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Urban land policy and housing in an endogenously growing monocentric city

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Abstract

We examine the long-run effects of urban land policy on housing investment/pricing and city development. Housing is introduced through a socially constant-returns household production technology with uncompensated positive neighborhood externalities. We prove the existence/uniqueness of and characterize the balanced growth spatial equilibrium. Both a control of the housing price at the urban fringe and a zoning policy that relaxes more-than-proportionately the floor area ratio in favor of locations toward the city center are growth-enhancing. The long-run rate of growth is unambiguously lower in a regime where zoning does not differentiate land-use intensity, compared to the conventional setup.

JEL classification: R11; R14; D90
Keywords: Housing capital and pricing; Monocentric city; Floor-area-ratio policy; Endogenous growth

1. Introduction

Over the past two decades, the study of the dynamics of housing investment and prices has been one of the fundamental issues at the center of the regional science and urban economics platform.[1] This is not only because housing capital account for a lion

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[1] Attempts along these research lines include the now-classic works by Anas (1978) and Fujita (1982), both conducting steady-state analysis under the exogenous growth framework.

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share of nonhuman wealth, but because housing price movements are essential to macroeconomic performance and stability. For example, more than 40% of an average American's nonhuman wealth during the post-WWII period was in forms of residential housing. While the exorbitant housing prices in Japan caused significantly lower standard of living than in the US despite their comparable per capita income figures, the 1997 Asian financial crisis led to approximately a 45% reduction in Hong Kong family wealth within a short period of 2 years. Thus, the chief objectives of this paper are: (i) to examine the dynamic inter-relationships between housing capital and prices and city growth and development; and, (ii) to conduct a positive analysis on the long-run effects of housing price control and urban zoning policies.

Led by Romer (1986), Lucas (1988) and Stokey (1991), endogenous growth theorists have developed analytic frameworks, allowing for the determination of the economy's rate of growth within the system rather than relying on the exogenous technical progress factor. This is important because we have observed that some cities rise and some fall, which cannot be simply driven by exogenous city-specific technologies. However, not until recently has there been a tenuous literature applying the endogenous growth theory to urban economic issues. On the one hand, Ioannides (1994), Palivos and Wang (1996) and Black and Henderson (1999) study spatial agglomeration within the endogenous growth framework in which the stock of housing capital is taken as exogenously given.[2] On the other hand, Anas et al. (1995) focuses exclusively on the stability property of dynamic spatial equilibrium, illustrating the possibility of dynamic indeterminacy (in the sense that there is a continuum of transition paths converging to the unique balanced growth path). Therefore, the issues concerning urban land policy and housing investment and pricing within a general-equilibrium endogenous growth framework still remain largely unexplored.

The importance of examining these issues in a dynamic general equilibrium framework is apparent: partial equilibrium models fail to account for the feedback effects from the endogenously determined housing price and city size, whereas static models ignore the dynamic effects from intertemporal housing price adjustment and housing stock accumulation. The considerations of endogenous growth also provide at least three valuable insights. First, it permits sustained growth in per capita city output, which is consistent with empirical evidence.[3] Second, policies that influence the level of output in exogenous growth models may now have permanent growth effects. This allows us to study the potential long-run effect of urban policies, such as housing price controls and zoning restrictions, in addition to other preference and technology parameters. Finally, those

[2] More specifically, Ioannides (1994) uses a product variety model within a two-period overlapping generations framework to study the optimal growth process in a system of cities. Under an infinite-horizon endogenous growth setup, Palivos and Wang (1996) develop a one-sector spatial agglomeration model with uncompensated positive externalities. In an independent work, Black and Henderson (1999) consider a similar endogenous growth framework but allow for two different types of cities in which the evolution of cities are governed by both exogenous population growth and endogenous human capital accumulation.

[3] It is well-documented by Romer (1986) that at the aggregate level, per capita output has exhibited perpetual growth. Moreover, it continues to observe urbanization based on the metropolitan area data (cf. Palivos and Wang (1996)). These two facts together imply sustained growth in per capita output for an average city.

affecting the rate of city growth may further feed back to influence the intertemporal price of housing and the accumulation of housing capital. In this paper, we take a step toward exploring this fertile research area by constructing an endogenous growth model with endogenously determined housing prices, housing stock evolution and household time allocation.

Specifically, we follow in the spirit of Becker (1976, Part 4) and regard the productive activity of the representative agent in our economy as 'household production.' There are two production factor inputs: the existing housing stock and the time devoted to production. Households allocate nonproductive time to 'home entertainment' (or leisure, in short) and the 'effective leisure' is measured by the housing capital-augmented time devoted to home entertainment, in a form analogous to the human-capital augmented leisure as in Heckman (1976). We follow the conventional wisdom of urban economics to account explicitly for the 'transportation cost' facing each resident at a particular location in the circular city. In contrast to the iceberg framework developed by Samuelson (1954), however, we allow the transportation cost schedule to depend on both the level of household consumption and the design of zoning policy.[4]

Under our endogenous growth framework, the housing capital stock, household production and consumption all grow unboundedly, reaching a constant growth rate along the balanced growth path. By transforming the system following the techniques developed by Bond et al. (1996), we prove the existence and uniqueness of a non-degenerate balanced growth spatial equilibrium. We then characterize the balanced growth spatial equilibrium by illustrating how changes in demand (household preferences), supply (household production technology) and spatial parameters (city border and transportation technology) may affect housing accumulation and pricing, time allocation as well as the endogenously determined balanced growth rate of the city economy. Our model is rich enough to enable a complete examination of the long-run growth consequences of urban land policies. In particular, we consider two types of urban land policies: housing price controls and zoning restrictions on the floor area ratio. We find that a control of the housing price at the urban fringe reduces non-productive use of housing capital and enhances city growth. A uniformly loose restriction on the floor area ratio encourages non-productive use of housing capital, raises equilibrium housing prices and lowers city growth. Yet, by relaxing more than proportionately the floor area ratio in favor of locations toward the city center, one may increase productive use of housing capital and reduce housing prices, thus fostering city growth. In a regime where zoning does not differentiate land-use intensity, the long-run rate of growth is unambiguously lower than that with a decreasing floor-area-ratio schedule in the distance away from the city center.

The remainder of the paper is organized as follows. At the end of this introductory section, we review briefly the broader literature related to our work and discuss the limitation of our study. The basic environment of the economy is delineated in Section 2. While Section 3 analyzes individual optimization, Section 4 defines and proves the existence and uniqueness of the balanced growth spatial equilibrium. In Section 5, we

[4] Thus, the transportation cost is not measured exclusively by the traditionally defined commuting cost.

characterize the balanced growth spatial equilibrium and draw selectively policy implications based on the comparative static analysis. Finally, Section 6 concludes the paper.

1.1. Review of the broader literature and limitations of this study

Our paper also contributes to a large literature on durable housing in a spatial context. In early studies by Anas (1978), Arnott (1980) and Fujita (1982), housing cannot be demolished and redeveloped. While Brueckner (1981) allows housing to be demolished and redeveloped, the housing stock is assumed constant over time. Turnovsky and Okuyama (1994) develop an optimal growth model with a composite good sector and a housing construction sector where housing is only for household consumption. In Arnott et al. (1983), a partial-equilibrium model of housing quality and maintenance is used to establish a stationary state based on profit maximization of the representative landlord. In an independent work, Arnott et al. (1999) construct a general-equilibrium housing model where households choose both quality and quantity of housing consumption and developers decide the structure density and time path of housing. While all these papers consider steady states in the traditional exogenous growth framework, our paper contributes to the field by allowing for an endogenous determination of a non-degenerate rate of balanced growth of consumption, output, and housing capital in a monocentric city.

Due to the complexity of incorporating both time and spatial dimensions in a general-equilibrium framework, it is necessary to simplify the structure in order to obtain analytic solution. The resultant limitations are as follows. First, the spatial structure is purely monocentric, under which the amount of floor area per unit of land, the number of lots per unit of land and the amount of floor area per housing unit are all tied to distance from the city center. Thus, the more general urban land use patterns in the durable housing literature, such as Arnott (1980), Brueckner (1981) and Fujita (1982), cannot be examined under our stylized framework. Second, to be consistent with the balanced growth setup, we cannot model demolition and redevelopment as in the stationary environment of Arnott et al. (1983). Instead, we can only focus on the hypothetical balanced growth path of housing quality, which dismisses the short-run periodic changes in residential dwellings. Third, we follow the spirit of Romer (1986) to restrict our attention to a general productive household capital that greatly simplifies the analysis. This simplification leaves the potential interactions between non-housing and housing capitals unexplored.

Despite these limitations, we would like to point out that some of which may be innocuous for the purpose of this study. First, the stylized spatial pattern in our model can already result different effects of price control and floor area ratio policies at different locations. Such a conclusion would remain in a city with nonmonotone bid rents. Second, the issue of demolition and redevelopment of dwellings is less important in the long-run analysis, as long as the net depreciation rate is asymptotically constant and the reconstruction can be captured by housing quality improvements along a balanced growth path. Finally, even by formally constructing a model with differentiated non-housing and housing capitals, results in Bond et al. (1996) suggest that the characteristics of the balanced growth equilibrium remain qualitatively unchanged when the non-housing capital production is non-housing capital intensive and the housing sector is housing capital intensive.

2. The basic environment

Time is continuous. A fixed number (N) of infinitely lived households reside in a monocentric, circular city in which production and market sites are located in the central business district (CBD) and the border (b) is determined at zero bid rent for land.[5] The residential zone is thus indexed over the range of $[0, b(t)]$ at any point in time t. We decompose the housing stock h into two components: the quality component q and the quantity component η. Thus, $h = q\eta$. As in Palivos and Wang (1996), bounded city limits the growth of lot size and housing quantity. However, the present paper allows the quality of housing to grow perpetually. Thus, under this decomposition, the dwelling unit fixed to the lot size is called 'quantity', whereas any interior/exterior improvements are regarded as 'quality.'

In our simple dynamic spatial environment, there is only one single 'household good' produced with labor and housing capital inputs. The amount of the household good, subtracting the transportation cost, can be used for household consumption (c) and housing capital investment ($\dot{H} + \delta H$). To motivate, consider an Eskimo household's consumption–production behavior with 'fresh water' as the household good. This household could drink water (for consumption) or freeze it as ice. While the ice could be used to build an igloo (housing investment), it may also be used to produce fresh water in the future by defrosting (production input). The transportation cost can be measured by the loss of ice from melting in the consumption–production process. Thus, fresh water could be used as consumption good, housing capital and production input. Of course, the allocation of the household good may in general capture resource trade-offs between current and future consumption with regard to many household commodities.

The household good is produced using a well-behaved technology, $Af(s(z,t)h(z,t), \bar{h}(z,t))$, where $A > 0$ is the production technology parameter, s is the fraction of time devoted to production, $\bar{h}$ is the *average* level of the neighborhood's housing stock (which is, by symmetry, equal to h in equilibrium at each location and each point in time), and the household productivity of this labor effort is augmented by the current housing stock. Thus, housing is used for both household consumption and production purposes. The incorporation of $\bar{h}$ into the production function is designed to measure the neighborhood effect, that is, the average level of the neighborhood's housing stock contributes positively to individual household's home production.[6] This captures the Marshallian externality emphasized by Jacobs (1969). The role of neighborhood externality effects within the

[5] For an endogenous determination of the city size in a system of cities, the reader is referred to Abdel-Rahman (1990). See also Wang (1990) for an endogenous determination of the location of the CBD in a linear city.

[6] Alternatively, one may measure the neighborhood externality by household income in the area, which implies a production function of the following form: $Af(s(z,t)h(z,t), \bar{s}(z,t)\bar{h}(z,t))$, where $\bar{s}(z,t)\bar{h}(z,t)$ captures the spillover effects from neighbors' income. The main findings of our paper are invariant to this modification.

hedonic-pricing framework has been documented empirically by Bond and Coulson (1989), who conclude that neighborhood income (rather than racial composition) generates a significantly positive externality effect. Since household income in equilibrium is a monotone increasing function of housing capital, our consideration of the neighborhood effect is consistent with this empirical finding.

Assumption 1. (Household production technology) The household production function f is strictly increasing and strictly concave in private inputs, s and h, and linearly homogeneous in reproducible factors, h and $\overline{h}$, taking the Cobb–Douglas form: $f(sh,\overline{h}) = A(sh)^{1-a}\overline{h}^{a}$, where $A>0$ and $a\in[0,1)$.

This setup contrasts with the traditional optimal growth model in which the steady-state rate of growth of the economy is pinned down by the exogenous technical progress rate. It allows for endogenous growth such that the effects of deep structure and policy parameters on the economy-wide rate of growth can be examined. The production technology extends that of Romer (1986) and Lucas (1988). Rather than having knowledge or human capital in the entire society to generate positive spillovers as in their works, it is the housing capital in the neighborhood that creates such an externality within our framework. When $a=0$, the neighborhood effect is absent and the production technology degenerates to that in the so-called AK-model without Marshallian externalities as in Rebelo (1991).[7]

Households allocate nonproductive time $(1-s)$ to home entertainment and effective leisure (x) is measured by housing capital-augmented leisure time in a multiplicative form analogous to human capital-adjusted leisure as in Heckman (1976): $x(z,t)=(1-s(z,t))h(z,t)$. We then consider,

Assumption 2. (Household preference) Household's preference is time-separable with a constant rate of time preference $\rho>0$ and an instantaneous (point-in-time) utility function $U(c(z,t),x(z,t))$ that is strictly increasing and strictly concave in c and x, exhibiting constant elasticity of intertemporal substitution: $U=\{c^{\alpha}[(1-s)h]^{1-\alpha}\}^{1-\sigma^{-1}}/(1-\sigma^{-1})$, where $\sigma>0$ measures the elasticity of intertemporal substitution and $\alpha\in(0,1)$.

This functional form is standard in the optimal growth literature with two consumables accepting endogenous growth. Moreover, we assume bounded lifetime utility to ensure sensible household intertemporal optimization:

Condition U. (Bounded lifetime utility) $\rho>(1-\sigma^{-1})\max\gamma_c$, $\gamma_c\equiv\dot{c}/c$.

This is in analogy to the Brock–Gale condition (Brock and Gale, 1969), requiring that the time preference rate is sufficiently high to dominate rate of increase of utility from consumption growth. Notice that although the maximum consumption growth rate is endogenous, it is well-defined and depends only on economic primitives in equilibrium

[7] Notably, to allow for perpetual growth, it is necessary to have a non-decreasing-returns-to-scale household production. On the other hand, increasing-returns-to-scale production is excluded since it is inconsistent with balanced growth given time-additive preferences.

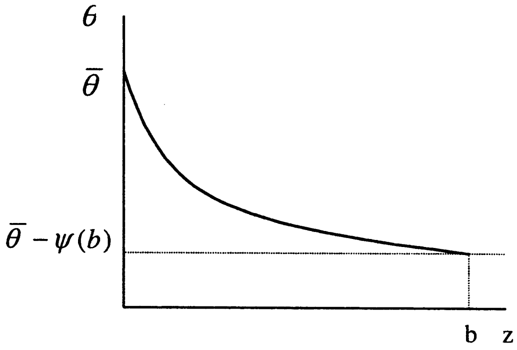

Fig. 1. The floor area ratio schedule.

as it can be seen from the dynamic system to be presented below (in particular, Eqs. (16)–(18)).[8]

Moreover, this paper permits the building height to vary, depending on the zoning policy. Denote the distance away from the CBD as z and the floor area ratio (FAR) as θ.[9] We postulate:

Assumption 3. (Floor area ratio) The floor area ratio θ depends negatively on z with the value at the CBD normalized to $\overline{\theta}$: $\theta(z) = \overline{\theta} - \psi(z)$, where $\psi(0) = 0, \psi' > 0, \psi'' < 0$, and $\overline{\theta} > \psi(b)$.

That is, the floor area ratio at the CBD reaches the maximum value; it decreases in the distance away from the CBD monotonically. Fig. 1 plots this floor area ratio schedule. For comparative-static analysis, it may be interesting to examine the effects of a zoning policy that maintains the FAR at urban fringe, $\theta(b)$, but changes the slope of the FAR schedule. In order to accomplish this task, one may rewrite the FAR schedule as: $\theta(z) = [\overline{\theta} - \psi(b)] - \psi_0 \Psi(z)$, where $\psi_0 > 0,\ \Psi' > 0,\ \Psi'' < 0,\ \Psi(0) = -\psi(b)/\psi_0 < 0$, and $\Psi(b) = 0$. Thus, a change in ψ_0 implies a steeper FAR schedule, whereas a change in $\overline{\theta}$ results in a parallel shift.

To account for the limited space toward the CBD, the zoning policy allows for buildings with greater height to be constructed near the city center. Under the consideration of symmetry, each household residing at a location z is provided with a land lot of size η/θ. At each location z, land density equals $2\pi z$, and the endogenously determined population density is $m(z) = 2\pi z/(\eta/\theta)$. As can be seen later, the assumption of negative dependence of θ on z ensures a positive relationship between the FAR and the degree of congestion in the city, corroborating with empirical observations. Moreover, this setup implies lower buildings with larger lot size away from the CBD, capturing the pattern of

[8] See Bond et al. (1996) for a detailed discussion. Alternatively, one may follow Lucas (1988) to impose a sufficient condition $\sigma \leq 1$ under which Condition U is automatically satisfied. This sufficient condition requires that the intertemporal elasticity of substitution is not higher than the benchmark of log-linear preferences.

[9] In the context of a city, the FAR is a more relevant measure than the coverage ratio because the latter fails to account for the height of each building.

traditional monocentric cities. For simplicity, we consider that land is provided by the city developer and that there is no infrastructure cost in city development. Since the city is bounded, the zoning restrictions on the FAR imply that the quantity of housing is limited and hence there is a congestion externality through the aggregation of the population at each location.

It is not the purpose of the paper to study urban population distribution. However, by allowing for variable floor area ratios it creates generic indeterminacy in urban configurations. Specifically, for any given (unbounded) path of housing quality (q), the conventional population density in a two-dimensional monocentric city cannot be uniquely pinned down in this model because housing quantity increases with the number of floors of each building at any arbitrary location. This problem is generic unless there is a preference over the height of the building together with a housing construction cost schedule that depends on the number of floors of the building. Such a task would complicate the analysis dramatically, which is beyond the purposes of the present paper. Among all possibilities, we, for the sake of convenience, restrict our attention to that with a uniform population distribution:[10]

Assumption 4. (Population distribution) The population distribution is uniform, i.e., $m(z) = m$ for all z.

Under Assumption 4, population identity requires: $N(t) = \int_0^{b(t)} m\mathrm{d}z = mb(t)$, which implies

$$m = \frac{N(t)}{b(t)} = \frac{2\pi z}{\eta(z,t)/\theta} \tag{1}$$

That is, the border will be enlarged as the population of the city increases, while the size of buildings increases with the distance away from the CBD given a constant FAR.[11]

Further, we generalize the Samuelson (1954) iceberg framework to postulate a transport cost schedule to be compatible with endogenous growth and variable floor area ratios:

Assumption 5. (Transportation cost) The transportation cost per unit of household consumption per unit of distance, $\tau > 0$, is constant over time and depends positively on the FAR, θ.

Thus, a household located at z (simply referred to as household z) who consumes $c(z,t)$ will pay transportation expenses of $\tau(\theta)zc(z,t)$. Intuitively, a higher FAR can result in more severe traffic congestion, thereby increasing the transportation cost. This transportation cost setup is more general than the traditionally used commuting cost.[12]

[10] For each population distribution, one may obtain a spatial equilibrium. Should the population density be decreasing in the distance away from the CBD (as observed in reality), it can be easily verified that the long-run effects of zoning policies will remain qualitatively unchanged but quantitatively stronger (see a discussion at the end of Section 5).

[11] At the CBD ($z = 0$), a uniform population distribution requires the quantity of housing approach zero asymptotically (i.e., $\eta(0,t) \to 0$ as $t \to \infty$). This is innocuous because in a continuum setup, each location is of measure zero.

[12] An alternative is to consider: $\tau(\theta)zf(z,t)$. That is, the transportation cost is higher as the amount of production increases. This alternative setup, however, would not change the main conclusions of the paper.

It is noted, however, that congestion may be nonpecuniary, damaging productivity or lowing utility. Such type of congestion on the production side can be easily included—in this case, the positive neighborhood externality specified previously can be regarded as one net of the negative externality from congestion. Congestion on the preference side, unfortunately, cannot be considered under our framework, as it makes the balanced growth solution unsolvable.

Let δ denote the sum of the depreciation rate of the quality adjusted housing stock, the population growth rate and the rate of change of the relative price of housing. In our model, household consumption and housing investment are not perfect substitutes. Thus, there is in general a well-defined housing price schedule, $\{p(z,t)\}_{z\in[0,b(t)]}$, which is regarded as parametrically given by individual households residing at each location. In the absence of non-housing assets, all household savings are channeled to housing capital purchases.[13] Then, our representative household z faces the following optimization problem (PH):

$$\max_{\{c,\,s\}} \int_0^\infty U[c(z,t),(1-s(z,t))h(z,t)]e^{-\rho t}\mathrm{d}t$$

subject to

$$\dot{h}(z,t) = \frac{1}{p(z,t)}[f(s(z,t)h(z,t),\overline{h}(z,t)) - (1+\tau(\theta)z)c(z,t)] - \delta h(z,t) \tag{2}$$

and the initial condition $[h(0) = h_0 > 0]$, the non-negativity constraints on c and gross housing investment $[\dot{h}(z,t) + \delta h(z,t)]$, and the interiority condition for time allocation $[s\in(0,1)]$.

3. Optimization

Under Assumptions 1–3 and 5 and Condition U, lifetime utility is bounded and the optimal control problem presented above is well-defined. Denoting the costate variables associated with (2) as λ, we can write the current-value Hamiltonian Λ as (time and location indices are dropped whenever they do not cause any confusion):

$$\Lambda = U[c,(1-s)h] + \frac{\lambda}{p}[f(s,h) - (1+\tau(\theta)z)c] - \lambda\delta h.$$

Straightforward application of Pontryagin's Maximum Principle yields,

[13] As discussed in the introduction, our model thus ignores potential portfolio choice between housing and non-housing capital.

Lemma 1. (Necessary and sufficient conditions for PH) *Under Assumptions 1–3 and 5, the necessary and sufficient (transversality) conditions for household optimization (PH) are:*

$$U_c = \frac{\lambda}{p}(1+\tau(\theta)z) \tag{3}$$

$$U_x = \frac{\lambda}{p}[(1-a)As^{-a}] \tag{4}$$

$$\frac{\dot{\lambda}}{\lambda} = (\rho+\delta) - \frac{1}{p}(1-a)As^{-a} \tag{5}$$

$$\lim_{t\to\infty} \lambda(t)h(t)\mathrm{e}^{-\rho t} = 0. \tag{6}$$

Eqs. (3) and (4) describe intertemporal efficiency for household consumption and leisure, respectively. Eq. (5) governs the efficient accumulation of the housing stock, whereas (6) is transversality condition for housing capital, ensuring sufficiency for the well-behaved household optimization problem.. Now, we can apply the utility functional forms and combine (3) and (4) into:

$$\frac{1-\alpha}{\alpha}\frac{c}{(1-s)h} = \frac{(1-a)As^{-a}}{1+\tau(\theta)z} \tag{7}$$

which equates the marginal rate of substitution between household consumption and effective leisure with the marginal product of labor (discounted by the transportation factor, $1+\tau z$). Next, we can rewrite (2) as:

$$\frac{\dot{h}}{h} = \frac{1}{p}[As^{1-a} - (1+\tau(\theta)z)\frac{c}{h}] - \delta \tag{8}$$

Totally differentiating (3) and (4) and utilizing (5), we get:

$$[\alpha(1-\sigma^{-1})-]\frac{\dot{c}}{c} + (1-\sigma^{-1})(1-\alpha)\left[\frac{\dot{h}}{h} - \frac{s}{1-s}\frac{\dot{s}}{s}\right]$$
$$= (\rho+\delta) - \frac{1}{p}(1-a)As^{-a} - \frac{\dot{p}}{p} \tag{9}$$

$$\frac{\dot{c}}{c} = \frac{\dot{h}}{h} - \left(a + \frac{s}{1-s}\right)\frac{\dot{s}}{s} \tag{10}$$

which are the modified Keynes–Ramsey equations within our endogenous growth framework. Differently from conventional optimal growth models, the dynamic paths of the two control variables (c and s) become entangled with the price and stock of the state variable (h) even after substituting out the costate variable (λ). Eqs. (8)–(10) are the fundamental equations governing the dynamical system of (c, s, h), *given* the dynamics of the housing price, p. It is clear that, in general equilibrium, we need to determine the equilibrium evolution of the housing price in order to close the system.

4. Dynamic spatial equilibrium and balanced growth spatial equilibrium

We are now ready to define the *dynamic spatial equilibrium* and to establish the existence and uniqueness of the *balanced growth spatial equilibrium*. It is important to introduce a *locational equilibrium condition* that ensures, in equilibrium, no household in any location would have an incentive to change its residence at any point in time:[14]

$$U[c(z,t),(1-s(z,t))h(z,t)]$$
$$= U[c(b,t),(1-s(b,t))h(b,t)] \quad \forall z \in [0,b(t)]. \tag{11}$$

Definition 1. (Dynamic spatial equilibrium) A dynamic spatial equilibrium (DSE) is a tuple of quantities $\{c(z,t), s(z,t), h(z,t), q(z,t), \eta(z,t)\}_{z\in[0,b(t)];t\geq 0}$ together with a sequence of positive prices $\{p(z,t)\}_{z\in[0,b(t)];t\geq 0}$ such that for any sequence of positive city size measures $\{b(t)\}_{t\geq 0}$,

(i) each representative household $z\in[0,b(t)]$ $(t\geq 0)$ chooses c, s and h to solve the optimization problem (PH) subject to the evolution Eq. (2);
(ii) both population identity (1) and housing capital identity $h = q\eta$ hold; and,
(iii) the locational equilibrium condition (11) is met.

Total differentiation of (11) with respect to time leads to:

$$(1-\sigma^{-1})\left\{\alpha\frac{c(\dot{z})}{c(z)} + (1-\alpha)\left[\frac{h(\dot{z})}{h(z)} - \frac{s(z)}{1-s(z)}\frac{s(\dot{z})}{s(z)}\right]\right\}$$
$$= (1-\sigma^{-1})\left\{\alpha\frac{c(\dot{b})}{c(b)} + (1-\alpha)\left[\frac{h(\dot{b})}{h(b)} - \frac{s(b)}{1-s(b)}\frac{s(\dot{b})}{s(b)}\right]\right\} \tag{12}$$

Denote the rate of change of the housing stock as γ (i.e., $\dot{h}/h = \gamma$). Normalizing the housing price on the urban fringe to $p(b) = \overline{p} > 0$ and utilizing Assumption 4, we can derive:

$$p(z,\ t) = \frac{(1+\tau b)c(b,t) - (1+\tau z)c(z,t)}{(\gamma+\delta)h(z,t)}$$
$$= \frac{N[(1+\tau b)c(b,t) - (1+\tau z)c(z,t)]}{(\gamma+\delta)2\pi z b\theta} \tag{13}$$

We next introduce the concept of balanced growth following Bond et al. (1996) and Palivos et al. (1997).

[14] We do not consider adjustment costs in dwelling investment or relocation costs in residential choice. Inclusion of such possibilities would complicate the analysis without affecting the evaluation of urban land policy qualitatively.

Definition 2. (Balanced growth spatial equilibrium) A balanced growth spatial equilibrium (BGSE) path is a DSE path along which,

(i) perpetually growing quantities, c, h and q, grow at some constant rates;
(ii) the share s converges to a constant within the interior of the unit interval; and,
(iii) both housing quantity η and price p converge to a positive constant.

A non-degenerate BGSE is a BGSE such that the rates of growth of c, h and q are all positive.

Thus, along a BGSE path, $\dot{s} = 0$ and, from (7), $\dot{c}/c = \dot{h}/h = \gamma$, implying common growth between household consumption and housing. Moreover, non-degenerate balanced growth requires $\dot{p} = 0$ (cf. Bond et al., 1996), whereas in the context of urban economics, steady-state migration is consistent with bounded city and hence $\dot{\eta} = 0$ (cf., Palivos and Wang, 1996).

Manipulating (8) along a BGSE path yields:

$$\gamma(z) = \frac{1}{p(z)}[As(z)^{1-a} - (1 + \tau(\theta)z)v(z)] - \delta \tag{14}$$

where $v(z) \equiv c(z)/h(z)$. Thus, from (7) we have $v(z) = \alpha(1-a)As^{-a}(1-s)/[(1-\alpha)(1+\tau z)]$, which can be solved recursively once s is obtained. Next, apply (9) to get:

$$\gamma(z) = \sigma\left[\frac{1}{p(z)}(1-a)As^{-a}(z) - (\rho + \delta)\right] \tag{15}$$

To focus on the case of nondegenerate (positive) growth, we impose:

Condition G. (Nondegenerate growth) $(1-a)As^{-a}/p > \rho + \delta$.

This type of condition is usually referred to as the Jones–Manuelli condition (Jones and Manuelli, 1990) in the endogenous growth literature. It requires the productive use of time and the price of housing not too high (i.e., $ps^a < (1-a)A/(\rho+\delta)$). Thus, a severe housing price bubble or a macroeconomic stagnation (such that s is sufficiently low) may lead to a degenerate equilibrium path. Notably, if consumption and housing investment are perfect substitute ($p = 1$) and the neighborhood externality effect is absent ($a = 0$), Condition G reduces to the familiar inequality in the AK model ($A > \rho + \delta$), independent of any endogenous variables.

From (12), it is straightforward that the rate of growth must be common for every location:

$$\gamma(z) = \gamma(b) = \gamma \tag{16}$$

Substituting (7) into (14), we have

$$\gamma(z) = \frac{1}{p(z)}\left[As(z)^{1-a} - \frac{\alpha}{1-\alpha}(1-a)As(z)^{-a}(1 - s(z))\right] - \delta \tag{17}$$

In locational equilibrium, duality implies individual households at any location must have identical minimum expenditure, which can be used with (1), (7), (14) and (16) to obtain:

Lemma 2. *(Housing price schedule) Under Assumptions 1–5, the balanced growth housing price schedule satisfying locational equilibrium is given by:*

$$p(z) = \bar{p} + \frac{\frac{\alpha}{1-\alpha}(1-a)A}{\gamma+\delta}[B(b,\ z)-1]s^{-a}(z)(1-s(z)) \tag{18}$$

where $B(b,\ z) \equiv h_0(b)/h_0(z) = (b/z)[\theta(b)/\theta(z)][q_0(b)/q_0(z)], B(z;b) > B(b;b) = 1.$

Proof. By duality, we can obtain the minimum expenditure function from:

$$\min\ E = (1+\tau(z))c + p(z)[\gamma(z)+\delta]h \quad \text{s.t.} U(c,(1-s)x) = U_0,$$

which implies $E(z) = E(b)\ \forall z$. Using (1), we get:

$$\begin{aligned}&[1+\tau(\theta(z))z]v(z) + p(z)[\gamma(z)+\delta]\\ &\quad = (1+\tau(\theta(b))b)v(b)\frac{h(b)}{h(z)} + p(b)[\gamma(b)+\delta]\frac{h(b)}{h(z)}\end{aligned}$$

or, the following locationally no-arbitrage condition for all $z \in [0,b]$:

$$\begin{aligned}&p(z)(\gamma(z)+\delta)h(z) - \bar{p}(\gamma(b)+\delta)h(b)\\ &\quad = \frac{\alpha}{1-\alpha}(1-a)A[s^{-a}(b)(1-s(b))h(b) - s^{-a}(z)(1-s(z)h(z)]\end{aligned}$$

This together with (1), (7), and (14) imply

$$p(z) = \bar{p} + \frac{\frac{\alpha}{1-\alpha}(1-a)A[Bs^{-a}(b)(1-s(b)) - s(z)^{-a}(1-s(z))]}{\gamma(z)+\delta}$$

By (7), (16), and the locationally no-arbitrage condition, we have

$$s(b) = s(z)$$

Combining the last two equations and manipulating yield (18), which completes the proof. □

To simplify the analysis, we restrict our attention to the case where $\mathrm{d}\eta(z)/\mathrm{d}z = d[\theta(z)z]/\mathrm{d}z > 0$ for all z. That is, the quantity of housing increases in the distance away from the CBD. Therefore, the BGSE can be characterized by a block-recursive 3×3 system, (15), (17) and (18), which jointly determines the balanced growth values of γ^*,

$s(z)*$ and $p(z)*$. Then, from (7) we solve $v(z)*$ and, finally, from (1) and (PF) one obtains $\eta(z)* = \theta(z)z/\Delta$ and $k(z)* \equiv h(z)/y(z) = (1/A)s(z)^{-(1-a)}$, respectively, where $\Delta = N/(2\pi b)$ measures the population density of the spatial economy, depending crucially on the magnitude of the urban fringe b given the fixed population N.

Utilizing Gantmacher (1960), Lemmas 1 and 2, Proposition 1 in Bond et al. (1996) and the arguments above, we can conclude,

Proposition 1. (Existence and uniqueness of the BGSE) *Under Assumptions 1–5 and Conditions U and G, there is a unique BGSE path along which both c, h and q grow at a common rate* $\gamma > 0$ *and* $\dot{s} = \dot{p} = \dot{\eta} = 0$.

5. Equilibrium characterization and policy implications

We are now ready to perform the comparative-static analysis with respect to autonomous shifts in: (i) preference, technology and spatial parameters (including the household productivity scaling factor, A, the population growth-adjusted depreciation rate, δ, the relative consumption share, $\alpha/(1-\alpha)$, the time preference rate, ρ, the elasticity of intertemporal substitution, σ, the autonomous component of the unit transportation cost, τ_0, and the city size, b, or the inverse of the population density, $1/\Delta$); and, (ii) urban land policies (including the urban fringe housing price, $\overline{p}$, the gradient of the FAR schedule, ψ_0, and the maximum FAR at the CBD, $\overline{\theta}$).[15] It is easily seen that both the unit transportation cost and zoning policies only affect the ratio of housing stock at the border to that at an arbitrary location z (i.e., B). Thus, in the comparative static analysis below, we only summarize their effects via B.

Define $S(s(z); A, \alpha/(1-\alpha)) \equiv [\alpha/(1-\alpha)](1-a)As^{-a}(z)(1-s(z))$, which measures the ratio of consumption and transportation spending to the housing stock. Total differentiation of this 3×3 system, (15), (16) and (17), leads to:

$$J\begin{bmatrix} d\gamma(z) \\ ds(z) \\ dp(z) \end{bmatrix} = \begin{bmatrix} dA \\ d\delta \\ d\left(\frac{\alpha}{1-\alpha}\right) \\ \rho \\ d\sigma \\ db \\ d\overline{p} \\ dB \end{bmatrix}$$

[15] Regarding the urban fridge housing price as a policy variable is innocuous, as one may consider both an endogenous and an exogenous components without altering the comparative static results.

where the pre-multiplied matrix J and K are defined as:

$$J \equiv \begin{bmatrix} 1 & \frac{\sigma a(1-a)A}{p} s^{-a-1} & \frac{\sigma}{p^2}(1-a)As^{-a} \\ 1 & -\frac{1}{p}[(1-a)As^{-a} - \frac{\partial S}{\partial s}] & \frac{1}{p^2}(As^{1-a} - S) \\ \frac{B-1}{(\gamma+\delta)^2} S & -\frac{B-1}{\gamma+\delta}\frac{\partial S}{\partial s} & 1 \end{bmatrix}$$

and

$$K = \begin{bmatrix} \frac{\sigma(1-a)}{p} s^{-a} & -\sigma & 0 & -\sigma & \frac{\gamma}{\sigma} & 0 & 0 & 0 \\ -\frac{1}{p}\frac{\partial S}{\partial A} & -1 & -\frac{1}{p}\frac{\partial S}{\partial(\frac{\alpha}{1-\alpha})} & 0 & 0 & 0 & 0 & 0 \\ \frac{B-1}{\gamma+\delta}\frac{\partial S}{\partial A} & -\frac{B-1}{(\gamma+\delta)^2} S & \frac{B-1}{\gamma+\delta}\frac{\partial S}{\partial(\frac{\alpha}{1-\alpha})} & 0 & 0 & \frac{S}{\gamma+\delta}\frac{\partial B}{\partial b} & 1 & \frac{S}{\gamma+\delta} \end{bmatrix},$$

and, using Eq. (2) along the balanced growth path, the effects of τ_0 and zoning policies $(\psi_0, \overline{\theta})$ on B are given by:

$$\partial B/\partial \tau_0 = (b/z)[\theta(b)/\theta(z)]\{d[q_0(b)/q_0(z)]/\mathrm{d}\tau_0\} > 0,$$

$$\partial B/\partial \psi_0 = (b/z)[q_0(b)/q_0(z)]\theta(b)[\Psi(z) - \Psi(b)]/\theta(z)^2 < 0,$$

$$\partial B/\partial \overline{\theta} = (b/z)[q_0(b)/q_0(z)][\psi(b) - \psi(z)]/\theta(z)^2 \geq 0.$$

Due to the complexity of the model, there is no straightforward condition to determine the sign of the determinant of J (denoted D). However, one may restrict the parameter space in such a way that any autonomous changes in the housing price on the border raise the balanced growth value of housing prices at other locations:[16]

Condition P. *(Housing price normality)* $\mathrm{d}p(z)*/\mathrm{d}\overline{p} > 0$.

It can be easily shown that under Condition P, $\mathrm{sign}\{\mathrm{d}p(z)*/\mathrm{d}\overline{p}\} = -\mathrm{sign}\{D\}$, implying that under this plausible restriction, $D < 0$.

By examining the pre-multiplied matrix on the LHS, we can see that the comparative-static results of σ must be opposite to those of ρ. Next, denote $\varepsilon_\eta \equiv (z/\eta)(\mathrm{d}\eta/\mathrm{d}z) > 0$ and $\varepsilon_q \equiv -(z/q)(\mathrm{d}q/\mathrm{d}z) > 0$. Using the standard 'land abundance' argument, one would

[16] This restriction is realistic especially under the assumption of uniform population distribution within the monocentric city. We shall see below that under this restriction, we can obtain very sensible comparative-static properties.

Table 1
Comparative-static results

Effect on	A	δ	$\alpha/(1-\alpha)$	ρ	σ	b	τ_0	$\bar{p}$	ψ_0	$\bar{\theta}$
$\gamma(z)$	?	$-$*	$-^{\#}$	$-$	+	+	$-$	$-$	+	$-$
$s(z)$	?	0*	?	$-^{\Delta}$	$+^{\Delta}$	+**	$-$**	$-$**	+**	$-$**
$p(z)$	?	0*	$+^{\#}$	+	$-$	$-$	+	+	$-$	+

*Under $\sigma = 1$; **for $\sigma > \delta/(\delta + \bar{p})$; $^{\#}$for sufficiently small s or sufficiently large a; $^{\Delta}$for z sufficiently close to b.

expect that for a given population, an increase in urban fringe drives down housing prices (i.e., $\mathrm{d}p/\mathrm{d}b < 0$). This is guaranteed by:[17]

Condition L. (Land supply regularity) $\varepsilon_q > \varepsilon_\eta$.

Under this condition, we can easily verify that $\partial B/\partial b < 0$, which together with $\partial B/\partial \psi_0 < 0$, $\partial B/\partial \bar{\theta} > 0$ and the pre-multiplied matrix on the LHS imply that the comparative-static results of b and ψ_0 must be opposite to those of $\bar{p}$, while the effects of $\bar{\theta}$ are similar to those of $\bar{p}$.

By tedious but straightforward comparative-static analysis, we can establish the following three propositions (see Appendix A for the mathematical expressions and Table 1 for a summary of the analytic results). In performing these exercises, we impose throughout Assumptions 1–5 as well as Conditions U, G, P and L.

The underlying intuition can be elaborated as follows. First, the effect of a high rate of time preference (ρ) is to suppress economic growth. Similarly, the higher the elasticity of intertemporal substitution (σ) is, the greater the rate of economic growth will be. Both results are consistent with conventional findings in the endogenous growth literature, indicating that Condition P is quite sensible. Moreover, when time preference is high or intertemporal substitution is more difficult, savings are lower, as is the housing supply, thereby leading to a higher housing price.

Second, in contrast with most of endogenous growth models, the level of the household production scaling factor (A) need not have a positive long-run effect on the economy-wide rate of growth. This is mainly due to a positive direct effect conflicting with a negative free-rider effect from external neighborhood spillovers. As a consequence, the housing stock accumulation pattern in response to household productivity is ambiguous, as are equilibrium housing prices. Should the degree of the neighborhood externality be low (small a), the free-rider problem is dampened. In this case, an improvement in household production (higher A) spurs economic growth and enhances housing capital accumulation.

Third, a larger urban fringe (b), under a given population, enables a greater quantity of housing. Such a level effect via the positive neighborhood externality encourages the productive use of housing capital and creates a permanent increase in the growth rate. By increasing the quantity of available lots, it is obvious that land prices decrease across the

[17] Precisely, $\mathrm{d}p/\mathrm{d}b$ is determined exclusively by

$$\frac{\mathrm{d}[\theta(z)q_0(z)]}{\mathrm{d}z} = q_0\frac{\mathrm{d}\theta(z)\theta z}{\mathrm{d}z} + \theta(z)z\frac{\mathrm{d}q_0(z)}{\mathrm{d}z} = \frac{q\cdot\eta}{z^2}(\varepsilon_\eta - \varepsilon_q),$$

which is negative if $\varepsilon_q > \varepsilon_\eta$.

entire city, as do housing prices. The resultant increase in housing demand therefore matches the higher supply in general equilibrium. This may provide a rationale, explaining why many cities have been expanded into metropolises by merging the surrounding suburbs over the past two centuries.

Fourth, an increase in the autonomous component of the unit transportation cost (τ_0) raises the ratio of housing stock at the border to that at any arbitrary location ($\partial B/\partial \tau_0 > 0$). This results in a higher housing prices at any location within the city, thus discouraging the productive use of housing capital. As a result, both the rate of housing capital accumulation and the rate of growth of the city decrease.

These arguments can be summarized by,

Proposition 2. (Characterization of the BGSE with respect to economic primitives) *(i) The effect of a lower time preference rate or a higher elasticity of intertemporal substitution is to promote economic growth rate and to suppress equilibrium housing prices (ii) An increase in the scaling factor of household production has an ambiguous effect, depending on the magnitude of the positive productivity force relative to that of the negative free-rider force (iii) An enlargement in the urban fringe tends to encourage productive use of housing capital and enhance economic growth (iv) An increase in the autonomous component of the unit transportation cost leads to higher housing prices and lower economic growth.*

We next discuss the comparative statics of a price control policy. Our result shows a negative effect of urban fringe land pricing ($\overline{p}$) on the long-run growth rate of the economy. This is due to more costly household production. The result suggests a potential growth-promoting role for public policies, including tax deductions on mortgage interest payments, tax holidays for urban land development, and urban housing price controls

Proposition 3. (Price control policy analysis along the BGSE path) *A price control that reduces the housing price at the urban fringe ($\overline{p}$) encourages productive use of housing capital and enhances the rate of economic growth.*

We are now ready to examine the effects of the two zoning policies. Consider first a zoning policy that induces a steeper FAR schedule (higher ψ_0). Recall that this zoning policy has an inverse effect on the ratio of housing stock at the border to that at any arbitrary location ($\partial B/\partial \psi_0 < 0$). Thus, by relaxing more than proportionately the FAR in favor of locations toward the CBD, the BGSE housing prices are lower. This policy therefore encourages the productive use of housing capital and promotes economic growth.

It is interesting to point out that a land policy in the form of raising the FAR uniformly (higher $\overline{\theta}$) has an adverse long-run effect on economic growth. Notably, this parallel shift of the FAR schedule increases the ratio of the housing capital at the fringe to that toward the CBD. As a consequence, the agglomeration effect and the effectiveness of neighborhood spillovers become less significant. Both imply less productive use of housing capital and lower growth of the city economy. Given the demand for housing (relative to consumption), the decumulation of housing capital lowers housing supply, thereby leading to higher housing prices. As one can see, the comparative-statics with respect to the two types of zoning policies are completely different.

Proposition 4. (Zoning policy analysis along the BGSE path) *(i) A loose restriction on the floor area ratio by relaxing it more than proportionately for locations near the city center (higher ψ_0) encourages productive use of housing capital, lowers equilibrium housing prices and fosters economic growth (ii) A uniformly loose restriction on the floor area ratio (higher $\overline{\theta}$) encourages non-productive use of housing capital, raises equilibrium housing prices and suppresses economic growth.*

One may wonder what happens if the city government does not impose the decreasing FAR schedule as specified in Assumption 3. To compare with our benchmark setup, it is natural to consider the case where the FAR is kept constant for all locations in such a way that the average housing quantity remain unchanged (referred to as a flat zoning policy). More specifically, we set $\theta(z) = \tilde{\theta}$ for all z. The average housing quantity in the benchmark model satisfying Assumption 3 with $\psi(z) = \psi_0 z$ can be computed as:

$$\frac{1}{N(t)}\int_0^b m\eta(z,t)\mathrm{d}z = \frac{1}{N(t)}\int_0^b 2\pi z(\overline{\theta} - \psi_0 z)\mathrm{d}z = \frac{\pi b^2}{N(t)}(\overline{\theta} - \frac{2}{3}\psi_0 b)$$

Under $\theta(z) = \tilde{\theta}$, the average housing quantity becomes:

$$\frac{1}{N(t)}\int_0^b 2\pi z\tilde{\theta}\mathrm{d}z = \frac{\pi b^2}{N(t)}\tilde{\theta}$$

Equating the two yields: $\tilde{\theta} = \overline{\theta} - (2/3)\psi_0 b$. Notice that this FAR schedule is equivalent to a reduction in the slope (ψ_0) to zero in conjunction with a uniform increase in the FAR of the magnitude $(1/3)\psi_0 b$ (i.e., from the original level of $\overline{\theta} - \psi(b)$ to $\tilde{\theta}$). Thus, this zoning policy change can be regarded as a combination of a change in the FAR by relaxing it more than proportionately for locations away from the city center and a uniformly loose restriction on the FAR. Utilizing Proposition 4, we can conclude:

Proposition 5. (Flat zoning policy) *Compared to the benchmark setup, a flat zoning policy discourages productive use of housing capital, increases equilibrium housing prices and lowers economic growth.*

The flat zoning policy is intended to capture the phenomenon where the city government does not design a zoning policy to differentiate land-use intensity based upon the underlying spatial structure. Our result suggests that such a non-differentiating zoning policy is always growth-retarding in the long run. Notably, it is difficult to contrast our benchmark model with one that exists no zoning policy. In particular, should we consider a city without any FAR restrictions and without imposing uniform population distribution, it is clear to see that all population will concentrate at the CBD in the absence of additional sources of negative externality such as pollution and traffic congestion. Thus, the monocentric city will be degenerated.

Recall that the neighborhood externality vanishes as a approaches zero. However, a measures not only the degree of the neighborhood externality but the degree of diminishing returns of private inputs, sh. Thus, it is difficult to isolate the pure neighborhood effect in this model. Instead, we consider an autonomous change in the neighborhood externality. This can be done by rewriting the production function as $A(sh)^{1-a}(\phi\overline{h})^a$ where $\phi > 0$ captures the exogenous component and $\overline{h}$ $(= h$ in equilibrium) the endogenous component of the neighborhood externality. When $\phi = 1$, it reduces to the original production function. Clearly, ϕ generates qualitatively identical comparative statics to A, as reported in Table 1. It is immediate that the degree of the neighborhood externality must have ambiguous long-run effects on housing prices, resource reallocation and economic growth. Such an ambiguity is due primarily to the presence of a positive direct effect offsetting against a negative free-riding effect.

Finally, we argue that our main findings are robust to the case with a decreasing population density function in the distance from the CBD. For example, take $m(z) = m_b e^{b-z}$ (with $m_b > 0$), which is decreasing exponentially in z. The housing stock now becomes: $h(z) = [2\pi z/m(z)]\theta(z)$. Since $\partial[z/m(z)]/\partial z > 0$ and $\partial B(z)/\partial\theta(z) < 0$, all the comparative statics remain qualitatively unchanged. Although similar arguments cannot apply to the case with an increasing population density function, this latter case is not frequently observed in reality and, thus, one may simply assume it away without loss of generality.

6. Concluding remarks

We have developed a dynamic household production model to examine the intertemporal relationships between urban land use and housing evolution within an endogenous growth framework. We have proved the existence and uniqueness of the balanced growth spatial equilibrium and illustrated how changes in demand, supply and spatial setting affect housing accumulation, pricing and city development. We have also studied the long-run growth consequences of housing price control and zoning policies.

Several related issues still remain open. For brevity, we will only elaborate on three of them. First, we have restricted our attention to balanced growth analyses. Due to the presence of neighborhood positive externalities, it may be interesting to examine the possibility of dynamics indeterminacy, which may generate interesting housing price dynamics and rich policy implications.[18] Second, if one is concerned with the urban area as a whole, it may be useful to characterize the aggregate housing stock ($H \equiv \int_0^b mh(z)\mathrm{d}z$) and the average housing price ($p = \int_0^b mp(z)\mathrm{d}z$). Of course, this would likely require numerical exercises because $h(z)$ and $p(z)$ may not be integrated analytically. Third, within the closed-city framework, we take the size and the distribution of the city population as given. Should the city developer be allowed to choose optimally the population size (in an open city setting) and the population distribution based on local public good arguments, one may examine the dynamic interactions between the size of the city, the housing price and the pattern of urban agglomeration.[19] Finally, our paper conducts a positive analysis on the long-run effects of urban policies. One may introduce pollution and traffic congestion externalities to undertake a normative analysis. For example, it may be interesting to inquire under what circumstances the zoning policy design is optimal (or welfare-maximizing).

[18] The possibility of dynamic indeterminacy in the presence of neighborhood externalities has been verified by Anas et al. (1995). Nevertheless, its implications for land policy and housing accumulation remain unexplored.

[19] For example, a city with an optimally distributed population may be in the form of Riley's (1974) gammaville. Although this consideration allows us to study the effect of land policy on the dynamic pattern of a monocentric city, it is at the expense of significant analytic complexity which would require further simplification of the model structure in other dimensions.

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Appendix A

The comparative statics are summarized as follows, where detailed mathematical expressions are eliminated whenever the derivations are straightforward:

(i)

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\rho}<0,\ \frac{\mathrm{d}p}{\mathrm{d}\rho}>0;\quad \frac{\mathrm{d}\gamma}{\mathrm{d}\sigma}>0,\ \frac{\mathrm{d}p}{\mathrm{d}\sigma}<0;\quad \frac{\mathrm{d}\gamma}{\mathrm{d}b}>0,\ \frac{\mathrm{d}p}{\mathrm{d}b}<0;$$

(ii)

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\overline{p}}<0;\quad \frac{\partial\gamma}{\partial\psi_0}>0,\ \frac{\partial p}{\partial\psi_0}<0;\quad \frac{\mathrm{d}\gamma}{\mathrm{d}\overline{\theta}}<0,\ \frac{\mathrm{d}p}{\mathrm{d}\overline{\theta}}>0;$$

(iii)

$$\frac{\mathrm{d}s}{\mathrm{d}\rho}\propto(p-\overline{p})-1<0\ \text{ if } z\rightarrow b;$$

(iv)

$$\frac{\mathrm{d}s}{\mathrm{d}\overline{p}}\propto\delta-\sigma(p+\delta)<0,\ \text{ if } \sigma>\frac{\delta}{\delta+\overline{p}};$$

(v)

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\left(\frac{\alpha}{1-\alpha}\right)}\propto ap-\varepsilon(p-\overline{p})<0\ \text{ if } s\rightarrow\ 0 \text{ or } a \text{ is large,}$$

$$\text{where } \varepsilon\equiv-\frac{s}{S}\frac{\partial S}{\partial s}\propto a\overline{p}-\frac{s}{1-s}(p-\overline{p})\geq 0;$$

(vi)

$$\frac{\mathrm{d}p}{\mathrm{d}\left(\frac{\alpha}{1-\alpha}\right)}\propto a[\sigma p+(\sigma-1)\delta]-(\gamma+\delta)\frac{s}{1-s}>0 \text{ if } s\rightarrow 0 \text{ or } a \text{ is large;}$$

(vii)

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\delta}=<0,\quad \frac{\mathrm{d}s}{\mathrm{d}\delta}=0,\quad \text{and}\quad \frac{\mathrm{d}p}{\mathrm{d}\delta}=0\ \text{ if} \sigma=1;\ \text{and,}$$

(viii)

$$\frac{d\gamma}{dA}, \quad \frac{ds}{dA} \quad \text{and} \quad \frac{dp}{dA} \text{ are generally ambiguous in sign.}$$

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