Chapter 5

Numerical Results: Pricing the DJ iTraxx

In order to compare the properties of the large homogeneous portfolio with Gaussian, NIG, CSN, mixture of NIG and CSN copula model, we use the DJ iTraxx Europe series 5 with five years maturity to search the numerical results.

The reference portfolio consists of 125 credit default swap instruments with equal weight. The standard CDS index tranches with attachment/detachment points at 3%, 6%, 9%, 12%, 22%. The investors for equity tranches receive the market quote up-front-fee paid initially and an annual spread of 500 bp quarterly. Others also receive the spread payments on the outstanding notional and compensate for the losses of the credit portfolio when they hit the tranche they invested in. The settlement date of the fifth series of this index is 20-March-2006 and maturity 20-June-2011. We get the market quotes of these tranches at 12-April-2006 and 23-June-2006. The average CDS spread of the corresponding CDS portfolio with 5 years maturity is 32 bp and 34.8 bp at these days. We assume the constant recovery rate is 40% and the risk neutral rate as the Euro LIBOR rate at the same date. Furthermore, we use the constant default intensity model to derive the marginal default distributions and estimate the default intensity from the average CDS spread of the reference portfolio. The only one parameter ρ of the Gaussian copula model needs to be estimated by setting the price of equity tranche equal to the market quote. The parameters α and β for double NIG copula have to be estimated by minimizing the sum of the absolute errors over all tranches. The absolute errors are calculated by summing up all of absolute differences of model generated tranche prices and market quotes for each model. For more details, see Kalemanove et al. (2007). We use the same method to estimate the parameters of one factor double CSN copula models and one factor double mixture distribution of NIG and CSN distribution copula model.

5.1. Pricing iTraxx Tranches with Four Models

The market quotes for iTraxx tranches and the model generated tranche prices are listed in Table 3. The NIG(1) is the one factor double NIG copula model with one free parameter and NIG(2) with two except the correlation ρ. NIG(1) sets the parameter β equal zero, which makes the distribution symmetric. For one factor double CSN copula model, we have to estimate two parameters, ρ and σ_1^2. It is interesting that the minimum absolute errors for this model are obtained by maximizing σ_1^2. In the "mixture" model we restrict the parameters of one factor double mixture distribution for NIG and CSN model with $\beta = 0$, $p = 0.5$ and the maximum value of σ_1^2 for reducing the number of parameters. Then estimate the parameters α and the correlation ρ which makes the absolute error from the market data minimum and equality tranche equal to the market quote.

Attachment/ Detachment points	Market quote	Gaussian	NIG(1)	NIG(2)	CSN	mixture
0%-3%	23.53%	23.53%	23.53%	23.53%	23.53%	23.53%
3%-6%	62.75 bp	135.22 bp	62.75 bp	62.75 bp	120.31 bp	61.42 bp
6%-9%	18 bp	28.02 bp	27.40 bp	27.22 bp	31.61 bp	18.80 bp
9%-12%	9.25 bp	6.81 bp	17.02 bp	16.72 bp	10.70 bp	9.69 bp
12%-22%	3.75 bp	0.72 bp	9.17 bp	8.91 bp	2.03 bp	5.94 bp
absolute error		87.95 bp	22.60 bp	21.87 bp	74.34 bp	4.76 bp
correlation ρ		15.78%	15.71%	15.27%	19.39%	20.95 %
α			0.5047	0.7531		0.1045
β			0	-0.3001		0
σ_1^2					2.7519	2.7519

Table 3: Pricing the iTraxx Europe tranches with the LHP assumption based on different copula models at 12-April-2006.

Attachment/ Detachment points	Market quote	Gaussian	NIG(1)	NIG(2)	CSN	mixture
0%-3%	26%	26%	26%	26%	26%	26%
3%-6%	87 bp	165.22 bp	84.33 bp	86.98 bp	146.44 bp	86.27 bp
6%-9%	24 bp	37.97 bp	34.09 bp	34.92 bp	41.83 bp	27.38 bp
9%-12%	11 bp	10.14 bp	19.99 bp	20.35 bp	15.18 bp	11.98 bp
12%-22%	5 bp	1.21 bp	9.89 bp	9.94 bp	3.16 bp	6.42 bp
absolute error		96.85 bp	26.63 bp	25.22 bp	83.28 bp	6.51 bp
correlation ρ		16.46%	16.14%	16.43%	20.42%	25.64 %
α			0.6013	0.6034		0.0256
β			0	0.0310		0
σ_1^2					2.7519	2.7519

Table 4: Pricing the iTraxx Europe tranches with the LHP assumption based on different copula models at 23-June-2006.

The value of parameter p is 0.5 because we had made the fitting result for one factor double mixture distributions copula models with different value of p, which were listed in the Table 10, and they do not receive better results. The results of the mixture model with small p are close to the one factor double CSN copula model, and when the p is close to 1, the fitting results of the mixture model are almost near that of the one factor double NIG copula model.

From Table 3 and 4 we find that the prices of the equity tranches for all models are the same because we calibrate them in order to fit the market quote. The Gaussian copula models only fit the 9%-12% tranches well, but extremely overprice the 3%-6% and 6%-9% tranches, and underprice the most senior tranches. Better than that, the double NIG factor copula models (both NIG(1) and NIG(2)) get accurate fits for the second tranche. However these two models overprice the other tranches besides the 3%-6% tranche. The results for the CSN factor copula models are similar to the Gaussian copula, but they indeed make an improvement in the most senior tranche. Because of these, we consider a mixture distribution for the NIG distribution and the CSN distributions. We hope that it can keep the advantages and properties for the NIG and CSN distributions. According to Table 3 and 4, the NIG(2) with one more free parameter β doesn't bring more improvement to the fitting results, and in order to reducing the parameters, we denote the skewness parameter as zero. We can find the results in the last column of the Table 3 and 4. The overall fitting results are better than the Gaussian, NIG, and CSN factor copula models apparently. However, such a good fit is at the expense of a large amount of calculation time. This is due to the non-stability under convolution in this mixture distribution.

5.2. The Loss Distributions for Four Models

All factor copula models except the Gaussian factor copula model redistribute the risk out of the lower end of the loss distribution to its higher end. That is, these models place lower probability to zero loss and get a fatter upper tail than Gaussian model. (See Figure 11).

We plot the density distribution functions of portfolio losses form the LHP models with four different one factor copula models in Figure 12. With this data the one factor double mixture distribution copula model shows an extremely sharper kurtosis. Figure 13 presents the differences between the modified (NIG, CSN and mixture) and Gaussian densities. The total risk difference within the equity tranche is zero since we calibrate all models to fit the market equity tranche price. So in Figure 13 we only exhibit the range between 0.03~0.09 on the horizontal axis. From this figure we note that the Gaussian model allocates more risk in junior and junior mezzanine tranches than other modified models.

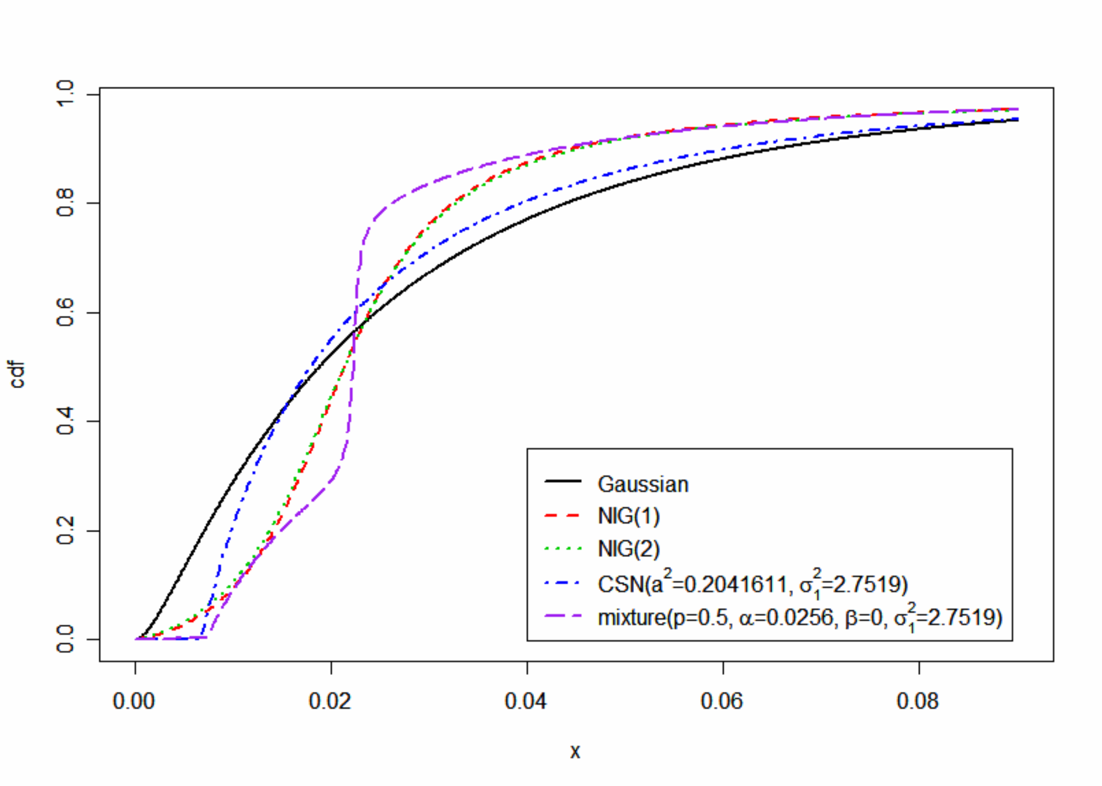

Figure 11: The cumulative loss distributions produced by the four models with the parameters given in Table 4.

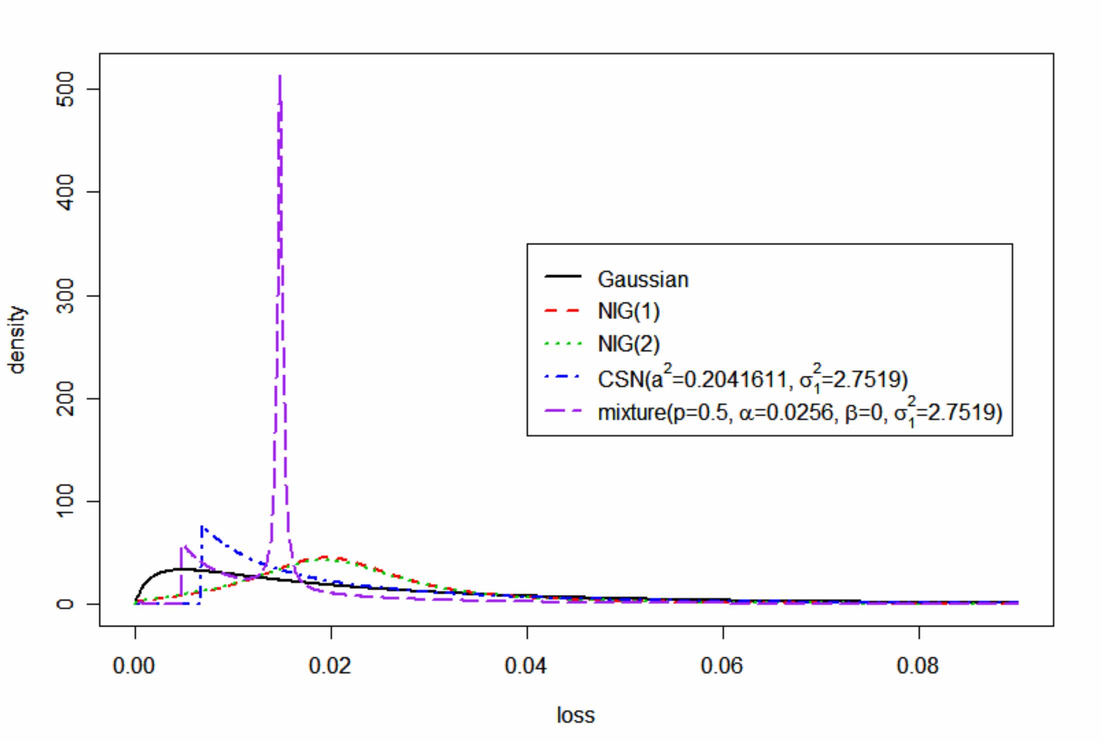

Figure 12: The densities for four loss functions with the parameters given in Table 4.

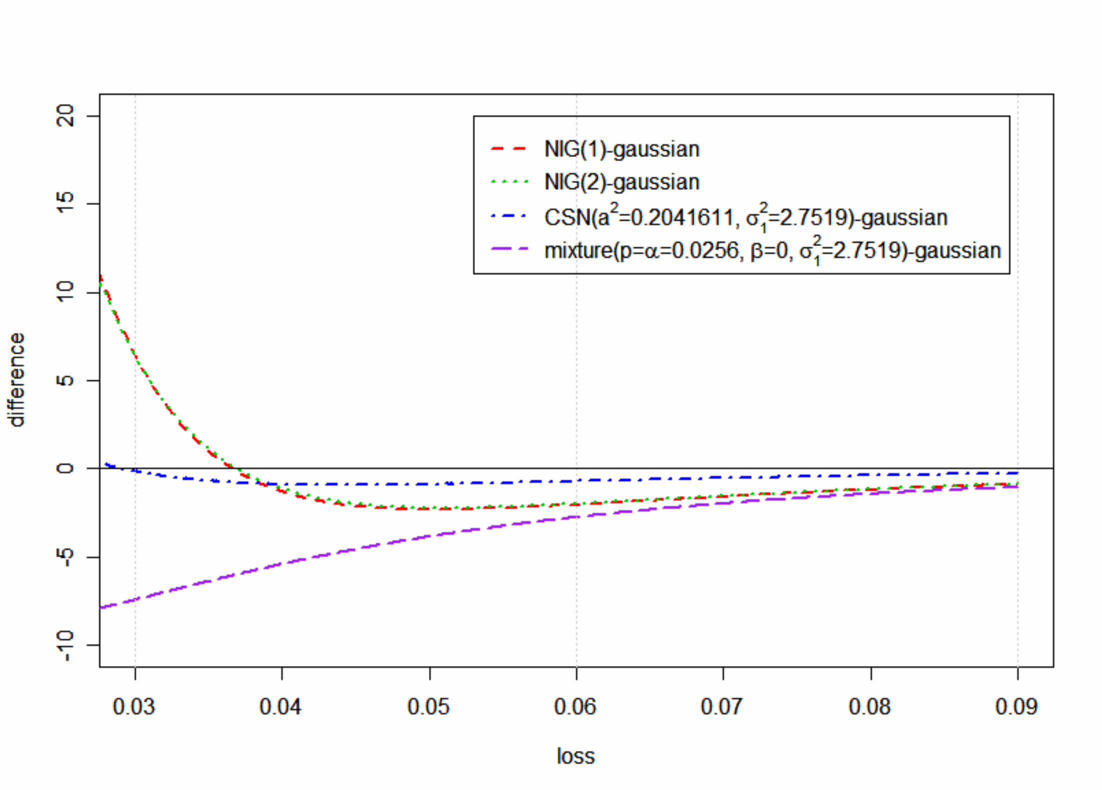

Figure 13: The difference between densities for three loss functions and Gaussian densities.

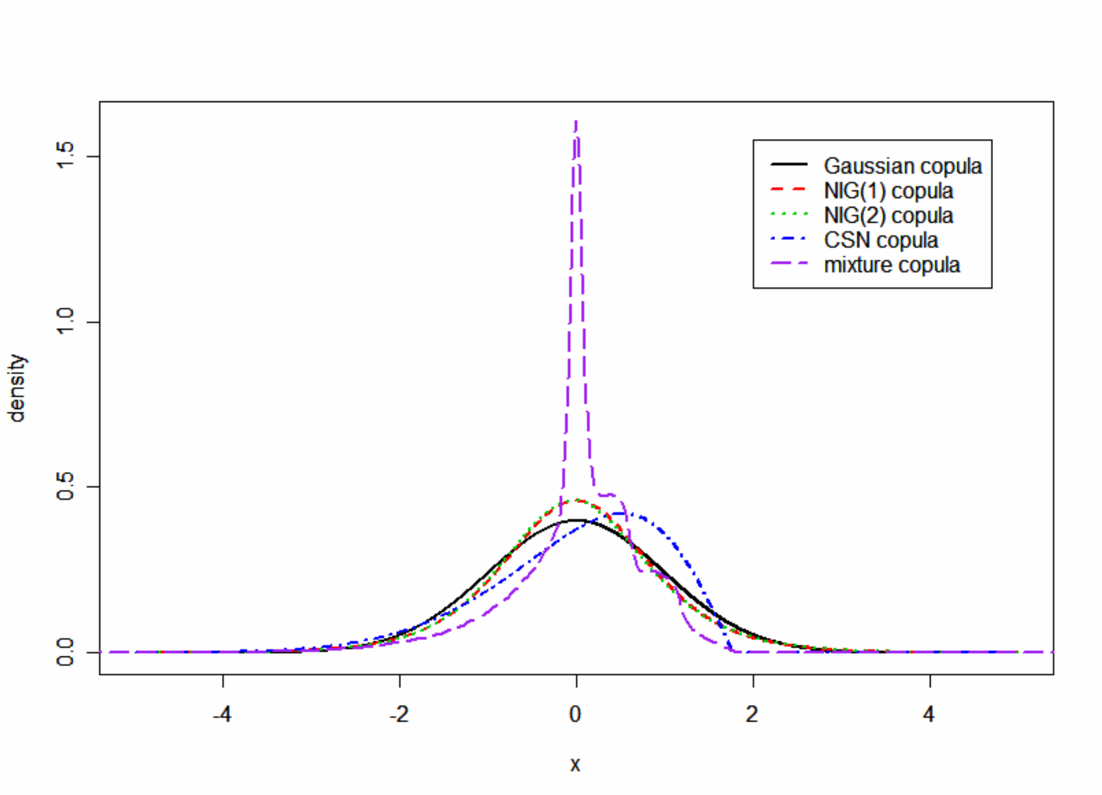

Figure 14: Densities of asset returns.

Figure 14 shows the distributions of the asset returns. The distribution for NIG(1) model is symmetric because it zero skewness parameter, the NIG(2) model is slightly skewed to the right. On the contrary, the CSN model is apparently skewed to the left. The mixture model has comparatively high kurtosis.

5.3. The Comparison between the Compound and Base Correlation

Compound correlation is the correlation found by calibrating the one factor copula model to the market quote of a CDO tranche (for example 3-6%), base correlation is found by calibrating to the market quote of a first loss tranche, i.e. to the sum of all tranches up to an attachment point (for example the 0-6% mezzanine tranche, the sum of 0-3% and 3-6%). The curve of correlations obtained by calibrating to first loss tranches turns out to be much smoother and more stable than that obtained by calibrating to plain tranches.

The most important advantage for base correlation is it can be used to produce consistent implied correlations for non-standardized tranches thus providing a process to investors to check the market quoted prices of tranches with non-standard attachment points. That is, we can use the market standard tranches to calibrate the model for base correlation inputs, and then to interpolate form these base correlations to value a CDO tranche.

As the review in Section 5.2, we calculate the compound and base correlations for four models. The Gaussian, NIG(1), NIG(2), CSN models have compound correlation smile evidently, it reveals that using these copula models may obey the LHP assumption. Differently, the compound correlation curve for the mixture distribution model is almost flat. The results for compound correlation at date 23-June-2006 are in Table 5 and the plot of compound correlation curves in Figure 15.

Attachment/ Detachment points	Gaussian	NIG(1)	NIG(2)	CSN	mixture
0%-3%	16.46%	16.14%	16.43%	20.42%	25.64%
3%-6%	8.18%	17.68%	16.44%	11.47%	25.90%
6%-9%	13.16%	10.74%	10.76%	15.27%	23.67%
9%-12%	16.91%	9.64%	9.80%	18.12%	24.71%
12%-22%	23.32%	10.28%	10.55%	23.16%	23.17%

Table 5: The implied compound correlation for each tranche of iTraxx Europe based on different copula models at 23-June-2006.

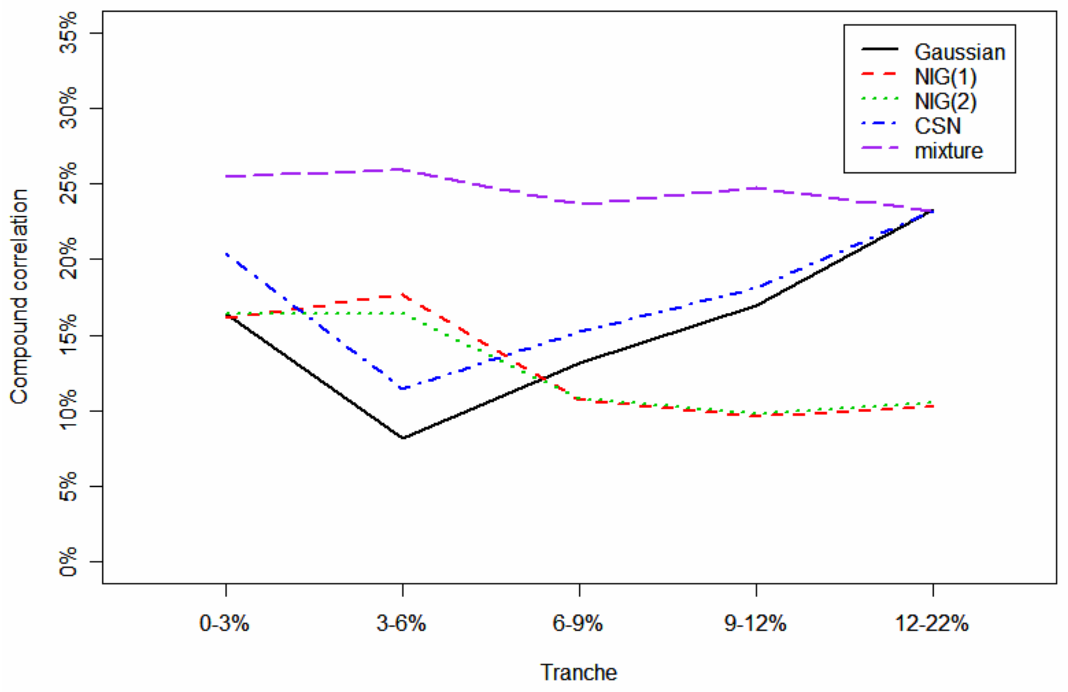

Figure 15: The implied compound correlation curve for iTraxx Europe series 5 based on different copula models at 23-June-2006.

We calculate the base correlation for four models in Table 6. The one factor double Gaussian copula and one factor CSN copula models are both observed as a correlation skew apparently. The NIG models are slightly upward sloping. Differently, the one factor double mixture distribution copula model has almost horizontal base correlation curve, see Figure 16. We put other data at 12-April-2006 in the appendix.

Attachment/ Detachment points	Gaussian	NIG(1)	NIG(2)	CSN	mixture
0%-3%	16.46%	16.14%	16.43%	20.42%	25.64%
0%-6%	25.42%	17.45%	17.66%	25.62%	25.36%
0%-9%	32.91%	19.01%	19.33%	30.15%	25.79%
0%-12%	39.38%	20.85%	22.12%	33.92%	25.94%
0%-22%	56.35%	27.27%	29.10%	43.31%	26.23%

Table 6: The implied base correlation for the iTraxx Europe tranches based on different copula models at 23-June-2006.

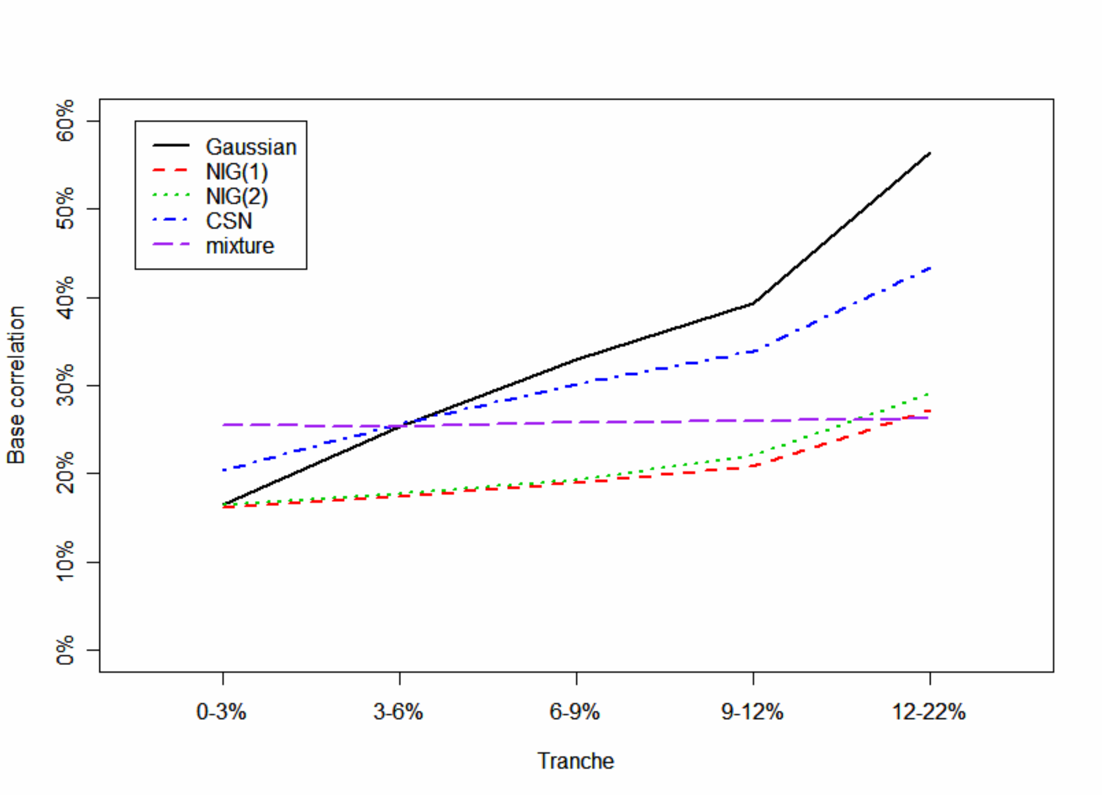

Figure 16: The implied base correlation curve for iTraxx Europe series 5 based on different copula models at 23-June-2006.

5.4. Conclusion

In this paper we present two extension models for the one factor double Gaussian copula model, one factor double closed skew normal (CSN) copula model and one factor double mixture distribution of NIG and CSN distribution copula model with Large Homogeneous Portfolio (LHP) assumptions for pricing a collateralized debt obligation (CDO). Although the CSN distribution has more parameters than Gaussian models to produce the kurtosis and asymmetry, its fitting results are similar to the Gaussian model. The NIG model fit the second tranches exactly, and it is simple and fast to make the computation of the default thresholds. However, it still can not provide very convincing results for mezzanine and senior tranches.

Hence we mix the NIG and CSN distributions and substitute it to the one factor copula model, and furthermore we find that setting the proportion parameter p to 0.5 can produce smaller absolute error. The main advantage of the one factor double mixture copula model is that its pricing results are apparently better than NIG models for each tranche. Although the computation of the mixture model is much slower than the NIG and Gaussian models due to its lack of stability under convolution of the mixture distribution, using right numerical method and variable change still can make it as fast as possible.

All of the models except the mixture model present inconstant compound correlations. This may obey the LHP assumption that instruments in the reference portfolio have the same correlations to the common factor, but the mixture model is different. The one factor double mixture distribution copula model doesn't cause the phenomenon of correlation skew.

It has been clear that the one factor Gaussian copula model was not the right model to price CDOs and many researchers are in desperate need for a convincing alternative. In this paper, we provide the one factor double mixture distribution copula model, which brings a more convincing result than NIG models and also gives the flexibility in the modeling of the dependence structure of a credit portfolio.