 (1-\tau)^2} \cdot \text{Var}(\Delta) \right]^2} \left( \frac{2\lambda(1+\lambda)}{(1-\lambda)^2 (1-\tau)^2} \cdot \text{Var}(\Delta) \right),$$

(A.20)

and $f_{\tau}$ denotes the partial derivative of $f$ with respect to $\tau$:
\[ f_{\tau} = -\frac{1}{\theta \left[ \frac{(1-\tau)}{\omega^2} \text{Var}(u) + \frac{\alpha^2 (1-\tau)}{\omega^2} \text{Var}(\varepsilon) + \frac{\lambda^2}{(1-\lambda)^4 (1-\tau)^2} \text{Var}(\Delta) \right]} \cdot \]
\[ \left[ -\frac{1}{\omega^2} \text{Var}(u) - \frac{\alpha^2}{\omega^2} \text{Var}(\varepsilon) + \frac{\lambda^2}{(1-\lambda)^4 (1-\tau)^2} \text{Var}(\Delta) \right]. \]

(A.21)

Thus, application of Cramer’s rule yields derivation of:

\[ \frac{d\lambda}{d\tau} = f_{\tau} \cdot \omega - f_{\lambda}. \quad (A.22) \]

**Appendix A4: Determination of \( \frac{d\lambda}{d\tau} \) under different types of shocks.**

(1) **The issuing shock**

Substituting \( \text{Var}(\varepsilon) = \text{Var}(\Delta) = 0 \) into \( f_{\lambda} \) and \( f_{\tau} \), we have:

\[ f_{\lambda} = 0, \text{ and } \quad f_{\tau} = \frac{1}{\theta (1-\tau)^2 \cdot \frac{1}{\omega^2} \cdot \text{Var}(u)} > 0. \quad (A.23) \]

Therefore,

\[ \frac{d\lambda}{d\tau} = \frac{f_{\tau}}{\omega - f_{\lambda}} = \frac{1}{\theta (1-\tau)^2 \cdot \frac{1}{\omega} \cdot \text{Var}(u)} > 0. \quad (A.24) \]

(2) **The dividend shock**

Substituting \( \text{Var}(u) = \text{Var}(\Delta) = 0 \) into \( f_{\lambda} \) and \( f_{\tau} \), we have:

\[ f_{\lambda} = 0, \text{ and } \quad f_{\tau} = \frac{1}{\theta (1-\tau)^2 \cdot \frac{\alpha^2}{\omega^2} \cdot \text{Var}(\varepsilon)} > 0. \quad (A.25) \]

Therefore,
\[
\frac{d\lambda}{d\tau} = \frac{f_\lambda}{\omega - f_\lambda} = \frac{1}{\theta(1 - \tau)^2 \cdot \frac{\alpha^2}{\omega} \cdot \text{Var}(\varepsilon)} > 0 .
\] (A.26)

(3) The margin-rate shock

Substituting \(\text{Var}(u) = \text{Var}(\varepsilon) = 0\) into \(f_\lambda\) and \(f_\tau\), we have:

\[
f_\lambda = -\frac{2(1 - \tau)(1 - \lambda)^3(1 + \lambda)}{\theta \lambda \text{Var}(\Delta)} < 0 , \quad \text{and} \quad f_\tau = \frac{-2(1 - \lambda)^4}{\theta \lambda^2 \text{Var}(\Delta)} < 0 .
\] (A.27)

Therefore,

\[
\frac{d\lambda}{d\tau} = \frac{f_\lambda}{\omega - f_\lambda} = \frac{-2(1 - \lambda)^4}{\theta \omega \lambda^2 \text{Var}(\Delta) + \frac{2(1 - \tau)(1 - \lambda)^3(1 + \lambda)}{\lambda}} < 0 .
\] (A.28)

**Appendix A5: Determination of \(\frac{dh_\tau}{d\tau}\) under different types of shocks.**

(1) The issuing shock

From Eq.(3.7) and Eq.(3.8), we have:

\[
h_\tau = \frac{(1 - \lambda)}{\theta \text{Var}(p_{t+1})(1 - \tau)} \cdot (\bar{p} - p_t) = \frac{(1 - \lambda)^2 u_t}{\theta \omega (1 - \tau) \text{Var}(p_{t+1})} .
\] (A.29)

Thus, differentiating \(h_\tau\) with respect to \(\tau\), we obtain:

\[
\frac{dh_\tau}{d\tau} = \frac{u_t}{\omega \theta} \cdot d \left[ \frac{1}{\text{Var}(p_{t+1})} \cdot \frac{(1 - \lambda)^2}{1 - \tau} \right]
\]

\[
= \frac{u_t}{\omega \theta} \cdot \left\{ \frac{-1}{\text{Var}^2(p_{t+1})} \cdot \frac{d\text{Var}(p_{t+1})}{d\tau} \cdot \frac{(1 - \lambda)^2}{1 - \tau} + \frac{1}{\text{Var}(p_{t+1})} \cdot \left[ \frac{-2(1 - \lambda)}{1 - \tau} \cdot \frac{d\lambda}{d\tau} + \frac{(1 - \lambda)^2}{(1 - \tau)^2} \right] \right\}
\]

\[
= \frac{u_t}{\omega \theta} \cdot \frac{(1 - \lambda)^2 \text{Var}(u)}{\omega^2} \cdot \frac{-1}{\omega^2} \cdot \text{Var}(u) \cdot \frac{2(1 - \lambda)}{1 - \tau} \cdot \frac{d\lambda}{d\tau} \cdot \text{Var}(u) \cdot \frac{(1 - \lambda)^2}{1 - \tau}
\]
\[- \frac{2(1 - \lambda)}{1 - \tau} \frac{d\lambda}{d\tau} + \frac{(1 - \lambda)^2}{(1 - \tau)^2} \]

\[= \frac{\omega \cdot u}{(1 - \tau)^2 \text{Var}(u)} > 0. \quad (A.30)\]

(2) The dividend shock

From Eq.(3.11) and Eq.(3.12), we have:

\[h_i = \frac{(1 - \lambda)}{\theta \text{Var}(p_{t+1}) (1 - \tau)} \cdot (P - p_i) = \frac{-\alpha (1 - \lambda)^2 \epsilon_i}{\theta \alpha (1 - \tau) \text{Var}(p_{t+1})}. \quad (A.31)\]

Thus, differentiating \(h_i \) with respect to \(\tau\), we obtain:

\[\frac{dh_i}{d\tau} = -\frac{\alpha \epsilon_i}{\omega \theta} \cdot \frac{d}{d\tau} \left[ \frac{1}{\theta \text{Var}(p_{t+1})} \cdot \frac{(1 - \lambda)^2}{1 - \tau} \right] \]

\[= -\frac{\alpha \epsilon_i}{\omega \theta} \cdot \left( \frac{1}{\theta \text{Var}(p_{t+1})} \cdot \frac{d\text{Var}(p_{t+1})}{d\tau} \cdot \frac{(1 - \lambda)^2}{1 - \tau} + \frac{1}{\theta \text{Var}(p_{t+1})} \cdot \left[ -2 \frac{1 - \lambda}{1 - \tau} \cdot \frac{d\lambda}{d\tau} + \frac{(1 - \lambda)^2}{(1 - \tau)^2} \right] \right) \]

\[= -\frac{\alpha \epsilon_i}{\omega \theta} \cdot \frac{\alpha^2 (1 - \lambda)^2 \text{Var}(\epsilon)}{\omega^2} \cdot \frac{1}{\theta \text{Var}(p_{t+1})} \cdot \frac{d\text{Var}(p_{t+1})}{d\tau} \cdot \frac{(1 - \lambda)^2}{1 - \tau} \]

\[- \frac{2(1 - \lambda)}{1 - \tau} \cdot \frac{d\lambda}{d\tau} + \frac{(1 - \lambda)^2}{(1 - \tau)^2} \]

\[= \frac{-\omega \epsilon}{\alpha (1 - \tau)^2 \text{Var}(\epsilon)} < 0. \quad (A.32)\]

(3) The margin-rate shock

From Eq.(3.16) and Eq.(3.21), we have:

\[h_i = \frac{\alpha^2 (1 - \lambda)^2 \text{Var}(\epsilon) - (1 - \lambda)^2}{\theta \text{Var}(p_{t+1}) \lambda (1 - \tau)} \cdot (P - p_i) = \frac{(\lambda - 1) \Delta_i}{\theta (1 - \tau)^2 \text{Var}(p_{t+1})}. \quad (A.33)\]

Similarly, let us differentiating the above expression with respect to \(\tau\), we obtain:

\[\frac{dh_i}{d\tau} = \frac{\Delta_i}{\theta} \cdot \frac{1}{\theta \text{Var}(p_{t+1})} \cdot \frac{\lambda - 1}{(1 - \tau)^2} \]
\[ \frac{\Delta_t}{\theta} \cdot \left\{ \frac{-1}{\text{Var}^2(p_{t,t+1})} \cdot \frac{d\text{Var}(p_{t,t+1})}{d\tau} \cdot \frac{\lambda - 1}{(1 - \tau)^2} + \frac{1}{\text{Var}(p_{t,t+1})} \left[ \frac{1}{(1 - \tau)^2} \cdot \frac{d\lambda}{d\tau} + \frac{2(\lambda - 1)}{(1 - \tau)^3} \right] \right\} \]

\[ = \frac{\Delta_t}{\theta} \cdot \frac{\lambda^2 \text{Var}(\Delta)}{(1 - \lambda)^2 (1 - \tau)^2} \cdot \frac{-1}{\text{Var}(\Delta)} \left[ \frac{2\lambda^2}{(1 - \lambda)^2 (1 - \tau)^3} + \frac{2\lambda}{(1 - \lambda)^3 (1 - \tau)^2} \cdot \frac{d\lambda}{d\tau} \text{Var}(\Delta) \right] \]

\[ = \frac{\lambda - 1}{(1 - \tau)^3} + \frac{1}{(1 - \tau)^3} \cdot \frac{d\lambda}{d\tau} + \frac{2(\lambda - 1)}{(1 - \tau)^3} \]

\[ = \frac{\Delta_t}{\theta \text{Var}(p_{t,t+1})} \cdot \frac{\lambda + 2}{\lambda (1 - \tau)^3} \cdot \frac{d\lambda}{d\tau} < 0, \quad (A.34) \]

since \( \frac{d\lambda}{d\tau} < 0 \) in this case.