行政院國家科學委員會專題研究計畫　成果報告

描點相關時多階製程管制方法之研究(2/2)

計畫類別：個別型計畫
計畫編號：NSC91-2118-M-004-008-
執行期間：91 年 08 月 01 日至 92 年 07 月 31 日
執行單位：國立政治大學統計學系

計畫主持人：楊素芬

計畫參與人員：何漢葳蘇惠君何志傑劉家銘蔡淑芳

報告類型：完整報告

處理方式：本計畫涉及專利或其他智慧財產權，2 年後可公開查詢

中　華　民　國 92 年 10 月 27 日

行政院國家科學委員會補助專題研究計畫 □＊成果報告
□期中進度報告

描點相關時多階製程管制之研究

計畫類別：□* 個別型計畫 □ 整合型計畫
計畫編號：NSC 91－2118－M －004－008－
執行期間： 90 年 8 月 1 日至 92 年 7 月 31 日

計畫主持人：楊素芬教授
共同主持人：
計畫參與人員： 何漢葳、蘇惠君、何志傑、劉家銘、蔡淑芳

成果報告類型(依經費核定清單規定繳交)：□精簡報告 *□完整報告

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

處理方式：除產學合作研究計畫、提升產業技術及人才培育研究計畫、列管計畫及下列情形者外，得立即公開查詢
□涉及專利或其他智慧財產權，□一年□*二年後可公開查詢

執行單位：國立政治大學統計系

中 華 民 國 92 年 10 月 25 日

STATISTICAL PROCESS CONTROL FOR THE AUTOCORRELATED OBSERVATIONS UNDER TWO SUB-PROCESSES

Su-Fen Yang
Department of Statistics
National Chengchi University
Taipei, 116, Taiwan

ABSTRACT

The observations from the process output are always assumed independent when using a control chart to monitor a process. However, for many processes the process observations are autocorrelated. This autocorrelation can have a significant effect on the performance of the control chart. This paper considers the problem of monitoring the mean of a quality characteristic X on the first sub-process and the mean of a quality characteristic Y on the second sub-process, in which the observations X can be modeled as an AR(1) model and observations Y can be modeled as an transfer function of X since the state of the second sub-process is dependent on the state of the first sub-process. To effectively distinguish and maintain the state of the two dependent sub-processes, the Shewhart control chart of residual and the cause-selecting control chart are proposed. The proposed control charts' performance is measured by the rate of true alarm or false alarm. From numerical analysis, it shows that the performance of the proposed control charts is much better than the Hotelling T^2 control chart and the individual X and Y charts.

Key Words: *Autocorrelated observations, control charts, process variation, residuals.*

1. INTRODUCTION

Control charts are first proposed by Shewhart (1931), and become effective tools for improving the process quality and productivity.

A basic assumption in applications of control charts is that observations from the process at different times are independent random variables. However, the independence assumption is often violated for processes in chemical and pharmaceutical industries. Observations from these processes are always autocorrelated. When the control charts developed under the independence assumption, the autocorrelated process results in decreasing the in-control average run length (ARL). For effective monitoring the autocorrelated processes, one popular developed approach is to constructing control charts using the residuals from the time series model to the process data (see Abraham and Kartha (1979), Alwan (1991), Alwan and Roberts(1988), Berthouex, Dooley, Kapoor, Dessouky and Delves (1986), Ermer (1980), Harris and Ross (1991), Montgomery (1996), Montgomery and Mastrangelo (1991), and Wardell, Moskowitz, and Plante (1992, 1994)). The properties of the proposed residual charts and their performance are investigated by Harris and Ross (1991), Longnecker and Ryan (1992),Yashchin (1993),Kramer and Schmid (1997), Schmid (1995),Lin and Adams (1996), Schmid (1997a), Padgett, Thombs and Padgett (1992), Runger, Willemain and Prabhu (1995), Vander Weil (1996), Timmer, Pignatiello and Longnecker (1998), Schmid (1997b) ,Zhang (1998), Schmid and Schone (1997), Alwan and Roberts (1988) and Lu and Reynolds (1999).

Much of the paper on the performance of control charts based on residuals has focused on the Shewhart control chart of residuals.

Today, many industrial products are produced by several dependent processes not just one process. Consequently, it is not appropriate to monitor these processes with a control chart for each individual process; what is needed is an appropriate method for controlling the processes.

Zhang(1984) proposes the simple cause-selecting chart to monitor the second process of the two dependent processes. Wade and Woodall (1993) review the basic principles of the cause-selecting chart for two dependent processes and suggest a modification to the use of simple cause-selecting chart. They also examine the relationship between the simple cause-selecting chart and the multivariate T^2 control chart. In their opinion, the simple cause-selecting control chart has some advantages over the T^2 control chart. However, the statistical process control approach to effectively distinguish and monitor the dependent sub-processes states for autocorrelated observations has not been addressed. In this paper, Shewhart chart of residuals and cause-selecting control chart are developed to monitor the large mean shift of the first process and the large mean shift of the second process, respectively. The performance of the proposed control charts for monitoring the two sub-processes is measured by the probability of alarms. Finally, a numerical example illustrates the application of the proposed control charts and its performance.

2. AUTOCORRELATED OBSERVATIONS FOR TWO SUB-PROCESSES

In this paper, we consider two dependent sub-processes, which may have two types of failure mechanisms. One type of the failure mechanisms occurs only in the first process and causes the mean shift of the quality variable (X), while the other type occurs only in the last process and causes the mean shift of the quality variable (Y). Two types statistical control charts will be derived to effectively distinguish and monitor the dependent sub-processes states for autocorrelated observations. Before describing how to derive the statistical control charts, the assumptions of the production processes behavior are given as follows.

2.1 Assumptions and Notation

Assumptions

(1) The production has two dependent processes. The first process is called the sub-process 1 and the second process is called the sub-process 2. The sub-process 1 and the sub-process 2 are dependent. So the quality variable X produced by the sub-process 1 will affect the quality variable Y produced by the sub-process 2. A pair of observations (x_t, y_t) are sampled from the end of the sub-process 2 every h time unit of sampling interval, $t = 1,2,3,\ldots.$

(2) For autocorrelated observations x_t at sub-process 1, it is assumed that quality variable X_t can be written as an AR(1) model at time t with process mean ξ_X, that is

$$X_t = (1-\phi)\xi_X + \phi X_{t-1} + a_t, t = 1,2,\ldots, \qquad (1)$$

where ϕ is the AR parameter satisfying $|\phi|<1$. The a_t's are assumed to be independent normal random variables with mean 0 and variance σ_a^2.

Since X affects Y, the model relating the two variables can be written as a transfer function, that is

$$Y_t = C_Y + V_0 X_t + V_1 X_{t-1} + N_t, t = 1,2,\ldots, \qquad (2)$$

where C_Y is a constant and where N_t's are assumed to be independent normal random variables with mean 0 and variance σ_N^2.

(3) When one failure mechanism occurs only in the sub-process 1, it will shift the mean of X. This will also cause the mean of Y shifts. When the other failure mechanism occurs only in the sub-process 2, then it will shift the mean of Y and the mean of X is unchanged.

(4) The time to sampling and charting one item is very small and negligible.

3. THE TIME SERIES MODEL FOR AUTOCORRELATED PROCESS

Time series model, especially AR(1) model, has been widely used to model many types of processes. When the sub-process 1 is in control the minimum mean square error forecast (Box, Jenkins, and Reinsel (1994)) made at time $t-1$ for time t is

$$\hat{X}_t = (1-\hat{\phi})\xi_{X0} + \hat{\phi} X_{t-1} \qquad (3)$$

The residual at time t is

$$e_{Xt} = X_t - \hat{X}_t \qquad (4)$$

Suppose that a failure mechanism would cause a step change from ξ_{X0} to ξ_{X1} in the process mean between time $t=\tau-1$ and τ. The expectations of the residual for various times are

$$E(e_{Xt}) = \begin{cases} 0 & t=\tau-1,\ \tau-2,\ldots, \\ \xi_{X1}-\xi_{X0} & t=\tau, \\ \phi_1^l(\xi_{X1}-\xi_{X0}) & t=\tau+l, l=1,2,\ldots. \end{cases} \qquad (5)$$

The residuals are uncorrelated and normally distributed with variance σ_a^2. We may find that the expectation of a residual after the shift occurs is a decreasing function of the time after the shift. Hence, the probability of an alarm by a control chart of residuals in the sub-process 1 is the highest for the sample immediately after the shift, and this probability continually decreases over time as the forecast adapts to the shift.

In sub-process 2, a transfer function is used to express the relationship between quality variables Y and X. The estimate for equation (2) is

$$\hat{Y}_t = \hat{C}_y + \hat{V}_0 X_t + \hat{V}_1 X_{t-1} \qquad (6)$$

The residual at time t is

$$e_{Yt} = Y_t - \hat{Y}_t \qquad (7)$$

Suppose that another failure mechanism would cause a step change from ξ_{Y0} to ξ_{Y1} in the process mean between time $t = \tau' - 1$ and τ'. The expectations of the residual for various times are

$$E(e_{Yt}) = \begin{cases} 0 & t = \tau' - 1,\ \tau' - 2, \ldots, \\ \xi_{Y1} - \xi_{Y0} & t = \tau', \\ 0 & t = \tau' + l, l = 1,2,\ldots. \end{cases} \qquad (8)$$

The residuals are uncorrelated and normally distributed with variance σ_N^2. Note that the shift of process mean only appears at time τ' after the failure mechanism occurs between time $\tau' - 1$ and τ'.

4. CONTROL CHARTS CONSTRUCTION

A general form of control charts is represented as follows.

$$\mu_W \pm k\sigma_W, \qquad (9)$$

where μ_W and σ_W are the mean and standard deviation of a control statistic, say W_t, when the process is in control. The constant, k, is chosen to give a specified in control false alarm probability. The k is frequently taken to be 3 to give an false alarm probability 0.0027 for a Shewhart type control chart.

To monitor the sub-process 1, the Shewhart control chart of residuals plots the control statistic e_{Xt}, $t = 1,2,\ldots.$. For this control statistic, the control limits in equation (9) are

$$\pm 3\sigma_a \qquad (10)$$

To monitor the sub-process 2, it is incorrect to construct the control chart based on the distribution of quality variable Y, since quality variable Y is affected by quality variable X. The proposed approach is to monitor the specific quality in the sub-process 2 by adjusting the effect of Y from X; that is the specific quality is presented by the cause-selecting values ($e_{Yt} = Y_t - \hat{Y}_t$). When the specific quality is in control, the cause-selecting control chart is constructed by the in-control distribution of cause-selecting values.

For sub-process 2, the cause-selecting control chart plots the control statistic e_{Yt}, $t = 1,2,....$ For this control statistic, the control limits in equation (9) are

$$\pm 3\sigma_N \quad (11)$$

Consequently, the Shewhart control chart and cause-selecting control chart are derived to effectively distinguish and monitor the process states of sub-process 1 and sub-process 2.

The variance of the control statistic e_{Xt} is usually unknown. The average moving range divided by coefficient d_2 is always taken as its estimate, that is $\hat{\sigma}_a = \frac{\overline{MR}_X}{d_2}$, where $\overline{MR_X} = \frac{\sum_{t=1}^{n-1} MR_{X(t-1)}}{n-1}$, and where $MR_{X(t-1)} = | e_{Xt} - e_{X(t-1)} |, t = 2,3,..n.$ Similar to the variance of the control statistic e_{Yt}, the estimate of σ_N is $\hat{\sigma}_N = \frac{\overline{MR_Y}}{d_2}$, where $\overline{MR_Y} = \frac{\sum_{t=1}^{n-1} MR_{Y(t-1)}}{n-1}$, and where $MR_{Y(t-1)} = | e_{Yt} - e_{Y(t-1)} |, t = 2,3,..n.$. Thus the control limits of Shewart control chart and cause-selecting control chart in equations (10) and (11) can be expressed as

$$\pm 3\frac{\overline{MR_X}}{d_2}, \qquad (12)$$

and

$$\pm 3\frac{\overline{MR_Y}}{d_2}, \qquad (13)$$

respectively. Note that the value of d_2 is dependent on the sample size $n(=2)$.

5. MEASUREMENT OF THE PERFORMANCE OF CONTROL CHARTS

The performance of a control chart can be measured by the probability of alarm. The probability of false alarm is the rate of false alarm occurred on the control chart before a failure mechanism occurs in the process, and the probability of true alarm is the rate of true alarm occurred on the control chart after the failure mechanism occurs in the process and before it is removed. When the process is out of control it is desirable to have a high rate of true alarm so that the change of process mean will be detected quickly, and when the process is in control it is desirable to have a low rate of false alarm. The probability of alarm for the developed two control charts will be calculated when both the sub-processes are in control and when either one of or both the sub-processes means shift.

5.1 Calculating the Probability of Alarm for the Two Proposed Control Charts

The probability of false alarm for the Shewhart chart of residuals is 370.4, and same to the cause-selecting chart. To monitor the two dependent sub-processes, two developed charts are used simultaneously. Hence the probability of at least one false alarm for the two charts is 0.0054, that is 1-0.9973x0.9973. To calculate the probability of true alarm for the two charts, we have to compute the power of the Shewhart chart of residuals and cause-selecting chart, respectively. Let the power of the Shewhart chart at time t is Ps_t, and the power of the cause-selecting chart at time t is Pc_t, then

$$Ps_t = \Pr(e_{Xt} > 3\frac{\overline{MR_X}}{d_2}\ or\ e_{Xt} < -3\frac{\overline{MR_X}}{d_2} \mid \phi_1^l(\xi_{X1}-\xi_{X0})) = 2\Phi_S(-3-\frac{d_2\phi_1^l(\xi_{X1}-\xi_{X0})}{\overline{MR_X}}),\ \ t=\tau+l,\ ,$$

$$l = 0,1,2,.... \quad (14)$$

$$Pc_t = \Pr(e_{Yt} > 3\frac{\overline{MR_Y}}{d_2}\ or\ e_{Yt} < -3\frac{\overline{MR_Y}}{d_2} \mid (\xi_{Y1}-\xi_{Y0})) = 2\Phi_C(-3-\frac{d_2(\xi_{Y1}-\xi_{Y0})}{\overline{MR_Y}}),\ \ t=\tau'. \quad (15)$$

where Φ_S and Φ_C are the cumulative standard normal probabilities, respectively.

Note that the power of the Shewhart chart of residuals decreasing over time, but not for the cause-selecting chart. Hence the power for the two charts will be decreasing over time, once the sub-process 1 is out of control.

There are four situations for the out-of-control process. The four situations and power of the two charts are described as follows.

(1) The sub-process 1 is out of control but the sub-process 2 is in control. The power (Psc_t) of the two charts is

$$Psc_t = 1-(1-Ps_t)\bullet(0.9973), \qquad t=\tau+l,\ \ l=1,2,3,....$$

(2) The sub-process 1 is in control but the sub-process 2 is out of control. The power (Psc_t) of the two charts is

$$Psc_t = 1-(0.9973)\bullet(1-Pc_t), \qquad t=\tau'.$$

(3) The sub-process 1 is out of control after time $\tau-1$, and the sub-process 2 is out of control after time $\tau'-1$, where $\tau<\tau'$. The power (Psc_t) of the two charts is

$$\begin{aligned} Psc_t = \ & 1-(1-Ps_t)\bullet(0.9973), && \tau\le t<\tau' \\ & 1-(1-Ps_t)\bullet(1-Pc_t), && t=\tau', \\ & 1-(1-Ps_t)\bullet(0.9973), && \tau'+l<t, \quad l=1,2,3.... \end{aligned}$$

(4) The sub-process 1 is out of control after time $\tau''-1$, and the sub-process 2 is out of control after

time $\tau^{'}-1$, where $\tau^{''}>\tau^{'}$. The power (Psc_t) of the two charts is

$$Psc_t = \begin{cases} 1-0.9973\bullet(1-Pc_t) & t=\tau^{'}, \\ 1-(1-Ps_t)\bullet(0.9973) & t\geq\tau^{''}. \end{cases}$$

6. A Numerical Example and Some Comparison Results

A quality engineer found that there is a large variability for the thickness of the thin golden films. From the quality data analysis, he found that the thickness of the thin golden films (Y) in the second sub-process was primarily affected by gold concentration (X) in the first sub-process. Two independent machines, say machine 1 and machine 2, may failure and influence the mean of the gold concentration and thickness respectively. Since the unacceptable mean of the thickness may be influenced by machine 1 or gold concentration. To effectively maintain the variability of the gold concentration and thickness and distinguish which sub-process is out of control, two control charts should be constructed as described before.

To construct the proposed control charts, 100 paired observations (X_t, Y_t) are sampled from the end of the second sub-processes (Table 1). The 100 observations for X_t, t=1,2,…,100, is found autocorrelated and a time series model AR(1) is fitted. The fitted model is

$$\hat{X}_t = 4.0562 + 0.6102X_{t-1}. \qquad (16)$$

The residual (e_{X_t}) is calculated by $X_t - \hat{X}_t$, and the Shewhart chart of the residuals is constructed (Figure 1). All plotted points (e_{X_t}, t=1, 2,…,100) are within the control limits of the chart. Hence, the Shewhart chart of the in-control residuals, with upper control limit=5.39576, lower control limit=-5.39576 and center line=0, can be used to monitor the future variability of the gold concentration in the first sub-process. Then, the relationship between (X_t, Y_t) is investigated (Figure 2). It shows that they are related on time or sampling number. Hence a time series model, transfer function, is fitted. The fitted transfer function is

$$\hat{Y}_t = 3.16 + 1.0023X_t + 0.0967X_{t-1}. \quad (17)$$

The cause-selecting value (e_{Y_t}) is calculated by $Y_t - \hat{Y}_t$, and the cause-selecting control chart is constructed (Figure 3). All plotted points (e_{Y_t}, t=1, 2,…,100) are within the control limits of the cause-selecting control chart. Hence, the cause-selecting chart of the in-control cause-selecting value, with upper control limit=2.9432, lower control limit=-2.9432 and center line=0, can be used to monitor the future variability of the thickness in the second sub-process (Figure 4).

To measure the performance or detecting ability of the two proposed control charts, 51 additional paired samples (X_t, Y_t), t=1, 2, …, 51, are taken from the end of the second sub-process (Table 2). The 51 paired values ($e_{X_t} = X_t - \hat{X}_t$, $e_{Y_t} = Y_t - \hat{Y}_t$), are calculated using equations (16) and (17), and then plot on the constructed Shewhart chart of the residuals and cause-selecting control chart respectively (Figure 5 and 6). It is found that points 21 and 22 fall outside of the control limits of the Shewhart chart of the residuals and point 19 falls outside of the control limits of the cause-selecting control chart. It indicates that the first sub-process is out of control on point 21 and 22 and the machine 1 has to be adjusted, and the second sub-process is out of control on point 19 and the machine 2 has to be adjusted.

The performance of the two proposed control charts can be evaluated by comparing with Hotelling T^2 control chart and Shewhart individual X and Y control charts. First, we construct

Hotelling T^2 chart for the 100 paired observations (X_t, Y_t) (eg, see Montgomery 2001). The upper control limit of the T^2 chart is 11.829 with false alarm probability 0.0027. The 100 values of T^2 statistic are calculated and plotted on the T^2 chart (Table 1 and Figure 7). All points are within the control limits, so the T^2 chart with upper control limit=11.829, can be used to control the future state of the second sub-process with bivariate quality characteristics. The additional 51 values of T^2 statistic are calculated and plotted on the constructed T^2 chart (Figure 8). We found that only point 19 is out of control. Secondly, we construct Shewhart individual X chart for the first 100 observations (X_t) and Shewhart individual Y chart for the first 100 observations (Y_t) (Figure 9 and 10). Since some points fall outside of the control limits of the Shewhart individual X and Shewhart individual Y chart respectively, hence the outliers are removed and the control limits of the individual X and individual Y chart are re-calculated until all points fall within the control limits. Consequently, the control limits of the individual X and individual Y chart are (UCL=14.44, CL=10.45, LCL=6.47) and (UCL=19.37, CL=14.71, LCL=10.05), respectively (Figure 11, and 12). The additional 51 values of X_t and Y_t are thus plotted on the constructed individual X and individual Y chart (Figure 13 and 14). We found that many points are out of control on the two individual charts.

It is obvious that the performance of the proposed control charts is better than the other two type control charts. Our proposed control charts effectively distinguish the out-of-control points on the first sub-process and the second sub-process. The T^2 chart can only detect out one out-of-control point, and it cannot distinguish which sub-process is out of control for the autocorrelated observations on the two dependent sub-processes. The individual X chart and individual Y chart show up many false alarms, respectively. Hence, they cannot effectively detect the autocorrelated observations on the two dependent sub-processes.

7. SUMMARY

In the paper, the statistical process control approach to effectively distinguish and monitor the two dependent sub-processes states for autocorrelated observations is proposed. Shewhart chart of residuals is used to monitor the shift of the process mean in the first sub-process and the cause-selecting control chart are developed to monitor the mean shift of the second sub-process. The performance of the proposed control charts for monitoring the two sub-processes is measured by the probability of alarms. Finally, a numerical example illustrates the application of the proposed control charts, and their performance is demonstrated better than the Hotelling T^2 control chart and two individual X and Y control chart when the observations on the two dependent sub-processes are autocorrelated. Several important extensions of the developed approach can be developed. It is straightforward to extend the proposed approach to study other control charts for small shift of process means on the dependent sub-processes, like EWMA or charts for attributes. The differences between the approaches lie in the derivation of the probabilities of Type I and Type II errors. One particularly interesting research area for future research involves the dependent processes control for correlated observations with ARMA model.

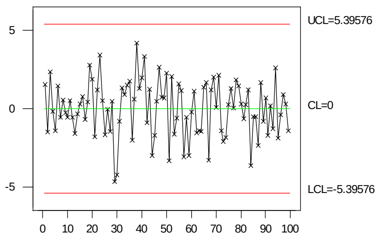

Figure 1: Shewhart chart of residual

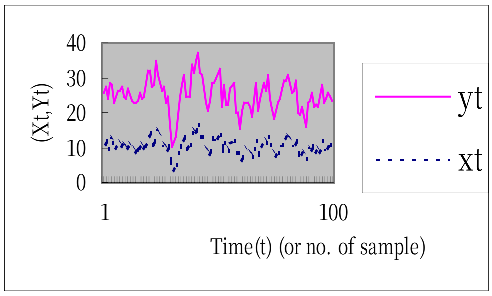

Figure 2: The relationship between Xt and Yt on time

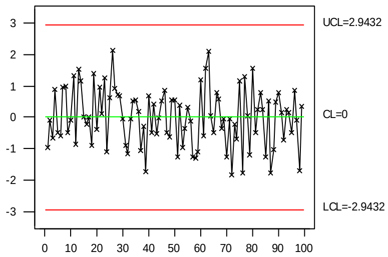

Figure 3: Cause-selecting Chart

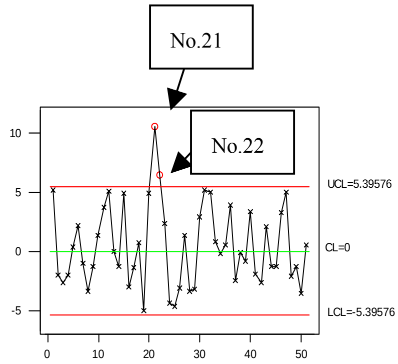

Figure 4: monitoring result of Shewhart chart of residual

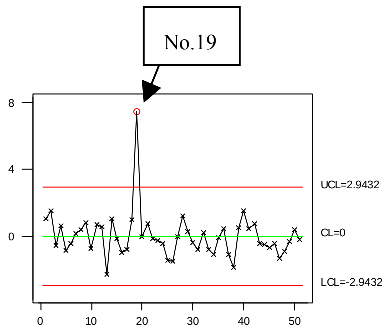

Figure 5: monitoring result of Cause-selecting Chart

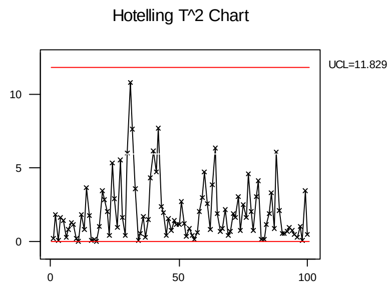

Figure 6: Hotelling T2 chart

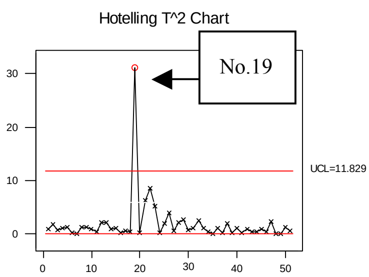

Figure 7: Monitoring Result of Hotelling T2 chart

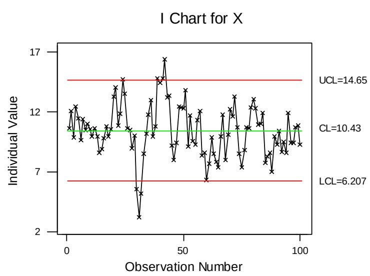

Figure 8: Individual X control chart

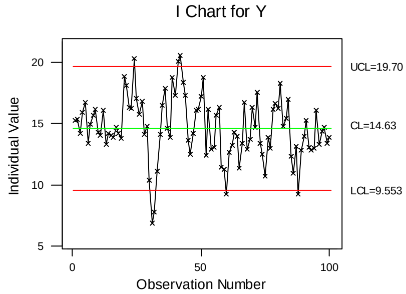

Figure 9: Individual Y control chart

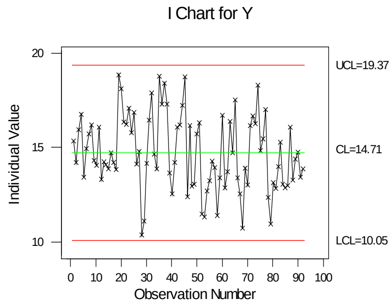

Figure 11: In-control Individual Y control chart

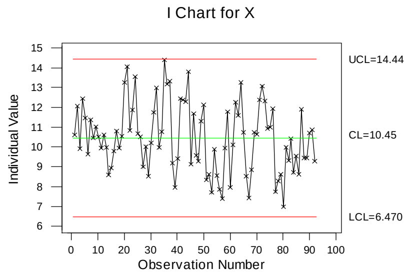

Figure 10: In-control Individual X control chart

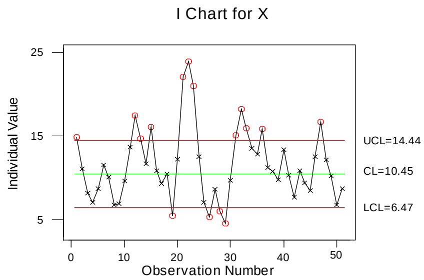

Figure 12: monitoring result of Individual X control chart

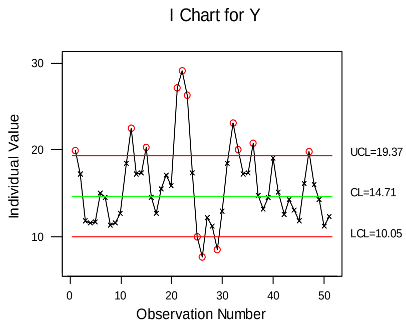

Figure 13: monitoring result of Individual Y control chart

Table 1: The 100 observations for (Xt,Yt)

No.	X_t	Y_t	$\hat{X}_t$	e_{Xt}	MR_{Xt}	$\hat{Y}_t$	e_{Yt}	MR_{Yt}	Hotelling T^2
1	10.628	15.299							0.261961
2	12.08	15.352	10.541	1.539		16.2954	-0.943		1.82844
3	9.921	14.198	11.427	-1.506	3.045	14.2721	-0.074	0.869	0.067479
4	12.443	15.936	10.11	2.333	3.839	16.5909	-0.655	0.581	1.66109
5	11.476	16.743	11.649	-0.173	2.506	15.8662	0.877	1.532	1.43551
6	9.626	13.43	11.059	-1.433	1.26	13.9179	-0.488	1.365	0.275604
7	11.381	14.921	9.93	1.451	2.884	15.498	-0.577	0.089	0.803632
8	10.461	15.699	11.001	-0.54	1.991	14.7455	0.953	1.53	1.28982
9	11.015	16.205	10.439	0.576	1.116	15.2125	0.992	0.039	1.16441
10	10.53	14.304	10.778	-0.248	0.824	14.7796	-0.476	1.468	0.225106
11	9.934	14.06	10.482	-0.548	0.3	14.1358	-0.076	0.4	0.052497
12	10.623	16.082	10.118	0.505	1.053	14.768	1.314	1.39	1.87722
13	9.979	13.322	10.538	-0.559	1.064	14.1894	-0.867	2.181	0.862852
14	8.575	14.227	10.145	-1.57	1.011	12.7199	1.507	2.374	3.65364
15	8.933	14.082	9.289	-0.356	1.214	12.9428	1.139	0.368	1.76329
16	9.805	13.857	9.507	0.298	0.654	13.8517	0.005	1.134	0.094166
17	10.802	14.724	10.039	0.763	0.465	14.935	-0.211	0.216	0.136329
18	9.954	14.186	10.648	-0.694	1.457	14.1815	0.004	0.215	0.050987
19	10.539	13.815	10.13	0.409	1.103	14.6856	-0.871	0.875	1.03923
20	13.28	18.863	10.487	2.793	2.384	17.4905	1.373	2.244	3.42938
21	14.046	18.126	12.16	1.886	0.907	18.5224	-0.396	1.769	2.83427
22	10.838	16.351	12.627	-1.789	3.675	15.3817	0.969	1.365	2.02715
23	11.867	16.23	10.67	1.197	2.986	16.1029	0.127	0.842	0.433099
24	14.73	20.316	11.297	3.433	2.236	19.0716	1.244	1.117	5.32191
25	13.549	17.072	13.044	0.505	2.928	18.1647	-1.093	2.337	2.90852
26	10.665	15.765	12.324	-1.659	2.164	15.1601	0.605	1.698	0.955134
27	10.503	16.852	10.564	-0.061	1.598	14.7191	2.133	1.528	5.52425
28	9	14.111	10.465	-1.465	1.404	13.1965	0.914	1.219	1.61905
29	10.027	14.791	9.548	0.479	1.944	14.0806	0.71	0.204	0.450309
30	5.527	10.363	10.175	-4.648	5.127	9.6699	0.693	0.017	6.00457
31	3.182	6.8451	7.429	-4.247	0.401	6.88414	-0.039	0.732	10.824
32	5.189	7.7834	5.998	-0.809	3.438	8.66838	-0.885	0.846	7.62446
33	8.542	11.077	7.223	1.319	2.128	12.2236	-1.147	0.262	3.60659
34	10.186	14.153	9.268	0.918	0.401	14.1953	-0.042	1.105	0.068234

35	11.759	16.465	10.272	1.487	0.569	15.9315	0.533	0.575	0.581365
36	12.995	17.886	11.232	1.763	0.276	17.3224	0.564	0.031	1.69991
37	9.992	14.632	11.986	-1.994	3.757	14.4319	0.2	0.364	0.302128
38	10.77	13.878	10.153	0.617	2.611	14.9208	-1.043	1.243	1.50637
39	14.828	18.779	10.628	4.2	3.583	19.0643	-0.285	0.758	4.30783
40	14.394	17.287	13.104	1.29	2.91	19.0218	-1.735	1.45	6.1185
41	14.791	20.068	12.839	1.952	0.662	19.3772	0.691	2.426	4.70693
42	16.439	20.57	13.082	3.357	1.405	21.0674	-0.497	1.188	7.66888
43	13.185	18.394	14.087	-0.902	4.259	17.9651	0.429	0.926	2.41323
44	13.339	17.293	12.102	1.237	2.139	17.8049	-0.512	0.941	1.95985
45	9.187	13.627	12.196	-3.009	4.246	13.6578	-0.031	0.481	0.43837
46	7.964	12.536	9.662	-1.698	1.311	12.0312	0.505	0.536	1.58162
47	9.408	14.211	8.916	0.492	2.19	13.3595	0.851	0.346	0.747548
48	12.457	16.071	9.797	2.66	2.168	16.5556	-0.485	1.336	1.45152
49	12.397	16.183	11.657	0.74	1.92	16.7907	-0.608	0.123	1.13994
50	12.294	17.234	11.621	0.673	0.067	16.6818	0.552	1.16	1.18754
51	13.809	18.75	11.558	2.251	1.578	18.1899	0.56	0.008	2.69616
52	9.138	12.392	12.482	-3.344	5.595	13.6548	-1.263	1.823	1.24111
53	11.672	16.144	9.632	2.04	5.384	15.7428	0.401	1.664	0.364923
54	9.558	12.92	11.178	-1.62	3.66	13.869	-0.949	1.35	0.891356
55	9.297	13.047	9.888	-0.591	1.029	13.4024	-0.355	0.594	0.435517
56	11.311	15.71	9.729	1.582	2.173	15.3965	0.314	0.669	0.186217
57	12.117	16.292	10.958	1.159	0.423	16.3985	-0.106	0.42	0.606532
58	8.349	11.451	11.45	-3.101	4.26	12.6997	-1.249	1.143	2.01656
59	8.611	11.309	9.151	-0.54	2.561	12.5985	-1.289	0.04	2.99173
60	6.333	9.2373	9.311	-2.978	2.438	10.3403	-1.103	0.186	4.75739
61	7.71	12.683	7.921	-0.211	2.767	11.4999	1.183	2.286	2.58507
62	9.881	13.232	8.761	1.12	1.331	13.8099	-0.578	1.761	0.856342
63	8.554	14.261	10.086	-1.532	2.652	12.6892	1.572	2.15	3.88636
64	7.856	13.941	9.276	-1.42	0.112	11.8615	2.079	0.507	6.37424
65	7.393	11.388	8.85	-1.457	0.037	11.3303	0.058	2.021	1.89787
66	9.952	13.376	8.567	1.385	2.842	13.8502	-0.474	0.532	0.712481
67	11.785	16.721	10.129	1.656	0.271	15.9344	0.787	1.261	0.892642
68	7.955	12.87	11.247	-3.292	4.948	12.2735	0.597	0.19	2.16575
69	10.105	13.716	8.91	1.195	4.487	14.0577	-0.342	0.939	0.40841
70	12.249	16.361	10.222	2.027	0.832	16.4146	-0.054	0.288	0.730034
71	11.606	14.716	11.53	0.076	1.951	15.9773	-1.261	1.207	1.90121
72	13.263	17.526	11.138	2.125	2.049	17.5757	-0.05	1.211	1.66926

73	10.734	13.361	12.149	-1.415	3.54	15.2011	-1.84	1.79	3.05212
74	8.514	12.519	10.606	-2.092	0.677	12.7323	-0.213	1.627	0.76164
75	7.407	10.708	9.251	-1.844	0.248	11.4076	-0.7	0.487	2.48831
76	8.834	13.891	8.576	0.258	2.102	12.7313	1.16	1.86	1.62151
77	10.729	12.996	9.447	1.282	1.024	14.7686	-1.773	2.933	4.58101
78	10.645	16.161	10.603	0.042	1.24	14.8675	1.294	3.067	2.051
79	12.395	16.656	10.552	1.843	1.801	16.6135	0.043	1.251	0.80049
80	13.064	16.264	11.62	1.444	0.399	17.4533	-1.189	1.232	3.04521
81	12.313	18.307	12.028	0.285	1.159	16.7649	1.542	2.731	4.11424
82	10.933	14.818	11.57	-0.637	0.922	15.3094	-0.491	2.033	0.194493
83	10.984	15.465	10.727	0.257	0.894	15.2269	0.238	0.729	0.137442
84	11.954	16.993	10.759	1.195	0.938	16.204	0.789	0.551	1.14789
85	7.733	12.331	11.35	-3.617	4.812	12.0667	0.264	0.525	1.879
86	8.273	10.935	8.775	-0.502	3.115	12.1996	-1.265	1.529	3.32225
87	8.611	13.124	9.104	-0.493	0.009	12.5909	0.533	1.798	0.898021
88	6.975	9.2197	9.311	-2.336	1.843	10.9837	-1.764	2.297	6.09336
89	9.978	12.828	8.312	1.666	4.002	13.8353	-1.007	0.757	2.1332
90	9.311	13.957	10.145	-0.834	2.5	13.4579	0.499	1.506	0.576574
91	10.411	15.282	9.738	0.673	1.507	14.4952	0.787	0.288	0.544448
92	8.701	13.029	10.409	-1.708	2.381	12.8881	0.141	0.646	0.681223
93	9.532	12.844	9.366	0.166	1.874	13.5557	-0.712	0.853	0.995131
94	8.606	12.964	9.873	-1.267	1.433	12.7081	0.256	0.968	0.772487
95	11.91	16.068	9.308	2.602	3.869	15.93	0.138	0.118	0.472838
96	9.451	13.283	11.324	-1.873	4.475	13.7845	-0.501	0.639	0.309009
97	9.43	14.392	9.823	-0.393	1.48	13.5253	0.867	1.368	1.0219
98	10.717	14.733	9.81	0.907	1.3	14.8136	-0.081	0.948	0.065452
99	10.878	13.413	10.596	0.282	0.625	15.0999	-1.687	1.606	3.47223
100	9.282	13.861	10.694	-1.412	1.694	13.5151	0.346	2.033	0.514682

Table2: Monitoring data for 51 observations (Xt,Yt)

No.	X_t	Y_t	e_{Xt}	e_{Yt}	Hotelling T^2
1	14.8498	19.963	5.12973	1.02119	0.91592

2	11.0785	17.242	-2.03904	1.54197	1.85225
3	8.1424	11.869	-2.67391	-0.52364	0.77915
4	7.03365	11.6	-1.99104	0.60294	1.14299
5	8.67484	11.702	0.32671	-0.83271	1.27424
6	11.4983	15.081	2.1487	-0.44229	0.280616
7	10.0398	14.494	-1.03264	0.15952	0.133074
8	6.79646	11.333	-3.38604	0.39033	1.25318
9	6.94004	11.603	-1.26336	0.82952	1.22572
10	9.6339	12.773	1.34289	-0.71428	0.898015
11	13.6541	18.495	3.71926	0.71815	0.385109
12	17.4463	22.534	5.0584	0.56693	2.17794
13	14.6894	17.292	-0.01255	-2.27823	2.15758
14	11.6981	17.334	-1.32156	1.02873	0.937439
15	16.0649	20.263	4.87048	-0.13013	1.13374
16	10.826	14.579	-3.033	-0.98555	0.186723
17	9.32026	12.745	-1.34195	-0.80381	0.658145
18	10.4651	15.511	0.72163	0.96057	0.358845
19	5.41638	17.061	-5.0256	7.46	31.2008
20	12.2433	15.921	4.88207	-0.03475	0.246105
21	22.0627	27.179	10.5356	0.72133	6.28831
22	23.9243	29.128	6.40544	-0.14516	8.65909
23	21.0305	26.304	2.37572	-0.24834	5.30496
24	12.5512	17.328	-4.33782	-0.44572	0.168742
25	7.09018	10.02	-4.62477	-1.45987	2.01101
26	5.32633	7.6617	-3.0563	-1.52251	4.02275
27	8.62114	12.277	1.31482	-0.03859	0.662756
28	5.98238	11.199	-3.33444	1.20951	2.16352
29	4.52262	8.5559	-3.18403	0.28443	2.68066
30	9.69682	12.971	2.88092	-0.34538	0.73004
31	15.0966	18.451	5.12338	-0.7777	1.2181
32	18.2478	23.131	4.97965	0.22152	2.59025
33	15.9519	20.1	0.760898	-0.81316	1.08825
34	13.5524	17.185	-0.2376	-1.10083	0.483099
35	12.8579	17.305	0.53198	-0.05297	0.105075
36	15.8267	20.742	3.92465	0.47576	1.17349
37	11.222	14.831	-2.49172	-1.10705	0.263398
38	10.7933	13.2	-0.11051	-1.8632	1.94094
39	9.77218	14.538	-0.8701	0.53974	0.286646

40	13.3735	19.007	3.35434	1.49785	1.09341
41	10.2868	15.206	-1.92995	0.44244	0.289896
42	7.68878	12.602	-2.64441	0.74115	1.01933
43	10.8339	14.315	2.08599	-0.44718	0.384301
44	9.35976	13.115	-1.30728	-0.47404	0.403899
45	8.48772	11.907	-1.2798	-0.665	0.89113
46	12.514	16.082	3.27862	-0.44192	0.354617
47	16.6993	19.763	5.007	-1.34428	2.35491
48	12.1259	16.016	-2.12015	-0.91217	0.115966
49	10.1841	14.25	-1.27134	-0.28985	0.12498
50	6.72467	11.253	-3.54587	0.36829	1.28809
51	8.69571	12.351	0.536117	-0.17507	0.640824

ACKNOWLEDGMENTS

Support for this research was provided in part by the National Science Council of the Republic of China, grant No. NSC-91-2118-M-004-008.

REFERENCE

1. Abraham, B. and Kartha, C.P.(1979). "Forecast stability and control charts”. ASQC Technical Conference Transactions. American Society for Quality Control, Milwaukee, WI. Pp. 675-685.
2. Alwan, L.C. (1991),”Autocorrelations: Fixed and Versus Variable Control Limits”. Quality Engineering 4, pp.167-188.
3. Alwan, L.C. and Radson, D. (1992). “ Time-Series investigation of subsample mean chart”. IIE Transactions 24, pp.66-80.
4. Alwan, L.C. and Roberts, H. V. (1988). " Time-Series modeling for statistical process control”. Journal of Business and Economic Statistics 6, pp.87-95.
5. Berthouex, P. M., Hunter, W. G. and Pallesen, L. (1978). “ Monitoring sewage treatment plants: Some quality control aspects”. Journal of Quality Technology 10. pp.139-149.
6. Delves, L. M. and Mohamed, J.L. (1985). Computational Methods for Integral Equations. Cambridge University Press, New York, NY.
7. Dooley, K.J. Kapoor, S. G., Dessouky, M. I., and Devor, R. E. (1986). “ An integrated quality systems approach to quality and productivity improvement in continuous manufacturing processes”. Transactions of the ASME Journal of Engineering for Industry 108. pp. 322-327.
8. Ermer, D.S. (1980). "A control chart for dependent data". ASQC Technical Conference Transactions. American Society for Quality Control, Milwaukee, WI. pp.121-128.
9. Harris, Y. J. and Ross, W. H. (1991). “Statistical process control procedures for correlated

observations". The Canadian Journal of Chemical Engineering 69, pp. 48-57.

10. Kramer, H. and Schmid, W. (1997). " Control charts for time series". Nonlinear Analysis 30, pp. 4007-4016.
11. Lin, W.S. and Adams, B. M. (1996). " Combined control charts for forcast-based monitoring schemes". Journal of Quality Technology 28, pp. 289-301.
12. LONGNECKER, M. T. and RYAN, T. P. (1992). "Charting Correlated Process Data". Technical Report No.166. Department of Statistics. Texas A & M University. Collage Station, TX.
13. Lu, C. and Reynolds, M. (1999). "EWMA control charts for monitoring the mean of autocorrelated processes.", Journal of Quality Technology, Vol.31, pp.166-188.
14. Montgomery, D. C. (1996). *Introduction to Statistical Quality Control*, 3rd ed. John Wiley & Sons, New York, NY.
15. Montgomery,, D. C. and Mastrangelo, C. M. (1991). "Some Statistical Process Control Methods for Autocorrelated data". *Journal of Quality Technolog*y 23, pp. 179-193.
16. Padgett, C. S.; Thombs, L. A.; and Padgett, W.J. (1992). "On the α-risks for Shewhart Control Charts". *Communications in Statistics-Simulation and Computation* 21, pp. 1125-1147.
17. Runger, G. C.; Willemain, T.R.; and Prabhu, S. (1995). "Average Run Lengths for CUSUM Control Charts Applied to Residuals". *Communications in Statistics-Theory and Methods* 24, pp. 273-282.
18. Schmid, W. (1995). "On the Run Length of a Shewhart Chart for Correlated Data". *Statistical Papers* 36, pp. 111-130.
19. Schmid, W. (1997a). "CUSUM Control Schemes for Gaussian Processes". *Statistical Papers* 38, pp. 191-217.
20. Schmid, W. (1997b). "On EWMA Charts for Time Series" in *Frontiers of Statistical Quality Control* edited by H. J. Lenz and P.-Th. Wilrich. Physica-Verlag, Heidelburg.
21. Sshmid, W. and Schone, A. (1997). "Some Properties of the EWMA Control Chart in the Presence of Autocorrelation". *Annals of Statistic*s 25, pp. 1277-1283.
22. Timmer, D. H.; Pigantkiello. J. JR.; and Longnecker, M. (1998). "The Development and Evaluation of CUSUM-Based Control Charts for an AR(1) Process". *IIE Transactions* 30, pp. 525-534.
23. Vander Well, S. A. (1996). "Modeling Processes That Wander Using Moving Average Models". *Technometrics* 38, pp. 139-151.
24. Wade, R. and Woodall, W., "A Review and Analysis of Cause-Selecting Control Charts," *Journal of Quality Technology*, **25**,.161-169 (1993).
25. Wardell, D. G.; Moskowitz, H.; and Plante, R. D. (1992). "Control Charts in the Presence of Data Correlation". *Management Science* 38, pp. 1084-1105.
26. Wardell, D. G.; Moskowitz, H.; and Plante, R. D. (1994) "Run Length Distributions of Special-Cause Control Charts for Correlated Processes". *Technometrics* 36, pp. 3-17.
27. Yashchin, E. (1993). "Performance of CUSUM Control Schemes for Serially Correlated Observations". *Technometrics* 35, pp. 37-52.
28. Zhang, N. F. (1997). "Detection Capability of Residual Control Chart for Stationary Process

Data". *Journal of Applied Statistics* 24, pp. 363-380.

29. Zhang, G., "A New Type of Control Charts and a Theory of Diagnosis with Control Charts," *World Quality Congress Transactions, American Society for Quality Control, Milwaukee, WI*, 75-85 (1984).

CONTROL FOR THE AUTOCORRELATED OBSERVATIONS

Su-Fen Yang
Department of Statistics
National Chengchi University
Taipei, 116, Taiwan
E-mail: yang@nccu.edu.tw

Chung-Ming Yang
Department of Insurance
Ling-Tung College
Taichung, 408,Taiwan

ABSTRACT

The observations from the process output are always assumed independent when using a control chart to monitor a process. However, for many processes their observations are autocorrelated and including the measurement error due to the measurement instrument. This autocorrelation and measurement error can have a significant effect on the performance of the processes control. This paper considers the problem of monitoring the mean of a quality characteristic X on the first process and the mean of a quality characteristic Y on the second process, in which the observations X can be modeled as an ARMA model and observations Y can be modeled as an transfer function of X since the state of the second process is dependent on the state of the first process. To effectively distinguish and maintain the state of the two dependent processes with measurement errors, the Shewhart control chart of residual and the cause-selecting control chart based on residuals are proposed. The performance of the proposed control charts is evaluated by the rate of true alarm or false alarm. From numerical analysis, it shows that the performance of the proposed control charts is significantly influenced by the variation of measurement errors. Application of the proposed control charts is illustrated through a numerical example.

Key Words: *Autocorrelated observations, dependent processes, cause-selecting control chart, measurement error variation.*

1. INTRODUCTION

Control charts are first proposed by Shewhart (1931), and become effective tools for improving the process quality and productivity.

A basic assumption in applications of control charts is that observations from the process at different times are independent random variables. However, the independence assumption is often violated for processes in chemical and pharmaceutical industries. Observations from these processes are always autocorrelated. When the control charts developed under the independence assumption, the autocorrelated process results in decreasing the in-control average run length (ARL). For effective monitoring the autocorrelated processes, one popular developed approach is to constructing control charts using the residuals from the time series model to the process data (see Abraham and Kartha (1979), Alwan (1991), Alwan and Roberts(1988), Berthouex, Dooley, Kapoor, Dessouky and Delves (1986), Ermer (1980), Harris and Ross (1991), Montgomery (1996), Montgomery and Mastrangelo (1991), and Wardell, Moskowitz, and Plante (1992, 1994)). The properties of the proposed residual charts and their performance are investigated by Harris and Ross (1991), Longnecker and Ryan (1992),Yashchin (1993),Kramer and Schmid (1997), Schmid (1995),Lin and Adams (1996), Schmid (1997a), Padgett, Thombs and Padgett (1992), Runger, Willemain and Prabhu (1995), Vander Weil (1996), Timmer, Pignatiello and Longnecker (1998), Schmid (1997b) ,Zhang (1998), Schmid and Schone (1997), Alwan and Roberts (1988) and Lu and Reynolds (1999).

Much of the paper on the performance of control charts based on residuals has focused on the Shewhart control chart of residuals.

The above autocorrelated processes articles assume that the imprecise measurement devices on the process measurements are impossible. However, the performance of control charts and other statistical process control tools could be seriously affected when the process measurement includes the error due to the measurement instrument. The effect of measurement error on the operating characteristics of an $\overline{X}$ chart, in cases where only the process mean shifts, is discussed by Bennett (1954), Mizuno (1961), Abraham(1977), Mittag (1993) and Mittag and Stemmann (1993). Kanazuka (1986) and Mittag (1995) investigate the effect of measurement error on the power characteristics of the $\overline{X}$-R control chart where both the process mean and process spread change. Mittag and Stemann (1998) extend the results of Mittag (1995), referring to the $\overline{X}$-S control chart. Rahim (1985) analysis the effects of imprecise measurement devices on the design parameters of the economic $\overline{X}$ control chart. Yang (2002) investigates the effect of measurement error on the design parameters of the economic asymmetric $\overline{X}$ and S control charts.

The above control charts articles consider a single process control. Today, many industrial products are produced in several dependent processes. Consequently, it is not appropriate to monitor these processes by utilizing a control chart for each individual process. Zhang (1984) proposes the simple cause-selecting chart to monitor the second process of the two dependent processes. Wade and Woodall (1993) review the basic principles of the cause-selecting chart for two dependent processes and modify Zhang's approach, and give an example to illustrate the use of the individual X chart and the simple cause-selecting chart. It is shown that their approach is better than that of Zhang for the dependent processes control. However, the statistical process control approach to effectively distinguish and monitor the dependent processes for autocorrelated observations including the process measurement error due to the measurement instrument has not been addressed.

In this paper, process measurements with measurement error used in construction of Shewhart control chart of residuals and cause-selecting control chart are developed. The proposed control charts can be effectively used to distinguish which process mean is out of control, and the effect of imprecise measurement on the performance of the proposed control charts for monitoring the two dependent processes is also investigated. Finally, application of the proposed control charts is demonstrated through an example. Numerical examples illustrate the effects of imprecise measurement on the proposed control charts.

2. **PROBLEM STATEMENT**

A possible industrial situation is taken to illustrate the effects of imprecise measurement on the performance of the two proposed control charts, which are respectively used to monitor the process means for autocorrelated observations on two dependent processes. In a production system, suppose that there are two dependent processes, which may have two types of failure mechanisms. One type of the failure mechanism occurs only on the first process and shifts the mean of the quality variable (X), while the other type occurs only on the second process and shifts the mean of the quality variable (Y). The quality variable Y is influenced by the quality variable X since the processes are dependent. The measurement process is considered, and it has a variance for a measurement device is employed for later measurements. Some quality engineers would like to develop appropriate control charts to monitor the process means on the two dependent processes and consider the effects of imprecise measurement. The problem here is what is the process control policy under the measurement error? That is, what are the control charts, how the measurement error affects the performance of the proposed control charts.

The solutions of these problems are as follows.

(1) A measurement device having measurement dispersion is investigated and the model of measurement error for the first process is determined. An appropriate time series model of the quality variable X is determined, since its observations are autocorrelated. Based on the in-control distribution of residuals ($X - \hat{X}$, the individual residual control chart can be set up to monitor the process mean of the first process.

(2) A measurement device having measurement dispersion is investigated and the model of measurement error for the second process is determined. An appropriate transfer function for quality variable Y and quality variable X is determined, since the quality variable Y is influenced by the quality variable X and correlated on time. Based on the in-control distribution of residual ($Y - \hat{Y}$) of the transfer function, the cause-selecting control chart can be set up to monitor the process mean of the second process.

(3) The two proposed control charts are used to distinguish and detect the shifts of process mean on the two dependent processes, and the detecting ability of the proposed control charts for different measurement dispersion is calculated and compared. Based on the analysis results, appropriate control policy can be proposed by the quality engineers.

3. DESCRIPTION OF PROCESSES

Two types statistical control charts will be derived to effectively distinguish and monitor the two dependent processes for autocorrelated observations with measurement errors. Before describing how to derive the two statistical control charts, the assumptions of the two dependent production processes behavior are given as follows.

3.1**Assumptions and Notation**

Assumptions

(5) The production has two dependent processes, say the process 1 and the second process is called the process 2. The process 1 and the process 2 are dependent. So the quality variable X_T produced by the process 1 will affect the quality variable Y_T produced by the process 2. A pair of true observations (x_{Tt}, y_{Tt}) are sampled from the end of the process 2 every h time unit of sampling interval, $t = 1,2,3,....$.

(6) For autocorrelated true observations x_{Tt} at process 1, it is assumed that quality variable X_T can be written as an AR(1) model at time t with process mean ξ_X, that is

$$X_{Tt} = (1-\phi_1)\xi_X + \phi_1 X_{T(t-1)} + a_t, t = 1,2,...., \qquad (1)$$

where ϕ_1 is the AR parameter satisfying $|\phi_1|<1$. The a_t's are assumed to be independent normal random variables with mean 0 and variance σ_a^2. The starting value, X_{T0}, is assumed following a normal distribution with mean ε_X and variance $\sigma_{XT}^2 = \dfrac{\sigma_a^2}{1-\phi_1^2}$.

Since X_T affects Y_T, we assume the model relating the two variables can be written as a simple transfer function. That is

$$Y_{Tt} = C_Y^{'} + V_0^{'} X_{Tt} + V_1^{'} X_{T(t-1)} + N_t^{'}, \quad t = 1,2,...., \qquad (2)$$

where $C_Y^{'}$ is a constant and where $N_t^{'}$'s are assumed to be independent normal random variables with mean 0 and variance $\sigma^2_{N'}$.

(7) When one failure mechanism occurs only in the process 1, it will shift the mean of X_T. This will also cause the mean of Y_T shifts. When the other failure mechanism occurs only in the process 2, then it will shift the mean of Y_T and the mean of X_T is unchanged.

(4) It is assumed that the measurement process for X_T has a variance, $\sigma^2_{X\varepsilon}$, for a measurement device is employed for later measurements. That is, the distribution of the measurement error (ε_X) is illustrated as ε_X~N(0, $\sigma^2_{X\varepsilon}$). Hence, the distribution of the observed process quality variable (X) with measurement error (ε_X) is illustrated as X= $X_T + \varepsilon_X$.

(5) It is assumed that the measurement process for Y_T has a variance, $\sigma^2_{Y\varepsilon}$, for a measurement device is employed for later measurements. That is, the distribution of the measurement error (ε_Y) is illustrated as ε_Y~N(0, $\sigma^2_{Y\varepsilon}$). Hence, the distribution of the observed process quality variable (Y) with measurement error (ε_Y) is illustrated as Y=$Y_T + \varepsilon_Y$.

(6) Two control charts are proposed to monitor the two dependent processes effectively.

(7) The time to sampling and charting one item is very small and negligible.

3.2. The Time Series Model For Autocorrelated Processes

Time series model, especially AR(1) model, has been widely used to model many types of processes. For process 1, the AR(1) process observed with measurement errors is equivalent to an ARMA(1,1) process (Box, Jenkins, and Reinsel (1994). That is, $X = X_T + \varepsilon_X$, X_T is independent of the ε_X, and X is an ARMA(1,1) model. When the process 1 is in-control, the ARMA(1,1) model at time t is expressed as

$$X_t = (1-\phi_1)\xi_{X0} + \phi_1 X_{t-1} + \gamma_t - \theta_1 \gamma_{t-1}, \qquad (3)$$

where $\gamma_t \sim NID(0,\sigma_r^2)$, θ_1 is MA parameter, and ϕ_1 is AR parameter.

Parameters ϕ_1 ,θ_1 and σ_r^2 in the ARMA(1,1) model in terms of the parameters ϕ_1 , σ_a^2 and $\sigma_{X\varepsilon}^2$ in the AR(1) model plus random error model. If ϕ_1 >0 and $\sigma_{X\varepsilon}^2$>0, then the ARMA(1,1) model parameters θ_1 and σ_r^2 can be obtained from the AR(1) plus error parameters using

$$\theta_1 = \frac{\sigma_a^2 + (1+\phi_1^2)\sigma_{X\varepsilon}^2}{2\phi_1\sigma_{X\varepsilon}^2} - \frac{1}{2}\sqrt{\left(\frac{\sigma_a^2 + (1+\phi_1^2)\sigma_{X\varepsilon}^2}{\phi_1\sigma_{X\varepsilon}^2}\right)^2 - 4} \qquad (4)$$

and

$$\sigma_r^2 = \frac{\phi_1\sigma_{X\varepsilon}^2}{\theta_1} \qquad (5)$$

(Reynold, Arnold, and Baik (1996)).

If σ_a^2, $\sigma_{X\varepsilon}^2$ and ϕ_1 are known, then θ_1 and σ_r^2 are known. When the process 1 is in control the minimum mean square error forecast made at time t-1 for time t is

$$\hat{X}_t = (1-\hat{\phi}_1)\xi_{X0} + \hat{\phi}_1 X_{t-1} - \hat{\theta}_1 \gamma_{t-1}, \qquad (6)$$

$$e_{Xt} = X_t - \hat{X}_t \qquad (7)$$

e_{Xt} is the residual at time t, $e_{Xt} \sim N(0,\sigma_r^2)$.

Suppose that a failure mechanism would cause a step change from ξ_{X0} to ξ_{X1} in the process mean between time $t=\tau-1$ and τ. The expectations of the residual for various times are

$$E(e_{Xt}) = \begin{cases} 0 & t=\tau-1,\ \tau-2,\ldots, \\ \dfrac{\theta_1^l(\phi_1-\theta_1)-\phi_1+1}{1-\theta_1}(\xi_{X1}-\xi_{X0}) & t=\tau+l,\ l=0,1,2,\ldots. \\ \dfrac{1-\phi_1}{1-\theta_1}(\xi_{X1}-\xi_{X0}) & t=\tau+l,\ l\to\infty \end{cases} \qquad (8)$$

The residuals are uncorrelated and normally distributed with variance σ_a^2. We may find that the expectation of a residual after the shift occurs is a decreasing function of the time after the shift. Hence, the rate of a true alarm from the control chart of residuals on the process 1 is the lowest for the sample immediately after the shift, and this rate of a true alarm continually increases and converges to a constant over time as the forecast adapts to the shift.

In process 2, a linear transfer function also presents the relationship between Y and X since they are dependent over time. The transfer function plus random error (ε_Y) is expressed as

$$Y_t = C_Y + V_0 X_t + V_1 X_{(t-1)} + N_t + \varepsilon_{Yt}, \quad t = 1,2,.... \qquad (9)$$

where $N_t \sim NID(0, \sigma_N^2)$, and $\varepsilon_{Y\varepsilon} \sim NID(0, \sigma_{Y\varepsilon}^2)$.

When the process 2 is in control, the estimate for equation (9) is

$$\hat{Y}_t = \hat{C}_y + \hat{V}_0 X_t + \hat{V}_1 X_{t-1}, \qquad (10)$$

and the residual at time t is

$$e_{Yt} = Y_t - \hat{Y}_t, \qquad (11)$$

where $e_{Yt} \sim N(0, \sigma_N^2 + \sigma_{Y\varepsilon}^2)$.

Suppose that another failure mechanism would cause a step change from ξ_{Y0} to ξ_{Y1} in the process mean between time $t = \tau' - 1$ and τ'. The expectations of the residual for various times are

$$E(e_{Yt}) = \begin{cases} 0 & t = \tau' - 1,\ \tau' - 2,..., \\ \xi_{Y1} - \xi_{Y0} & t = \tau', \\ 0 & t = \tau' + l, l = 1,2,.... \end{cases} \qquad (12)$$

The residuals are uncorrelated and normally distributed with variance $\sigma_N^2 + \sigma_{Y\varepsilon}^2$. Note that the shift of process mean only appears at time τ' after the shift.

4. CONSTRUCTING CONTROL CHARTS

To effectively distinguish and monitor the two dependent processes, two Shewhart control charts based on residuals, e_X and e_Y, are constructed. The control limits of the Shewhart control charts are dependent on the in-control distribution of e_X and e_Y, respectively. Since the in-control distribution of e_X is $e_{Xt} \sim N(0, \sigma_r^2)$, hence the control limits of the Shewhart control chart based on e_X, using to monitor the process 1, are as follows.

$$UCL_{e_X} = 3\sigma_r$$

$$CL_{e_X} = 0$$

$$LCL_{e_X} = -3\sigma_r .$$

From equation (5), we know that σ_γ is a function of measurement error variation, $\sigma_{X\varepsilon}$, for process 1. Hence, the control limits of the Shewhart control chart of e_X include the variation of measurement error. It indicates that the performance of the Shewhart chart is influenced by the variation of measurement error.

If the parameter σ_r is unknown then $\frac{\overline{MR_{eX}}}{d_2}$ is replaced, where $\overline{MR_{eX}} = \frac{\sum_{t=2}^{m} MR_{eXt}}{m-1}$,

$MR_{eXt} = |\, e_{Xt} - e_{X(t-1)} \,|$, $t = 2,3,\ldots,m$, d_2 is the factor of constructing control charts.

For process 2, the in-control distribution of e_Y is $e_{Yt} \sim N(0, \sigma_N^2 + \sigma_{Y\varepsilon}^2)$, hence the control limits of the Shewhart control chart based on e_Y are as follows.

$$UCL_{e_Y} = 3\sqrt{(\sigma_N^2 + \sigma_{Y\varepsilon}^2)}$$

$CL_{e_Y} = 0$

$LCL_{e_Y} = -3\sqrt{(\sigma_N^2 + \sigma_{Y\varepsilon}^2)}$.

The control limits of the Shewhart control chart of e_Y include the variation of measurement error, e_{Y_t}, for process 2. It indicates that the performance of the Shewhart chart of e_Y is influenced by the variation of measurement error.

If the parameter $\sigma_N^2 + \sigma_{Y\varepsilon}^2$ is unknown then $\frac{\overline{MR_{eY}}}{d_2}$ is replaced, where $\overline{MR_{eY}} = \frac{\sum_{t=2}^{m} MR_{eYt}}{m-1}$,

$MR_{eYt} = | e_{Yt} - e_{Y(t-1)} |, \ t = 2,3,...,m.$

To monitor the two processes using the proposed control charts, a pair of observations (X_t, Y_t) is sampled at the end of the process 2. The plotted statistics e_X and e_Y are calculated, and plotted on the proposed control charts, respectively. If no one signal any control charts, it indicates the two processes are all in control. If only a signal from the Shewhart control chart of e_X, it indicates process 1 is out of control. The quality engineer has to search and remove the occurred special cause. If only a signal from the Shewhart control chart of e_Y, it indicates process 2 is out of control. The quality engineer has to search and remove the occurred special cause. If both signals from the Shewhart control chart of e_X and Shewhart control chart of e_Y, it indicates process 1 and process 2 are out of control. The quality engineer has to search and remove the special causes on the out-of-control processes.

5. MEASURING THE PERFORMANCE OF CONTROL CHARTS

The performance of the proposed control charts is evaluated by the rate of false alarm when the processes are in control, and the rate of true alarm (or power) when the processes are out of control. For process 1, the rate of false alarm for the Shewhart chart of e_X is 370.4, and the same to the Shewhart chart of e_Y at process 2. To monitor the two dependent processes, two developed charts are used simultaneously. Hence the rate of at least one false alarm for the two charts is 0.0054, that is 1-0.9973x0.9973. To calculate the rate of true alarm for the two charts, we have to compute the rates of true alarm from the Shewhart chart of e_X and Shewhart chart of e_Y, respectively. Let the rate of true alarm from the Shewhart chart of e_X at time t is Ps_t, and the rate of true alarm from the Shewhart chart of e_Y at time t is Pc_t, then

$$\begin{aligned} Ps_t &= \Pr(e_{Xt} > 3\sigma_\gamma \, or \, e_{Xt} < -3\sigma_\gamma \mid \frac{\theta_1^l(\phi_1-\theta_1)-\phi_1+1}{1\text{-}\theta_1}(\xi_{X1}-\xi_{X0})) \\ &= \Phi_S(-3-\frac{\frac{\theta_1^l(\phi_1-\theta_1)-\phi_1+1}{1\text{-}\theta_1}(\xi_{X1}-\xi_{X0})}{\sigma_\gamma})+\Phi_S(-3+\frac{\frac{\theta_1^l(\phi_1-\theta_1)-\phi_1+1}{1\text{-}\theta_1}(\xi_{X1}-\xi_{X0})}{\sigma_\gamma}),\ t=\tau+l,\ l=0,1,2,.... \end{aligned} \tag{13}$$

$$\begin{aligned} Pc_t &= \Pr(e_{Yt} > 3(\sigma_N+\sigma_{Y\varepsilon}) \, or \, e_{Yt} < -3(\sigma_N+\sigma_{Y\varepsilon}) \mid (\xi_{Y1}-\xi_{Y0})) \\ &= \Phi_C(-3-\frac{(\xi_{Y1}-\xi_{Y0})}{(\sigma_N+\sigma_{Y\varepsilon})})+\Phi_C(-3+\frac{(\xi_{Y1}-\xi_{Y0})}{(\sigma_N+\sigma_{Y\varepsilon})}),\ \ t=\tau' \end{aligned} \tag{14}$$

Alternatively,

$$\begin{aligned} Ps_t &= \Pr(e_{Xt} > 3\frac{\overline{MR}_{eX}}{d_2} \, or \, e_{Xt} < -3\frac{\overline{MR}_{eX}}{d_2} \mid \frac{\theta_1^l(\phi_1-\theta_1)-\phi_1+1}{1\text{-}\theta_1}(\xi_{X1}-\xi_{X0})) \\ &= \Phi_S(-3-\frac{d_2\frac{\theta_1^l(\phi_1-\theta_1)-\phi_1+1}{1\text{-}\theta_1}(\xi_{X1}-\xi_{X0})}{\overline{MR}_{eX}})+\Phi_S(-3+\frac{d_2\frac{\theta_1^l(\phi_1-\theta_1)-\phi_1+1}{1\text{-}\theta_1}(\xi_{X1}-\xi_{X0})}{\overline{MR}_{eX}}),\ t=\tau+l,\ l=0,1,2,.... \end{aligned} \tag{15}$$

$$Pc_t = \Pr(e_{Yt} > 3\frac{\overline{MR_{eY}}}{d_2} \ or \ e_{Yt} < -3\frac{\overline{MR_{eY}}}{d_2} | (\xi_{Y1} - \xi_{Y0})) = \Phi_C(-3 - \frac{d_2(\xi_{Y1} - \xi_{Y0})}{\overline{MR_{eY}}}) + \Phi_C(-3 + \frac{d_2(\xi_{Y1} - \xi_{Y0})}{\overline{MR_{eY}}}), \quad t = \tau' \qquad (16)$$

where Φ_S and Φ_C are the cumulative standard normal probabilities, respectively.

Note that the rate of true alarm from the Shewhart chart of e_X increasing and converges to a constant over time, but not for the Shewhart chart of e_Y. Hence the rates of true alarm from the two charts will be increasing over time, once the process 1 is out of control.

The rate of true alarm from the Shewhart chart of e_X or the Shewhart chart of e_Y is also dependent on the variation of measurement error. From the equations (13) and (14), we find that the lager the variation of measurement errors leads to the larger rate of true alarm from the proposed control chart when any one of the processes is out of control.

There are four situations for the out-of-control process. The four situations and the rate of true alarm of the two charts are described as follows.

(5) The process 1 is out of control after time $\tau - 1$ but the process 2 is in control. The rate of true alarm (Psc_t) of the two charts is

$$Psc_t = 1 - (1 - Ps_t), \qquad t = \tau + l, \ l = 0,1,2,3,....$$

(6) The process 1 is in control but the process 2 is out of control after time $\tau' - 1$. The rate of true alarm (Psc_t) of the two charts is

$$Psc_t = 1 - (0.9973), \qquad t = \tau'.$$

(7) The process 1 is out of control after time $\tau - 1$, and the process 2 is out of control after time $\tau' - 1$, where $\tau < \tau'$. The rate of true alarm (Psc_t) of the two charts is

$$Psc_t = 1-(1-Ps_t), \quad \tau \le t < \tau'$$
$$1-(1-Ps_t)(1-Pc_t), \quad t=\tau',$$
$$1-(1-Ps_t), \quad \tau'+l<t, \quad l=1,2,3....$$

(8) The process 1 is out of control after time $\tau''-1$, and the process 2 is out of control after time $\tau'-1$, where $\tau''>\tau'$. The rate of true alarm (Psc_t) of the two charts is

$$Psc_t = 1-(1-Pc_t) \qquad t=\tau',$$
$$1-(1-Ps_t) \qquad t \ge \tau''.$$

甲、 NUMERICAL EXAMPLES

6.1 Performance of the Proposed Control Charts

The performance of the proposed control charts under various variations measurement error will be evaluated by the rate of false alarm or the rate of true alarm. The rate of false alarm for in-control processes or the rate of true alarm for out-of-control processes depends on the values of parameters ϕ_1, σ_a^2, $\sigma_{Y\varepsilon}^2$, $\sigma_{X\varepsilon}^2$, σ_N^2, $\xi_{X1}-\xi_{X0}$ and $\xi_{Y1}-\xi_{Y0}$. We fix ϕ_1=0.6, σ_a^2=0.5, and σ_N^2=0.5, but let $\xi_{X1}-\xi_{X0}$=0, 1, 2, $\xi_{X1}-\xi_{X0}$=0, 1, 2, $\sigma_{X\varepsilon}^2$=0.05, 0.1, 0.4, and $\sigma_{Y\varepsilon}^2$=0.05, 0.1, 0.4, respectively. The numerical values of rate of false alarm or rate of true alarm for the proposed charts are illustrated in the Table 1 ((1)~(6)). We find that the rate of false alarm keeps 0.005393 when both the processes are in control no matter what the variation of measurement error. For fixed variation of measurement error in the in-control process 2, the larger variations of measurement errors in the out-of-control process 1 lead to small rates of true alarm. That is, the imprecision measurement may seriously affect the ability of the proposed control charts to detect process disturbances quickly. For fixed variation of measurement error in the process 2 and fixed variation of measurement error in the out-of-control process 1, the rate of true alarm of the proposed charts decreases over time. That is, the detection ability of the proposed charts is largest after a failure mechanism occurs, but decreases and converges to constant over time. For fixed variation of measurement error in the process 1, the larger variations of measurement errors in the out-of-control process 2 lead to small rates of true alarm of the proposed charts. For fixed variation of measurement error in the process 1 and fixed variation of measurement error in the out-of-control process 2, the rate of true alarm of the proposed charts increases dramatically right the time after the failure mechanism (time 4 in Table 1(3), and time 11 in Table 1(6)) occurs but back to normal from next time (time 5 in Table 1(3), and time 12 in Table 1(6)). All indicates that the imprecision measurement may seriously affect the ability of the proposed control charts to detect process disturbances quickly.

6.2 Application of the Proposed Control Charts

A quality engineer found that there is a large variability for the thickness of the thin golden films. From the quality data analysis, he found that the thickness of the thin golden films (Y) in the process 2 was primarily affected by gold concentration (X) in the process 1. Two independent machines, say machine 1 and machine 2, may failure and influence the mean of the gold concentration and thickness respectively. However, for measuring X and Y, the process measurement includes the error due to the measurement instrument. Since the unacceptable mean of the thickness may be influenced by machine 1 or gold concentration. To effectively maintain the variability of the gold concentration and thickness and distinguish which process is out of control, two control charts should be constructed as described before. The performance of the proposed control charts is influence by the variation of measurement errors, since the variation of measurement errors is included in the control limits. To investigate the effects of variation of measurement errors on the performance of the proposed control charts, the detecting ability of the charts is compared under various variations of measurement errors.

Consider the dependent processes in which the gold concentration X follow the model in equation (1) with ϕ_1=0.6, σ_a^2=0.5, and $\psi_X = \dfrac{\sigma_{X\varepsilon}^2}{\sigma_{X\varepsilon}^2 + \sigma_{XT}^2}$=0.25. This implies that 25% of the variability of the process 1 is due to the variation in ε_X. The thickness of the thin golden films Y follow the model in equation (8) with C_Y=3.16, V_0=1, V_1=0.1, σ_N^2=0.5, and $\psi_Y = \dfrac{\sigma_{Y\varepsilon}^2}{\sigma_{Y\varepsilon}^2 + \sigma_N^2}$=0.35. This implies that 35% of the variability of the process 2 is due to the variation in ε_Y. Using equations (4) and (5), the corresponding parameters, θ_1 and σ_γ^2 in the ARMA(1,1) model in equation (3) are 0.19135 and 0.4083, respectively. Consequently, the control limits of the Shewhart control charts of e_X and e_Y are as follows.

UCL_{eX}=1.667

CL_{eX}=0

LCL_{eX}=-1.667

UCL_{eY}=3.115

CL_{eY}=0

UCL_{eY}=-3.115

If e_X of the paired plotted points (e_X ,e_Y) larger than the upper control limit 1.667 or less than lower control limit –1.667, it indicates process 1 is out of control. If ε_Y of the paired plotted points (e_X ,e_Y) larger than the upper control limit 3.115 or less than lower control limit –3.115, it indicates process 2 is out of control. If e_X and e_Y fall outside both of the control limits of the two proposed charts, it indicates process 1 and process 2 are out of control. The quality engineer has to search and repair the occurred special causes to let the processes maintain in-control state.

From history data, we found that the means of processes shift to ξ_{X1}=2, and ξ_{Y1}=2.5, when machine 1 and machine 2 are out of control, respectively.

100 observed paired data, (X, Y), is collected from the end of the second process, and based on equations (7) and (11) the plotted data (e_X , e_Y) are obtained and plotted on the Shewhart chart of e_X (Figure 1) and Shewhart chart of e_Y (Figure 2), respectively. Figure 1 shows that the first 40 points are in-control, but there is a signal from the point 41 and after that the mean shifts. Figure 2 shows that the first 70 points are in-control, but there is a signal on the point 71 and after that the mean unchanged. All these indicate that (1) the first process and the second process are both in control before point 40, but the out-of-control machine 1 occurs between point 40 and 41 which leads to a signal from point 41 and a small shift on mean for the first process; (2) The second process is in control before point 70, but the out-of-control machine 2 occurs between point 70 and point 71 which leads to a signal on point 71. After point 71, the second process backs to in-control state.

7. CONCLUSIONS

In chemical and pharmaceutical industries, observations from these processes are always autocorrelated. We propose statistical process control approach to effectively distinguish and monitor two dependent processes for autocorrelated observations including the process measurement error due to the measurement instrument. The Shewhart control chart of residuals and cause-selecting control chart based on residuals can be effectively used to monitor and distinguish the states of the first and the second process, respectively. The performance of the proposed control charts under various variations of measurement errors is evaluated by the rates of true alarm. The larger variations of measurement errors lead to small rates of true alarm when at least one of the processes is out of control. It indicates that the imprecision measurement may seriously affect the ability of the proposed control charts to detect process disturbances quickly. Application of the proposed control charts is illustrated through a numerical example, it shows that the states of the two dependent processes can be detected effectively.

If the measurement errors are inherent in the measurement devices and cannot be avoided, then the better way to reduce the effect of measurement errors is to give proper calibration or regular maintenance of measurement devices. The adaptive statistical process control approach may give an improvement for increasing detection ability of the proposed control charts under various variations of measurement errors.

Table 1: Performance of the Proposed Charts under Various Variations of Measurement Errors

(1) $\xi_{x1}-\xi_{x0}=0$ and $\xi_{y1}-\xi_{y0}=0$ (no failure mechanisms occur in the processes)

	$\sigma_{y\varepsilon}^2=0.05$			$\sigma_{y\varepsilon}^2=0.1$			$\sigma_{y\varepsilon}^2=0.4$		
time	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$
1	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
2	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
3	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
4	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
5	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
10	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
20	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
30	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393

(2) $\xi_{x1}-\xi_{x0}=1$ and $\xi_{y1}-\xi_{y0}=0$ (failure mechanism 1 occurs between time 2 and 3)

	$\sigma_{y\varepsilon}^2=0.05$			$\sigma_{y\varepsilon}^2=0.1$			$\sigma_{y\varepsilon}^2=0.4$		
time	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$
1	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
2	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
3	0.049916	0.043503	0.025811	0.049916	0.043503	0.025811	0.049916	0.043503	0.025811
4	0.011075	0.011067	0.011047	0.011075	0.011067	0.011047	0.011075	0.011067	0.011047
5	0.010275	0.010273	0.010223	0.010275	0.010273	0.010223	0.010275	0.010273	0.010223
10	0.010233	0.010148	0.009692	0.010233	0.010148	0.009692	0.010233	0.010148	0.009692
20	0.010233	0.010148	0.009691	0.010233	0.010148	0.009691	0.010233	0.010148	0.009691
30	0.010233	0.010148	0.009691	0.010233	0.010148	0.009691	0.010233	0.010148	0.009691

(3) $\xi_{x1}-\xi_{x0}=0$ and $\xi_{y1}-\xi_{y0}=1$ (failure mechanism 2 occurs between time 3 and 4)

	$\sigma_{y\varepsilon}^2=0.05$			$\sigma_{y\varepsilon}^2=0.1$			$\sigma_{y\varepsilon}^2=0.4$		
time	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$
1	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
2	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
3	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
4	0.029731	0.029731	0.029731	0.024225	0.024225	0.024225	0.01487	0.01487	0.01487
5	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
10	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
20	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
30	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393

(4) $\xi_{x1}-\xi_{x0}=2$ and $\xi_{y1}-\xi_{y0}=0$ (failure mechanism 1 occurs between time 2 and 3)

	$\sigma_{y\varepsilon}^2=0.05$			$\sigma_{y\varepsilon}^2=0.1$			$\sigma_{y\varepsilon}^2=0.4$		
time	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$
1	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
2	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
3	0.367704	0.317247	0.16444	0.367704	0.317247	0.16444	0.367704	0.317247	0.16444

4	0.038843	0.037455	0.036812	0.038843	0.037455	0.046812	0.038843	0.037455	0.036812
5	0.033154	0.033137	0.032792	0.033154	0.033137	0.032792	0.033154	0.033137	0.032792
10	0.032859	0.032272	0.02916	0.032859	0.032272	0.02916	0.032859	0.032272	0.02916
20	0.032859	0.032272	0.029157	0.032859	0.032272	0.029157	0.032859	0.032272	0.029157
30	0.032859	0.032272	0.029157	0.032859	0.032272	0.029157	0.032859	0.032272	0.029157

(5) $\xi_{x1}-\xi_{x0}=0$ and $\xi_{y1}-\xi_{y0}=2$ (failure mechanism 2 occurs between time 3 and 4)

	$\sigma_{y\varepsilon}^2=0.05$			$\sigma_{y\varepsilon}^2=0.1$			$\sigma_{y\varepsilon}^2=0.4$		
time	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$
1	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
2	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
3	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
4	0.19953	0.19953	0.19953	0.150188	0.150188	0.150188	0.068429	0.068429	0.068429
5	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
10	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
20	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
30	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393

(6) $\xi_{x1}-\xi_{x0}=1$ and $\xi_{y1}-\xi_{y0}=2$ (failure mechanism 1 occurs between time 2 and 3, and failure mechanism 2 occurs between time 10 and 11)

	$\sigma_{y\varepsilon}^2=0.05$			$\sigma_{y\varepsilon}^2=0.1$			$\sigma_{y\varepsilon}^2=0.4$		
time	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$	$\sigma_{x\varepsilon}^2=0.05$	$\sigma_{x\varepsilon}^2=0.1$	$\sigma_{x\varepsilon}^2=0.4$
1	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
2	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393	0.005393
3	0.052482	0.046085	0.028442	0.052482	0.046085	0.028442	0.052482	0.046085	0.028442
4	0.013745	0.013236	0.013114	0.013745	0.013236	0.013114	0.013745	0.013236	0.013114
5	0.012948	0.012945	0.012895	0.012948	0.012945	0.012895	0.012948	0.012945	0.012895
10	0.012905	0.012821	0.012366	0.012905	0.012821	0.012366	0.012905	0.012821	0.012366

11	0.207721	0.207653	0.207288	0.158884	0.158812	0.158424	0.077962	0.077883	0.077457
20	0.012905	0.012821	0.012365	0.012905	0.012821	0.012365	0.012905	0.012821	0.012365
30	0.012905	0.012821	0.012365	0.012905	0.012821	0.012365	0.012905	0.012821	0.012365

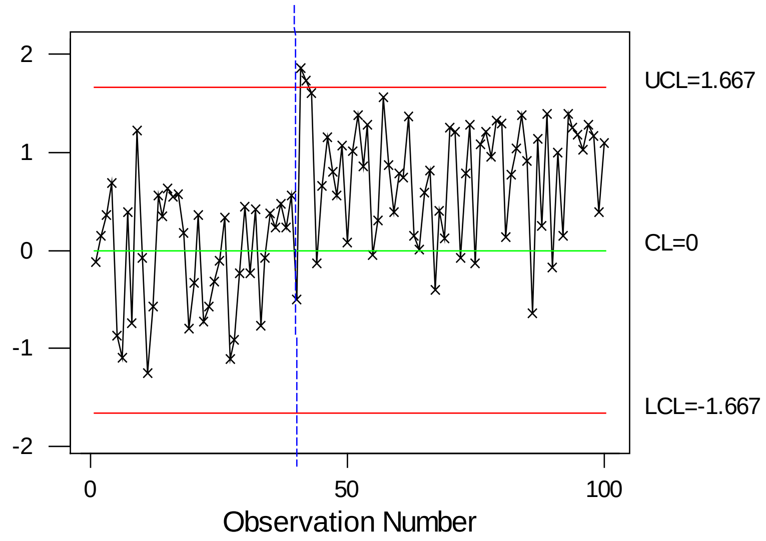

Figure 1: Shewhart chart of e_X

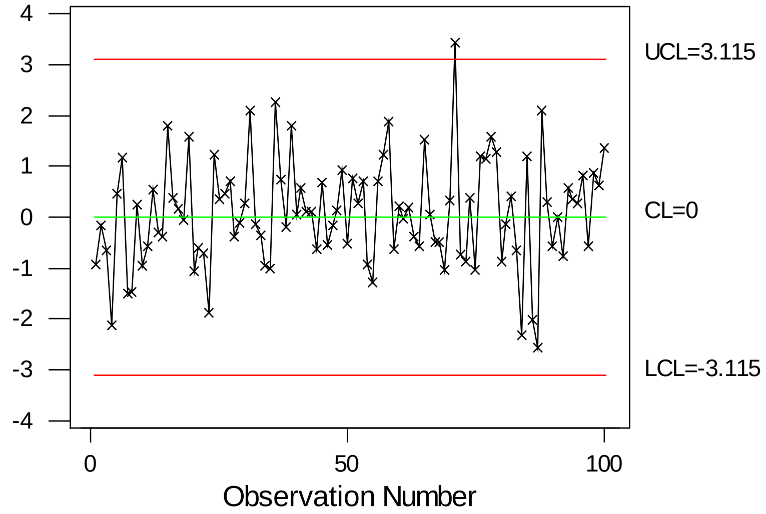

Figure 2: Shewhart chart of e_Y

ACKNOWLEDGMENTS

Support for this research was provided in part by the National Science Council of the Republic of China, grant No. NSC-91-2118-M-004-008.

REFERENCE

1. Abraham, B. and Kartha, C.P.(1979). "Forecast stability and control charts". ASQC Technical Conference Transactions. American Society for Quality Control, Milwaukee, WI. Pp. 675-685.
2. Alwan, L.C. (1991),"Autocorrelations: Fixed and Versus Variable Control Limits". Quality Engineering 4, pp.167-188.
3. Alwan, L.C. and Radson, D. (1992). " Time-Series investigation of subsample mean chart". IIE Transactions 24, pp.66-80.
4. Alwan, L.C. and Roberts, H. V. (1988). " Time-Series modeling for statistical process control". Journal of Business and Economic Statistics 6, pp.87-95.
5. Berthouex, P. M., Hunter, W. G. and Pallesen, L. (1978). " Monitoring sewage treatment plants: Some quality control aspects". Journal of Quality Technology 10. pp.139-149.
6. Delves, L. M. and Mohamed, J.L. (1985). Computational Methods for Integral Equations. Cambridge University Press, New York, NY.
7. Dooley, K.J. Kapoor, S. G., Dessouky, M. I., and Devor, R. E. (1986). " An integrated quality systems approach to quality and productivity improvement in continuous manufacturing processes". Transactions of the ASME Journal of Engineering for Industry 108. pp. 322-327.
8. Ermer, D.S. (1980). "A control chart for dependent data". ASQC Technical Conference Transactions. American Society for Quality Control, Milwaukee, WI. pp.121-128.
9. Harris, Y. J. and Ross, W. H. (1991). "Statistical process control procedures for correlated observations". The Canadian Journal of Chemical Engineering 69, pp. 48-57.
10. Kramer, H. and Schmid, W. (1997). " Control charts for time series". Nonlinear Analysis 30, pp. 4007-4016.
11. Lin, W.S. and Adams, B. M. (1996). " Combined control charts for forcast-based monitoring schemes". Journal of Quality Technology 28, pp. 289-301.
12. LONGNECKER, M. T. and RYAN, T. P. (1992). "Charting Correlated Process Data". Technical Report No.166. Department of Statistics. Texas A & M University. Collage Station, TX.
13. Lu, C. and Reynolds, M. (1999). "EWMA control charts for monitoring the mean of autocorrelated processes.", Journal of Quality Technology, Vol.31, pp.166-188.
14. Montgomery, D. C. (1996). *Introduction to Statistical Quality Control*, 3rd ed. John Wiley & Sons, New York, NY.
15. Montgomery,, D. C. and Mastrangelo, C. M. (1991). "Some Statistical Process Control Methods for Autocorrelated data". *Journal of Quality Technolog*y 23, pp. 179-193.

16. Padgett, C. S.; Thombs, L. A.; and Padgett, W.J. (1992). "On the α-risks for Shewhart Control Charts". *Communications in Statistics-Simulation and Computation* 21, pp. 1125-1147.
17. Runger, G. C.; Willemain, T.R.; and Prabhu, S. (1995). "Average Run Lengths for CUSUM Control Charts Applied to Residuals". *Communications in Statistics-Theory and Methods* 24, pp. 273-282.
18. Schmid, W. (1995). "On the Run Length of a Shewhart Chart for Correlated Data". *Statistical Papers* 36, pp. 111-130.
19. Schmid, W. (1997a). "CUSUM Control Schemes for Gaussian Processes". *Statistical Papers* 38, pp. 191-217.
20. Schmid, W. (1997b). "On EWMA Charts for Time Series" in *Frontiers of Statistical Quality Control* edited by H. J. Lenz and P.-Th. Wilrich. Physica-Verlag, Heidelburg.
21. Shewhart, W. (1931), Economic Control of Quality of Manufactured Product, D. Van Nostrand Company, Inc.
22. Sshmid, W. and Schone, A. (1997). "Some Properties of the EWMA Control Chart in the Presence of Autocorrelation". *Annals of Statistic*s 25, pp. 1277-1283.
23. Timmer, D. H.; Pigantkiello. J. JR.; and Longnecker, M. (1998). "The Development and Evaluation of CUSUM-Based Control Charts for an AR(1) Process". *IIE Transactions* 30, pp. 525-534.
24. Vander Well, S. A. (1996). "Modeling Processes That Wander Using Moving Average Models". *Technometrics* 38, pp. 139-151.
25. Wade, R. and Woodall, W., "A Review and Analysis of Cause-Selecting Control Charts," *Journal of Quality Technology*, **25**,.161-169 (1993).
26. Wardell, D. G.; Moskowitz, H.; and Plante, R. D. (1992). "Control Charts in the Presence of Data Correlation". *Management Science* 38, pp. 1084-1105.
27. Wardell, D. G.; Moskowitz, H.; and Plante, R. D. (1994) "Run Length Distributions of Special-Cause Control Charts for Correlated Processes". *Technometrics* 36, pp. 3-17.
28. Yashchin, E. (1993). "Performance of CUSUM Control Schemes for Serially Correlated Observations". *Technometrics* 35, pp. 37-52.
29. Zhang, N. F. (1997). "Detection Capability of Residual Control Chart for Stationary Process Data". *Journal of Applied Statistic*s 24, pp. 363-380.
30. Zhang, G., "A New Type of Control Charts and a Theory of Diagnosis with Control Charts," *World Quality Congress Transactions, American Society for Quality Control, Milwaukee, WI*, 75-85 (1984).

EFFECT OF MEASUREMENT ERROR ON TWO DEPENDENT PROCESSES CONTROL

Su-Fen Yang Han-Wei Ho
Department of Statistics
National Chengchi University
Taipei, 116, Taiwan
e-mail: yang@nccu.edu.tw

ABSTRACT

The presence of imprecise measurement may seriously affect the efficiency of process control and production cost. For two dependent processes, the quality variable in first process may influence the quality variable in the second process. To effectively monitor and distinguish the state of the two dependent processes with imprecise measurement, EWMA control chart and cause-selecting control chart are constructed including measurement errors. The EWMA control chart including imprecise measurement is constructed to monitor the small shift in the first process mean. The cause-selecting control chart including imprecise measurement is constructed to monitor the small shift in the second process mean. The effects of imprecise measurement on the performance of the two proposed control charts are examined for the case where the mean of each process may be changed by the occurred special cause. The performance of the proposed control charts is measured by the average run length. From sensitivity analysis, we found that the larger variation of imprecise measurement leads to larger average run length when the processes are out of control. It shows that the imprecision measurement may seriously affect the ability of the proposed control charts to detect processes disturbances quickly.

Keywords: Imprecise measurement; dependent processes; average run length.

1. INTRODUCTION

Process measurements are used in construction of control charts. The performance of control charts and other statistical process control tools could be seriously affected when the process measurement includes the error due to the measurement instrument.

The effect of measurement error on the operating characteristics of an $\overline{X}$ chart, in cases where only the process mean shifts, is discussed by Bennett (1954), Mizuno (1961), Abraham(1977), Mittag (1993) and Mittag and Stemmann (1993). Kanazuka (1986) and Mittag (1995) investigate the power of the $\overline{X}$-R control charts where both the process mean and process spread change. Mittag and Stemann (1998) extend the results of Mittag (1995), referring to the $\overline{X}$-S control chart. Rahim (1985) analysis the effects of imprecise measurement devices on the design parameters of the economic $\overline{X}$ control chart. Linna and Woodall (2001) address the effect of measurement error on the performance of $\overline{X}-S^2$ charts using a linear covariate. A criterion is suggested for the selection of the covariate. Yang (2002) investigates the effect of measurement error on the design parameters of the economic asymmetric $\overline{X}$ and S control charts. However, all the above papers discuss only a single process. In reality, it always takes several different processes to transform the raw materials into products. If the processes are mutually independent or uncorrelated, then we can use separate control charts to monitor the processes. However, the processes are often dependent. The former process will have influence on the latter process. If we apply the control charts to the observations at each process without removing the effect of former process then the state of the latter process might be misjudged. Zhang (1984) proposes the cause-selecting control chart to monitor the state of the second process. The cause-selecting control chart effectively distinguishes whether the second process is out of control. Wade and Woodall (1993) review the concepts of cause-selecting approach and propose using prediction limits instead of

Zhang's control limits of cause-selecting control chart. Yang and Chen (2003) propose economic X and cause-selecting control charts to monitor two dependent processes under two failure mechanisms. However, the performance measurement of two dependent processes control when the measurement errors existed has not been addressed. In this article, we derive the process model to consider the measurement errors on two dependent processes. We propose a EWMA control chart based on observations to monitor the small shift mean in the first process and a cause-selecting control chart based on residuals to monitor the small shift mean in the second process. The effect of measurement errors on the constructed EWMA and cause-selecting control charts is investigated. The performance of the proposed control charts is measured by average run length (ARL). It displays that the imprecise measurement will seriously affect the detection ability of the proposed control charts compared with proposed control charts excluding measurement errors. This advises manager to propose some appropriate strategies for measurement devices. Furthermore, applications of the proposed control charts are illustrated through an example.

2. DESCRIPTION OF TWO DEPENDENT PROCESSES

In this paper, it is assumed that the processes means may be shift when any special cause occurs in the processes. The two dependent production processes are described under the following assumptions.

2.1 Assumptions and Notation

Assumptions:

1. There are two dependent production processes, say the first process and the second process. The first process influences the second process since they are dependent.
2. Let X_T represent the true but unknown quality characteristic of interest for the first process and it is assumed that X_T follows the normal distribution with mean μ_x and variance σ_x^2 when the process is in statistical control. That is, when the first process is in control, $X_T \sim N\left(\mu_x,\ \sigma_x^2\right)$.

   Similarly, let Y_T stand for the true but unknown quality characteristic of interest for the second process. Since the first process influences the second process, Y_T is influenced by X_T. Hence, The relationship between Y_T and X_T can be expressed by a regression model, $Y_T \mid X_T = f(x_T) + \varepsilon_T$. The unconditional Y_T follows the normal distribution with mean μ_y and variance σ_y^2 when the process is in statistical control. That is, when the second process is in control, $Y_T \sim N(\mu_y, \sigma_{\mathrm{y}}^2)$.
3. There exist measurement errors due to measurement devices employed. The measurement process for the first process and the second process is assumed to increase the process variance by $\delta_x^2\sigma_x^2$ and $\delta_y^2\sigma_y^2$, respectively, where $\delta_x \neq 0$, $\delta_y \neq 0$. That is, the distribution of measurement error (ε_x) on the first process can be described as $\varepsilon_x \sim N\left(0,\ \delta_x^2\sigma_x^2\right)$, while the distribution of measurement error (ε_y) on the second process can be illustrated as $\varepsilon_y \sim N\left(0,\ \delta_y^2\sigma_y^2\right)$, where ε_x and ε_y are independent. In addition, it is also assumed that Y_T and ε_x, X_T and ε_y are also independent. The observed in-coming quality from the first process and out-going quality from the second process are defined as X_o and Y_o,

respectively, where $X_o = X_T + \varepsilon_x$ and $Y_o = Y_T + \varepsilon_y$. Hence, their distributions are illustrated as follows.

$$\begin{aligned} X_o &= X_T + \varepsilon_x \sim N\left(\mu_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right) \\ Y_o &= Y_T + \varepsilon_y \sim N\left(\mu_y,\ \sigma_y^2 + \delta_y^2\sigma_y^2\right) \end{aligned} \quad (1)$$

4. There is a special cause, say AC_1 that may only occur in the first process and shift the process mean from μ_x to $\mu_x + \delta_{(10)}\sigma_x$, $\delta_{(10)} \neq 0$. Another special cause, say AC_2, may only occur in the second process and shifts the process mean from μ_y to $\mu_y + \delta_{(01)}\sigma_y$, $\delta_{(01)} \neq 0$.

5. At the end of second process, a pair of observations ($X_{o_{(ijk)}}$, $Y_{o_{(ijk)}}$) are randomly drawn at a fixed time interval with sample size n = 1, where i is the indicator of the condition if AC_1 occurs in the first process, i = 0 or 1; j is the indicator of the condition if AC_2 occurs in the second process, j = 0 or 1, and k denote the number of samples taken, $k = 1, \ldots \infty$.

6. Since EWMA control chart is an effective control chart to detect small shifts of process mean when the sample size is one (Montgomery (2002)), it is thus constructed based on the observations, X_o, to control the first process. To effectively detect the state of the second process, the effects of X_o should be removed from Y_o (that is errors or residuals). We thus construct the cause-selecting control chart based on errors or residuals to control the second process.

Notation:

α_{e_1} : the probability that the EWMA chart based on observations gives a signal given the first process is in control.

β_{e_1} : the probability that the EWMA chart based on observations does not alert a signal given the first process is out of control.

α_{e_2} : the probability that the cause-selecting control chart based on residuals gives a signal given the second process is in control.

β_{e_2} : the probability that the cause-selecting control chart based on residuals does not alert a signal given the second process is out of control.

2.2 The Relationships Between X_o and Y_o

The model relating X_o and Y_o can take many forms. Let *i* represent the state of the first process, where *i* = 1 for the out-of-control first process, and *i* =0 for the in-control first process. Let *j* stands for the state of the second process, where *j* = 1 for the out-of-control second process, and *j* = 0 for the in-control second process. When pairs of observations ($X_{o_{(ijk)}}$, $Y_{o_{(ijk)}}$) are drawn, we assume that the regression model is suitable for describing the relationships between X_o and Y_o, namely,

$$Y_{o_{(ijk)}} \mid X_{o_{(ijk)}} = \beta_0 + \beta_1 X_{o_{(ijk)}} + \varepsilon_k + \varepsilon_Y \qquad (2)$$

where random errors and and $\varepsilon_y \sim N\left(0,\ \delta_y^2 \sigma_y^2\right)$.

The errors, $\varepsilon_k + \varepsilon_y$, are the effects of X_o removed from Y_o . If the regression model is unknown, the residuals, $e = Y_0 - \hat{Y}_0$, are used to replace $\varepsilon_k + \varepsilon_y$.

3. **THE POSSIBLE DISTRIBUTIONS OF X_o AND Y_o**

The construction of the EWMA chart based on in-coming quality and the cause-selecting control chart based on residuals are based on the distributions of the plotted statistics EWMA and residuals, respectively. There are four possible distributions of in-coming quality and out-going quality based on the possible states of the two dependent processes.

3.1 The First and The Second Processes Are Both In Control

The distributions of the true in-coming quality and measurement error on the in-control first process are assumed as follows.

$X_{T_{(00k)}} \sim N\left(\mu_x,\ \sigma_x^2\right)$ and $\varepsilon_x \sim N\left(0,\ \delta_x^2\sigma_x^2\right)$.

Therefore, the in-control distribution of the observed in-coming quality is

$$X_{o_{(00k)}} = X_{T_{(00k)}} + \varepsilon_x \sim N\left(\mu_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right) \tag{3}$$

From equation (2), the in-control distribution of the observed out-going quality, given the in-control observed in-coming quality, is

$$Y_{o_{(00k)}} \mid X_{o_{(00k)}} \sim N\left(\beta_0 + \beta_1 X_{o_{(00k)}},\ \sigma_{y|x}^2 + \delta_y^2\sigma_y^2\right) \tag{4}$$

The in-control expectation and variance of the observed out-going quality can be derived as follows.

$$\mathrm{E}\left(Y_{o_{(00k)}}\right) = \mathrm{E}\left(\mathrm{E}\left(Y_{o_{(00k)}} \mid X_{o_{(00k)}}\right)\right) = \mathrm{E}\left(\beta_0 + \beta_1 X_{o_{(00k)}}\right) = \beta_0 + \beta_1\mu_x$$

$$\begin{aligned}\mathrm{Var}\left(Y_{o_{(00k)}}\right) &= \mathrm{Var}\left(\mathrm{E}\left(Y_{o_{(00k)}} \mid X_{o_{(00k)}}\right)\right) + \mathrm{E}\left(\mathrm{Var}\left(Y_{o_{(00k)}} \mid X_{o_{(00k)}}\right)\right) = \mathrm{Var}\left(\beta_0 + \beta_1 X_{o_{(00k)}}\right) + \mathrm{E}\left(\sigma_{y|x}^2 + \delta_y^2\sigma_y^2\right) \\ &= \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma_{y|x}^2 + \delta_y^2\sigma_y^2\end{aligned}$$ Th

erefore, the distribution of the in-control observed out-going quality is expressed as

$$Y_{o_{(00k)}} \sim N\left(\beta_0 + \beta_1\mu_x,\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma_{y|x}^2 + \delta_y^2\sigma_y^2\right) \tag{5}$$

3.2 AC_1 Occurs in the First Process but the Second Process is In Control

When the first process is influenced by AC_1, the out-of-control distribution of the observed in-coming quality is

$$X_{o_{(10k)}} \sim N\left(\mu_x + \delta_{(10)}\sigma_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right). \qquad (6)$$

The out-of-control in-coming quality would influence the out-going quality, hence the distribution of the observed out-going quality is

$$Y_{o_{(10k)}} \sim N\left(\beta_0 + \beta_1\mu_x + \delta_{(10)}\sigma_X,\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma_{y|x}^2 + \delta_y^2\sigma_y^2\right). \qquad (7)$$

3.3 AC_2 Occurs in the Second Process but the First Process is In Control

When the second process is influenced by AC_2, the distribution of the observed in-coming quality is unchanged since AC_2 will not influence the first process. That is, distribution of the in-going quality is

$$X_{o_{(01k)}} \sim N\left(\mu_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right). \qquad (8)$$

When the second process is influenced by AC_2, the mean of the observed out-going quality, given the in-control observed in-coming quality, shifts to $\delta_{(01)}\sigma_y$.

Hence, the out-of-control distribution of the out-going quality is

$$Y_{(01k)} \sim N\left(\beta_0 + \beta_1\mu_x + \delta_{(01)}\sigma_y,\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma_{y|x}^2 + \delta_y^2\sigma_y^2\right). \qquad (9)$$

3.4 AC_1 Occurs in the First Process and AC_2 Occurs in the Second Process

When the first process is influenced by AC_1, the out-of-control distribution of the observed in-coming quality is

$$X_{o_{(11k)}} \sim N\left(\mu_x + \delta_{(10)}\sigma_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right). \qquad (10)$$

When the second process is influenced by AC_2, the mean of the observed out-going quality, given the out-of-control observed in-coming quality, shifts to $\beta_1\delta_{(10)}\sigma_X + \delta_{(01)}\sigma_y$.

Therefore, the out-of-control distribution of the out-going quality is

$$Y_{o_{(11k)}} \sim N\left(\beta_0 + \beta_1\left(\mu_x + \delta_{(10)}\sigma_x\right) + \delta_{(01)}(\sqrt{\sigma^2_{y|x} + \delta^2_y\sigma^2_y}),\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma^2_{y|x} + \delta^2_y\sigma^2_y\right). \quad (11)$$

The possible distributions for the four states of processes are summaries in Table 1.

Table 1: The possible distributions for the four states of the processes

State	AC_1	AC_2	Distribution of X_o	Distribution of Y_o
1	No	No	$N\left(\mu_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right)$	$N\left(\beta_0 + \beta_1\mu_x,\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma^2_{y\|x} + \delta^2_y\sigma^2_y\right)$
2	Yes	No	$N\left(\mu_x + \delta_{(10)}\sigma_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right)$	$N\left(\beta_0 + \beta_1\mu_x + \delta_{(10)}\sigma_x,\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma^2_{y\|x} + \delta^2_y\sigma^2_y\right)$
3	No	Yes	$N\left(\mu_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right)$	$N\left(\beta_0 + \beta_1\mu_x + \delta_{(01)}\sigma_y,\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma^2_{y\|x} + \delta^2_y\sigma^2_y\right)$
4	Yes	Yes	$N\left(\mu_x + \delta_{(10)}\sigma_x,\ \left(1+\delta_x^2\right)\sigma_x^2\right)$	$N\left(\beta_0 + \beta_1\left(\mu_x + \delta_{(10)}\sigma_x\right) + \delta_{(01)}\sqrt{\sigma^2_{y\|x} + \delta^2_y\sigma^2_y},\ \beta_1^2\left(1+\delta_x^2\right)\sigma_x^2 + \sigma^2_{y\|x} + \delta^2_y\sigma^2_y\right)$

4. CONSTRUCTIONS OF EWMA CONROL CHART AND CAUSE-SELECTING CONTROL CHART

To distinguish AC_2 that occurs in the second process from AC_1 that occurs in the first process, we construct the EWMA control chart based on the distribution of the observed in-coming quality and the cause-selecting control chart based on the distribution of the residuals to diagnose the states of the first process and the second process.

4.1 EWMA Control Chart for the First Process

When the first process is in control, the distribution of X_o, equation (3), is used to construct the EWMA control chart.

The plotted statistic, $Q_{(00k)}$, of the EWMA control chart is defined as

$$Q_{(00k)} = \lambda_1 X_{o_{(00k)}} + (1-\lambda_1) Q_{00(k-1)} \quad k = 1,\ldots, m, \qquad (12)$$

where λ_1 is a constant and $0 < \lambda_1 \le 1$.

Let the starting value $Q_{(000)}$ be the first process mean, μ_x.

Hence,

$$\begin{aligned}
\mathrm{E}\left(Q_{(00k)}\right) &= \mathrm{E}\left(\lambda_1 X_{o_{(00k)}} + (1-\lambda_1) Q_{00(k-1)}\right) = \mathrm{E}\left(\lambda_1 \sum_{l=0}^{k-1} (1-\lambda_1)^l X_{o_{(00k)}} + (1-\lambda_1)^k Q_{(000)}\right) \\
&= \left(\left(1-(1-\lambda_1)^k\right) + (1-\lambda_1)^k\right) \mathrm{E}\left(Q_{(000)}\right) = \mu_x \\
\mathrm{Var}\left(Q_{(00k)}\right) &= \mathrm{Var}\left(\lambda_1 X_{o_{(00k)}} + (1-\lambda_1) Q_{00(k-1)}\right) = \mathrm{Var}\left(\lambda_1 \sum_{l=0}^{k-1} (1-\lambda_1)^l X_{o_{(00k)}} + (1-\lambda_1)^k Q_{(000)}\right) \\
&= \lambda_1^2 \sum_{l=0}^{k-1} (1-\lambda_1)^{2l} \mathrm{Var}\left(X_{o_{(00k)}}\right) = \frac{\lambda_1}{2-\lambda_1}\left(1-(1-\lambda_1)^{2k}\right)\left(\left(1+\delta_x^2\right)\sigma_x^2\right)
\end{aligned}$$

That is, the distribution of the plotted statistic is

$$Q_{(00k)} \sim N\left(\mu_x,\ \frac{\lambda_1}{2-\lambda_1}\left(1-(1-\lambda_1)^{2k}\right)\left(\left(1+\delta_x^2\right)\sigma_x^2\right)\right). \qquad (13)$$

The control limits of the EWMA control chart are thus constructed as follows.

$$\begin{aligned}
\text{UCL} &= \mu_x + k_1 \sqrt{\frac{\lambda_1}{2-\lambda_1}\left(1-(1-\lambda_1)^{2k}\right)\left(\left(1+\delta_x^2\right)\sigma_x^2\right)} \\
\text{CL} &= \mu_x \\
\text{LCL} &= \mu_x - k_1 \sqrt{\frac{\lambda_1}{2-\lambda_1}\left(1-(1-\lambda_1)^{2k}\right)\left(\left(1+\delta_x^2\right)\sigma_x^2\right)}
\end{aligned} \qquad (14)$$

When $k \to \infty$, the control limits converges to $\mu_x \pm k_1 \sqrt{\frac{\lambda_1}{2-\lambda_1}\left(\left(1+\delta_x^2\right)\sigma_x^2\right)}$

4.2 Cause-Selecting Control Chart for the Second Process

Since the out-going quality Y_o is dependent on in-coming quality X_o, the real state of the second process cannot be identified until the effect of in-coming quality is removed. That is the real state of the second process can be specified by the cause-selecting values, or residuals.

The cause-selecting control chart is thus constructed by the in-control distribution of the cause-selecting values. Suppose the statistically significant fitted regression model between X_o and Y_o is

$$\hat{Y}_{o_{(ijk)}} = \hat{\beta}_0 + \hat{\beta}_1 X_{o_{(ijk)}} \qquad (15)$$

The cause-selecting values are $e_{(ijk)} = Y_{o_{(ijk)}} - \hat{Y}_{o_{(ijk)}}$.

Under the in-control process, the distribution of the cause-selecting values is

$$e_{(00k)} = Y_{o_{(00k)}} - \hat{Y}_{o_{(00k)}} \sim N\left(0,\ \sigma_e^2\right), \qquad (16)$$

where

$$\sigma_e^2 = \left(1 - \left(\frac{1}{m} + \frac{\left(X_{o_{(ook)}} - \overline{X}_{o_{(ook)}}\right)^2}{\sum_{k=1}^{m}\left(X_{o_{(ook)}} - \overline{X}_{o_{(ook)}}\right)^2}\right)\right)(\sigma_{y|x}^2 + \delta_y^2 \sigma_y^2)$$

The cause-selecting control chart is thus constructed based on equation (16).

The plotted statistic, $Z_{(00k)}$, of the cause-selecting control chart is defined as

$$Z_{(00k)} = \lambda_2 e_{(00k)} + (1-\lambda_2) Z_{00(k-1)}, k = 1,2,...m., \qquad (17)$$

where λ_2, and $0 < \lambda_2 \le 1$ is a constant.

Let the initial value be the second process mean, $Z_{(000)} = 0$.

For the in-control second process, the expectation and variance of $Z_{(00k)}$ are derived as follows.

$$E\left(Z_{(00k)}\right)=E\left(\lambda_2 e_{(00k)}+(1-\lambda_2)Z_{00(k-1)}\right)=E\left(\lambda_2\sum_{l=0}^{k-1}(1-\lambda_2)^l e_{(00k)}+(1-\lambda_2)^k Z_{(000)}\right)$$
$$=\left(\left(1-(1-\lambda_2)^k\right)+(1-\lambda_2)^k\right)E\left(Z_{(000)}\right)=0$$
$$\mathrm{Var}\left(Z_{(00k)}\right)=\mathrm{Var}\left(\lambda_2 e_{(00k)}+(1-\lambda_2)Z_{00(k-1)}\right)=\mathrm{Var}\left(\lambda_2\sum_{l=0}^{k-1}(1-\lambda_2)^l e_{(00k)}+(1-\lambda_2)^k Z_{(000)}\right)$$
$$=\lambda_2^2\sum_{l=0}^{k-1}(1-\lambda_2)^{2l}\mathrm{Var}\left(e_{(00k)}\right)=\frac{\lambda_2}{2-\lambda_2}\left(1-(1-\lambda_2)^{2k}\right)\left(\sigma_e^2\right)$$

That is, the distribution of the plotted statistic $Z_{(00k)}$ is

$$Z_{(00k)}\sim N\left(0,\ \frac{\lambda_2}{2-\lambda_2}\left(1-(1-\lambda_2)^{2k}\right)\left(\sigma_e^2\right)\right). \qquad (18)$$

Consequently, the control limits of the cause-selecting control chart are shown as follows.

$$\begin{aligned}\text{UCL}&=k_2\sqrt{\frac{\lambda_2}{2-\lambda_2}\left(1-(1-\lambda_2)^{2k}\right)\left(\sigma_e^2\right)}\\ \text{CL}&=0\\ \text{LCL}&=-k_2\sqrt{\frac{\lambda_2}{2-\lambda_2}\left(1-(1-\lambda_2)^{2k}\right)\left(\sigma_e^2\right)}\end{aligned} \qquad (19)$$

When $k\rightarrow\infty$, the control limits converges to $\pm k_2\sqrt{\frac{\lambda_2}{2-\lambda_2}\left(\sigma_e^2\right)}$.

The proposed EWMA control chart and cause-selecting control charts are thus used to monitor and distinguish if the first process or/and the second process is/are out of control. If there is one signal on the EWMA control chart but no signal on the cause-selecting control chart, it indicates that the first process is out of control but the second process is in control. If there is no signal on the EWMA control chart but one signal on the cause-selecting control chart, it indicates that the first process is in control but the second process is out-of-control. If there are signals on the EWMA control chart and cause-selecting control chart, it indicates that both processes are out of control. Once at least one of the processes is out of control, the occurred special cause(s) should be searched and removed from the process(es).

5. AVERAGE RUN LENGTH CALAULATION

The effects of imprecise measurement to the performance of the proposed control charts is evaluated by the average run length (ARL). When the processes are all in control, it is desirable to have a large ARL corresponding to a low false alarm rate. When at least one of the processes is out of control, it is desirable to have a small ARL corresponding to a high true alarm rate. The average run lengths for four possible states of processes are derived as follows.

5.1 The First and The Second processes Are All In Control

Let W_1 denote the sampling frequency until at least one signal flashes from the EWMA chart or cause-selecting control chart given the first and second processes are in control. The probability for each possible value of W_1 is calculated as follows

$$\begin{aligned}
&\Pr(W_1=1)=1-\left(1-\alpha_{e_1}\right)\left(1-\alpha_{e_1}\right)\\
&\Pr(W_1=2)=\left(1-\alpha_{e_1}\right)\left(1-\alpha_{e_1}\right)\left(1-\left(1-\alpha_{e_1}\right)\left(1-\alpha_{e_1}\right)\right)\\
&\vdots\\
&\Pr(W_1=k)=\left(1-\alpha_{e_1}\right)^{k-1}\left(1-\alpha_{e_1}\right)^{k-1}\left(1-\left(1-\alpha_{e_1}\right)\left(1-\alpha_{e_1}\right)\right)
\end{aligned} \qquad (20)$$

Hence, the average run length is the expectation of W_1, that is

$$\mathrm{E}(W_1)=\sum_{k=1}^{\infty}k\left(1-\alpha_{e_1}\right)^{k-1}\left(1-\alpha_{e_1}\right)^{k-1}\left(1-\left(1-\alpha_{e_1}\right)\left(1-\alpha_{e_1}\right)\right)=\frac{1}{1-\left(1-\alpha_{e_1}\right)\left(1-\alpha_{e_1}\right)}. \qquad (21)$$

5.2 AC_1 Occurs in the First Process but the Second Process Is In Control

Let W_2 be the sampling frequency until at least a true alarm is found from the EWMA control chart given AC_1 occurs in the first process but the second process is in control.

The probability for each possible value of W_2 is calculated as follows.

$$
\begin{aligned}
&\Pr(W_2 = 1) = (1-\beta_{e_1}) \\
&\Pr(W_2 = 2) = \beta_{e_1}(1-\beta_{e_1}) \qquad (22) \\
&\vdots \\
&\Pr(W_2 = k) = \beta_{e_1}^{k-1}(1-\beta_{e_1})
\end{aligned}
$$

Hence, the average run length is the expectation of W_2, that is (23)

$$
\mathrm{E}(W_2) = \sum_{k=1}^{\infty} k\beta_{e_1}^{k-1}(1-\beta_{e_1}) = \frac{1}{1-\beta_{e_1}}. \qquad (23)
$$

5.3 AC_2 Occurs in the Second Process but the First Process Is In Control

Let W_3 be the sampling frequency until at least one true alarm is found from the cause-selecting control charts given AC_2 occurs in the second process but the first process is in control.

The probability for each possible value of W_3 is calculated as follows.

$$
\begin{aligned}
&\Pr(W_3 = 1) = (1-\beta_{e_2}) \\
&\Pr(W_3 = 2) = \beta_{e_2}(1-\beta_{e_2}) \qquad (24) \\
&\vdots \\
&\Pr(W_3 = k) = \beta_{e_2}^{k-1}(1-\beta_{e_2})
\end{aligned}
$$

Hence, the average run length is the expectation of W_3, that is

$$
\mathrm{E}(W_3) = \sum_{k=1}^{\infty} k\beta_{e_2}^{k-1}(1-\beta_{e_2}) = \frac{1}{1-\beta_{e_2}}. \qquad (25)
$$

5.4 AC_1 Occurs in the First Process and AC_2 Occurs in the Second Process

Let W_4 stand for the sampling frequency until at least one true alarm is found from the EWMA and cause-selecting control charts given AC_1 occurs in the first process and AC_2 occurs in the second process.

The probability for each possible value of W_4 is calculated as follows.

$$\begin{aligned}&\Pr(W_4=1)=1-\beta_{e_1}\beta_{e_2}\\&\Pr(W_4=2)=\beta_{e_1}\beta_{e_2}\left(1-\beta_{e_1}\beta_{e_2}\right)\\&\vdots\\&\Pr(W_4=k)=\left(\beta_{e_1}\beta_{e_2}\right)^{k-1}\left(1-\beta_{e_1}\beta_{e_2}\right)\end{aligned}\qquad(26)$$

Hence, the average run length is the expectation of W_4, that is

$$\mathrm{E}(W_4)=\sum_{k=1}^{\infty}k\left(\beta_{e_1}\beta_{e_2}\right)^{k-1}\left(1-\beta_{e_1}\beta_{e_2}\right)=\frac{1}{1-\beta_{e_1}\beta_{e_2}}.\qquad(27)$$

The calculation of ARL in (20)~(27) is complicated. Crowder (1987a, 1987b, 1989) and Lucas and Saccucci (1990) propose approximated approach and Fortran program to calculate ARL for a single EWMA control chart. We modify the Fortran program proposed by Crowder (1987b) to obtain the numerical approximated values for α_{e_1}, β_{e_1}, α_{e_2} and β_{e_2}. Consequently, various ARL of the proposed control charts can be obtained.

6. SENSITIVITY ANALYSIS

The sensitivity analysis illustrates the effects of variation of measurement errors on the performance of the proposed control charts.

Let ψ_1 and ψ_2 be the ratios of measurement error variation contaminated in the total variation of the first process and the second process, respectively. That is

$$\psi_1=\frac{\delta_x^2}{1+\delta_x^2}$$
$$\psi_2=\frac{\delta_y^2\sigma_y^2}{\sigma_{y|x}^2+\delta_y^2\sigma_y^2}$$

It is usually assumed that ψ_1 and ψ_2 are less than 0.5, since the variations from the measurement errors are always less than or equal to all variation from the processes. The values of average run length under various combinations of ψ_1, ψ_2, $\delta_{(10)}$ and $\delta_{(01)}$ are shown in Table 1, which is organized into 36 separate panels, corresponding to $\delta_{(10)}$ = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0; $\delta_{(01)}$ = 0.0, 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0; ψ_1 = 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5; ψ_2 = 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5.

From Table 1, we find that when the first process is out of control but the second process is in control, $\delta_{(10)} \neq 0$ and $\delta_{(01)} = 0$, ψ_1 increases, leading to increase in ARL; while ψ_2 increases, ARL is unchanged. When the second process is out of control but the first process is in control, $\delta_{(10)} = 0$ and $\delta_{(01)} \neq 0$, we found that ψ_1 increases but the ARL is unchanged; while ψ_2 increases, leading to increase in ARL. When the first and the second processes are both out of control, $\delta_{(10)} \neq 0$ and $\delta_{(01)} \neq 0$, ψ_1 increases, leading to increase in ARL; while ψ_2 increases, leading to increase in ARL.

Therefore, the imprecision measurement devices seriously affected the performance of the proposed control charts. When ψ_1 and/or ψ_2 increase(s), the ability to detect the processes shift is poor. The results of sensitivity analysis are summarized in Table 2.

7. CONCLUSIONS

The EWMA control chart and the cause-selecting control chart are proposed to effectively monitor and distinguish the states of two dependent production processes with imprecise measurement devices. The EWMA control chart based on the observed in-coming quality is used to monitor the small mean shifts for the first process, whereas the cause-selecting control chart based on residuals is used to monitor the small mean shifts for the second process.

The performance of the proposed EWMA and cause-selecting control charts under various variations of measurement errors is measured by average run length. It is shown that the presence of measurement error may seriously affect the ability of the proposed control charts detect processes disturbance quickly. The larger variations of measurement errors lead to small rates of true alarm when at least one of the processes is out of control. If the measurement errors are inherent in the measurement devices and cannot be avoided, then the better way to reduce the effect of measurement errors is to give proper calibration or regular maintenance of measurement devices. The adaptive statistical process control approach may give an improvement for increasing detection ability of the proposed control charts under various variations of measurement errors.

Table 1 Sensitivity analysis

		ψ_2																				
		0.00							0.10							0.20						
		$\delta_{(01)}$							$\delta_{(01)}$							$\delta_{(01)}$						
ψ_1	$\delta_{(10)}$	0.00	0.50	1.00	1.50	2.00	2.50	3.00	0.00	0.50	1.00	1.50	2.00	2.50	3.00	0.00	0.50	1.00	1.50	2.00	2.50	3.00
	0.00	189.80	26.63	10.79	6.78	5.00	4.00	3.36	220.47	18.71	11.49	7.19	5.28	4.21	3.53	256.36	31.24	12.35	7.67	5.61	4.46	3.73
	0.50	26.63	13.57	7.89	5.57	4.35	3.60	3.09	26.63	14.07	8.06	5.64	4.39	3.62	3.10	26.63	14.63	8.22	5.72	4.43	3.65	3.12
	1.00	10.79	7.89	5.66	4.41	3.65	3.13	2.76	10.79	8.24	5.83	4.51	3.71	3.17	2.79	10.79	8.66	6.02	4.62	3.78	3.22	2.82
0.00	1.50	6.78	5.57	4.41	3.66	3.14	2.77	2.49	6.78	5.83	4.57	3.76	3.21	2.82	2.53	6.78	6.13	4.74	3.87	3.29	2.88	2.57
	2.00	5.00	4.35	3.65	3.14	2.78	2.50	2.28	5.00	4.55	3.78	3.24	2.84	2.55	2.32	5.00	4.78	3.93	3.34	2.92	2.61	2.37
	2.50	4.00	3.60	3.13	2.77	2.50	2.29	2.11	4.00	3.76	3.24	2.86	2.56	2.34	2.15	4.00	3.95	3.38	2.95	2.64	2.39	2.20
	3.00	3.36	3.09	2.76	2.49	2.28	2.11	1.97	3.36	3.22	2.86	2.57	2.34	2.16	2.01	3.36	3.38	2.98	2.66	2.41	2.22	2.06
	0.00	220.47	26.63	10.79	6.78	5.00	4.00	3.36	263.01	18.71	11.49	7.19	5.28	4.21	3.53	315.78	31.24	12.35	7.67	5.61	4.46	3.73
	0.50	28.71	14.07	8.06	5.64	4.39	3.62	3.10	28.71	14.61	8.42	5.91	4.59	3.79	3.24	28.71	15.21	8.60	6.00	4.64	3.82	3.27
	1.00	11.49	8.24	5.83	4.51	3.71	3.17	2.79	11.49	8.42	6.01	4.67	3.85	3.29	2.89	11.49	8.85	6.21	4.79	3.92	3.34	2.93
0.10	1.50	7.19	5.83	4.57	3.76	3.21	2.82	2.53	7.19	5.91	4.67	3.86	3.31	2.91	2.61	7.19	6.22	4.85	3.98	3.39	2.97	2.65
	2.00	5.28	4.55	3.78	3.24	2.84	2.55	2.32	5.28	4.59	3.85	3.31	2.92	2.62	2.39	5.28	4.83	4.00	3.42	3.00	2.68	2.43
	2.50	4.21	3.76	3.24	2.86	2.56	2.34	2.15	4.21	3.79	3.29	2.91	2.62	2.39	2.20	4.21	3.98	3.43	3.01	2.69	2.45	2.25
	3.00	3.53	3.22	2.86	2.57	2.34	2.16	2.01	3.53	3.53	2.89	2.61	2.39	2.20	2.06	3.53	3.41	3.01	2.70	2.46	2.26	2.10
	0.00	256.36	26.63	10.79	6.78	5.00	4.00	3.36	315.78	18.71	11.49	7.19	5.28	4.21	3.53	395.12	31.24	12.35	7.67	5.61	4.46	3.73
	0.50	31.24	14.63	8.22	5.72	4.43	3.65	3.12	31.24	15.21	8.60	6.00	4.64	3.82	3.27	31.24	15.87	9.06	6.32	4.89	4.02	3.43
	1.00	12.35	8.66	6.02	4.62	3.78	3.22	2.82	12.35	8.85	6.21	4.79	3.92	3.34	2.93	12.35	9.06	6.44	4.95	4.09	3.48	3.05
0.20	1.50	7.67	6.13	4.74	3.87	3.29	2.88	2.57	7.67	6.22	4.85	3.98	3.39	2.97	2.65	7.67	6.32	4.98	4.10	3.50	3.07	2.75
	2.00	5.61	4.78	3.93	3.34	2.92	2.61	2.37	5.61	4.83	4.00	3.42	3.00	2.68	2.43	5.61	4.89	4.09	3.50	3.08	2.76	2.51
	2.50	4.46	3.95	3.38	2.95	2.64	2.39	2.20	4.46	3.98	3.43	3.01	2.69	2.45	2.25	4.46	4.02	3.48	3.07	2.76	2.51	2.31
	3.00	3.73	3.38	2.98	2.66	2.41	2.22	2.06	3.73	3.41	3.01	2.70	2.46	2.26	2.10	3.73	3.43	3.05	2.75	2.51	2.31	2.15
	0.00	295.54	26.63	10.79	6.78	5.00	4.00	3.36	377.46	18.71	11.49	7.19	5.28	4.21	3.53	496.76	31.24	12.35	7.67	5.61	4.46	3.73
	0.50	34.43	15.27	8.40	5.81	4.48	3.68	3.14	34.43	15.91	8.81	6.09	4.70	3.85	3.29	34.43	16.68	9.29	6.43	4.95	4.05	3.46
	1.00	13.42	9.15	6.24	4.74	3.85	3.27	2.86	13.42	9.37	6.45	4.92	4.00	3.40	2.97	13.42	9.60	6.69	5.12	4.18	3.55	3.10
0.30	1.50	8.27	6.50	4.94	3.99	3.37	2.94	2.61	8.27	6.60	5.07	4.11	3.48	3.03	2.70	8.27	6.71	5.21	4.25	3.60	3.14	2.80
	2.00	6.03	5.07	4.11	3.46	3.01	2.67	2.41	6.03	5.13	4.19	3.55	3.09	2.75	2.49	6.03	5.19	4.28	3.64	3.18	2.83	2.57
	2.50	4.77	4.18	3.53	3.07	2.72	2.46	2.25	4.77	4.22	3.59	3.13	2.78	2.52	2.31	4.77	4.26	3.65	3.20	2.85	2.58	2.37
	3.00	3.98	3.58	3.12	2.76	2.49	2.28	2.11	3.98	3.61	3.16	2.81	2.54	2.33	2.16	3.98	3.63	3.21	2.87	2.60	2.39	2.21
	0.00	333.94	26.63	10.79	6.78	5.00	4.00	3.36	442.51	18.71	11.49	7.19	5.28	4.21	3.53	615.99	31.24	12.35	7.67	5.61	4.46	3.73
	0.50	38.56	16.00	8.61	5.90	4.53	3.71	3.17	38.56	16.71	9.03	6.20	4.75	3.89	3.31	38.56	17.51	9.54	6.54	5.01	4.09	3.48
	1.00	14.79	9.74	6.49	4.87	3.94	3.33	2.90	14.79	9.99	6.72	5.07	4.09	3.46	3.01	14.79	10.26	6.99	5.29	4.28	3.61	3.15
0.40	1.50	9.04	6.94	5.18	4.14	3.47	3.00	2.66	9.04	7.06	5.32	4.27	3.58	3.11	2.76	9.04	7.19	5.48	4.41	3.72	3.23	2.86
	2.00	6.55	5.42	4.33	3.60	3.10	2.74	2.47	6.55	5.49	4.42	3.70	3.19	2.83	2.55	6.55	5.56	4.52	3.80	3.29	2.92	2.63
	2.50	5.17	4.47	3.73	3.20	2.82	2.53	2.31	5.17	4.51	3.79	3.27	2.89	2.60	2.37	5.17	4.56	3.86	3.35	2.97	2.67	2.44
	3.00	4.29	3.82	3.29	2.89	2.59	2.35	2.17	4.29	3.85	3.34	2.94	2.64	2.41	2.22	4.29	3.88	3.39	3.00	2.70	2.47	2.28
	0.00	372.99	26.63	10.79	6.78	5.00	4.00	3.36	513.85	18.71	11.49	7.19	5.28	4.21	3.53	763.67	31.24	12.35	7.67	5.61	4.46	3.73
	0.50	44.15	16.85	8.83	6.00	4.58	3.75	3.19	44.15	17.64	9.28	6.31	4.81	3.92	3.34	44.15	18.54	9.82	6.66	5.08	4.14	3.51
	1.00	16.63	10.48	6.79	5.03	4.03	3.39	2.94	16.63	10.77	7.06	5.24	4.20	3.53	3.06	16.63	11.08	7.34	5.47	4.39	3.69	3.20
0.50	1.50	10.06	7.51	5.47	4.31	3.58	3.08	2.72	10.06	7.65	5.62	4.45	3.70	3.19	2.82	10.06	7.80	5.80	4.61	3.85	3.32	2.93
	2.00	7.24	5.87	4.59	3.77	3.22	2.83	2.53	7.24	5.95	4.69	3.88	3.32	2.92	2.62	7.24	6.03	4.81	3.99	3.43	3.02	2.71
	2.50	5.68	4.83	3.96	3.36	2.93	2.62	2.37	5.68	4.88	4.04	3.44	3.01	2.69	2.44	5.68	4.94	4.12	3.53	3.10	2.77	2.52
	3.00	4.71	4.13	3.50	3.04	2.70	2.44	2.24	4.71	4.17	3.56	3.11	2.77	2.50	2.30	4.71	4.21	3.62	3.17	2.84	2.57	2.36

(Continued) Table 1 Sensitivity analysis

		ψ_2																				
		0.30							0.40							0.50						
		$\delta_{(01)}$							$\delta_{(01)}$							$\delta_{(01)}$						
ψ_1	$\delta_{(10)}$	0.00	0.50	1.00	1.50	2.00	2.50	3.00	0.00	0.50	1.00	1.50	2.00	2.50	3.00	0.00	0.50	1.00	1.50	2.00	2.50	3.00
0.00	0.00	295.84	34.43	13.42	8.27	6.03	4.77	3.98	335.94	38.56	14.79	9.04	6.55	5.17	4.29	372.99	44.15	16.63	10.06	7.24	5.68	4.71
	0.50	26.63	15.27	8.40	5.81	4.48	3.68	3.14	26.63	16.00	8.61	5.90	4.53	3.71	3.17	26.63	16.85	8.83	6.00	4.58	3.75	3.19
	1.00	10.79	9.15	6.24	4.74	3.85	3.27	2.86	10.79	9.74	6.49	4.87	3.94	3.33	2.90	10.79	10.48	6.79	5.03	4.05	3.39	2.94
	1.50	6.75	6.80	4.94	3.99	3.37	2.94	2.61	6.75	6.94	5.15	4.14	3.47	3.00	2.66	6.75	7.51	5.47	4.31	3.58	3.06	2.72
	2.00	5.00	5.07	4.11	3.46	3.01	2.67	2.41	5.00	5.42	4.33	3.60	3.10	2.74	2.47	5.00	5.87	4.59	3.77	3.22	2.83	2.53
	2.50	4.00	4.15	3.53	3.07	2.72	2.46	2.25	4.00	4.47	3.73	3.20	2.82	2.53	2.31	4.00	4.83	3.96	3.36	2.93	2.62	2.37
	3.00	3.36	3.58	3.12	2.76	2.49	2.28	2.11	3.36	3.82	3.29	2.89	2.59	2.36	2.17	3.36	4.15	3.50	3.04	2.70	2.44	2.24
0.10	0.00	377.46	34.43	13.42	8.27	6.03	4.77	3.98	442.51	38.56	14.79	9.04	6.55	5.17	4.29	513.85	44.15	16.63	10.06	7.24	5.68	4.71
	0.50	28.71	18.91	8.81	6.09	4.70	3.85	3.29	28.71	16.71	9.03	6.20	4.75	3.89	3.31	28.71	17.64	9.28	6.31	4.81	3.92	3.34
	1.00	11.49	9.37	6.48	4.92	4.00	3.40	2.97	11.49	9.99	6.72	5.07	4.09	3.46	3.01	11.49	10.77	7.06	5.24	4.20	3.53	3.06
	1.50	7.19	6.60	5.07	4.11	3.48	3.03	2.70	7.19	7.06	5.32	4.27	3.58	3.11	2.76	7.19	7.58	5.62	4.45	3.70	3.19	2.82
	2.00	5.28	5.13	4.19	3.55	3.09	2.75	2.49	5.28	5.49	4.42	3.70	3.19	2.83	2.55	5.28	5.95	4.69	3.88	3.32	2.92	2.62
	2.50	4.21	4.22	3.59	3.15	2.78	2.52	2.31	4.21	4.51	3.79	3.27	2.89	2.60	2.37	4.21	4.88	4.04	3.44	3.01	2.69	2.44
	3.00	3.53	3.61	3.16	2.81	2.54	2.33	2.16	3.53	3.86	3.34	2.94	2.64	2.41	2.22	3.53	4.17	3.56	3.11	2.77	2.50	2.30
0.20	0.00	496.76	34.43	13.42	8.27	6.03	4.77	3.98	615.99	38.56	14.79	9.04	6.55	5.17	4.29	763.67	44.15	16.63	10.06	7.24	5.68	4.71
	0.50	31.24	16.63	9.29	6.43	4.95	4.05	3.46	31.24	17.51	9.54	6.54	5.01	4.09	3.48	31.24	18.54	9.83	6.66	5.08	4.14	3.51
	1.00	12.35	9.60	6.69	5.12	4.15	3.55	3.10	12.35	10.26	6.99	5.29	4.28	3.61	3.15	12.35	11.08	7.34	5.47	4.39	3.69	3.20
	1.50	7.67	6.71	5.21	4.25	3.60	3.14	2.80	7.67	7.19	5.48	4.41	3.72	3.23	2.86	7.67	7.80	5.80	4.61	3.85	3.32	2.93
	2.00	5.61	5.19	4.28	3.64	3.18	2.83	2.57	5.61	5.56	4.52	3.80	3.29	2.92	2.63	5.61	6.03	4.81	3.99	3.43	3.02	2.71
	2.50	4.46	4.26	3.65	3.20	2.85	2.58	2.37	4.46	4.56	3.86	3.35	2.97	2.67	2.44	4.46	4.94	4.12	3.53	3.10	2.77	2.52
	3.00	3.73	3.63	3.21	2.87	2.60	2.39	2.21	3.73	3.88	3.39	3.00	2.70	2.47	2.28	3.73	4.21	3.62	3.17	2.84	2.57	2.36
0.30	0.00	668.91	34.43	13.42	8.27	6.03	4.77	3.98	904.88	38.56	14.79	9.04	6.55	5.17	4.29	1264.21	44.15	16.63	10.06	7.24	5.68	4.71
	0.50	34.43	17.47	9.86	6.85	5.24	4.30	3.66	34.43	18.44	10.15	6.96	5.33	4.35	3.69	34.43	19.59	10.47	7.10	5.41	4.39	3.73
	1.00	13.42	9.86	6.97	5.36	4.39	3.73	3.26	13.42	10.56	7.29	5.54	4.50	3.80	3.31	13.42	11.44	7.68	5.75	4.63	3.89	3.38
	1.50	8.27	6.83	5.36	4.40	3.75	3.28	2.93	8.27	7.33	5.65	4.58	3.87	3.37	2.99	8.27	7.96	6.01	4.80	4.02	3.47	3.07
	2.00	6.03	5.26	4.39	3.75	3.29	2.94	2.66	6.03	5.64	4.63	3.92	3.41	3.03	2.74	6.03	6.13	4.94	4.13	3.56	3.14	2.82
	2.50	4.77	4.30	3.72	3.28	2.94	2.66	2.45	4.77	4.61	3.94	3.44	3.06	2.76	2.53	4.77	5.00	4.21	3.63	3.20	2.87	2.61
	3.00	3.98	3.66	3.26	2.93	2.66	2.45	2.28	3.98	3.92	3.45	3.07	2.78	2.54	2.35	3.98	4.28	3.69	3.26	2.92	2.65	2.44
0.40	0.00	904.88	34.43	13.42	8.27	6.03	4.77	3.98	1398.56	38.56	14.79	9.04	6.55	5.17	4.29	2494.48	44.15	16.63	10.06	7.24	5.68	4.71
	0.50	38.56	18.44	10.15	6.96	5.33	4.35	3.69	38.56	19.53	10.89	7.48	5.73	4.67	3.95	38.56	20.84	11.27	7.65	5.82	4.72	3.99
	1.00	14.79	10.56	7.29	5.54	4.50	3.80	3.31	14.79	10.89	7.68	5.86	4.76	4.03	3.51	14.79	11.83	8.09	6.09	4.91	4.13	3.58
	1.50	9.04	7.33	5.65	4.58	3.87	3.37	2.99	9.04	7.48	5.86	4.78	4.06	3.54	3.15	9.04	8.15	6.24	5.02	4.22	3.65	3.23
	2.00	6.55	5.64	4.63	3.92	3.41	3.03	2.74	6.55	5.73	4.76	4.06	3.55	3.16	2.86	6.55	6.23	5.09	4.28	3.71	3.28	2.95
	2.50	5.17	4.61	3.94	3.44	3.06	2.76	2.52	5.17	4.67	4.03	3.54	3.16	2.86	2.62	5.17	5.07	4.31	3.74	3.31	2.98	2.72
	3.00	4.29	3.92	3.45	3.07	2.78	2.54	2.35	4.29	3.95	3.51	3.15	2.86	2.62	2.43	4.29	4.30	3.77	3.34	3.01	2.74	2.53
0.50	0.00	1264.21	34.43	13.42	8.27	6.03	4.77	3.98	2494.48	38.56	14.79	9.04	6.55	5.17	4.29	11554.80	44.15	16.63	10.06	7.24	5.68	4.71
	0.50	44.15	19.59	10.47	7.10	5.41	4.39	3.73	44.15	20.84	11.27	7.65	5.82	4.72	3.99	44.15	22.33	12.28	8.55	6.34	5.14	4.34
	1.00	16.63	11.44	7.68	5.75	4.63	3.89	3.38	16.63	11.83	8.09	6.09	4.91	4.13	3.58	16.63	12.28	8.87	6.81	5.26	4.43	3.85
	1.50	10.06	7.96	6.01	4.80	4.02	3.47	3.07	10.06	8.15	6.24	5.02	4.22	3.65	3.23	10.06	8.35	6.81	5.29	4.47	3.88	3.44
	2.00	7.24	6.13	4.94	4.13	3.56	3.14	2.82	7.24	6.23	5.09	4.28	3.71	3.28	2.95	7.24	6.34	5.26	4.47	3.89	3.45	3.11
	2.50	5.68	5.00	4.21	3.63	3.20	2.87	2.61	5.68	5.07	4.31	3.74	3.31	2.98	2.72	5.68	5.14	4.45	3.88	3.48	3.11	2.88
	3.00	4.71	4.28	3.69	3.26	2.92	2.65	2.44	4.71	4.30	3.77	3.34	3.01	2.74	2.53	4.71	4.34	3.85	3.44	3.11	2.85	2.63

Table 2: Summary of sensitivity analysis

Shift parameter		Ratio of measurement error variation		ARL
$\delta_{(10)} \neq 0$	$\delta_{(01)} = 0$	ψ_1	↑	↑
		ψ_2	↑	-
$\delta_{(10)} = 0$	$\delta_{(01)} \neq 0$	ψ_1	↑	-
		ψ_2	↑	↑
$\delta_{(10)} \neq 0$	$\delta_{(01)} \neq 0$	ψ_1	↑	↑
		ψ_2	↑	↑

ACKNOWLEDGMENTS

Support for this research was provided in part by the National Science Council of the Republic of China, grant No. NSC-91-2118-M-004-008.

REFERENCES

Abraham, B. (1977), "Control Charts and Measurement Error," *Annual Technical Conference of the American Society for Quality Control* **31**, 370-374.

Bennett, C. (1954), "Effects of Measurement Error on Chemical Process Control," *Industrial Quality Control* **10**, No. 4, 17-20.

Crowder, S. V. (1987a), "A Simple Method for Studying Run Length Distributions of

Exponentially Weighted Moving Average Charts," *Technometrics* **29**, 401-407.

Crowder, S. V. (1987b), "Average Run Lengths of Exponentially Weighted Moving Average Charts," *Journal of Quality Technology* **18**, 203-210.

Crowder, S. V. (1989), "Design of Exponentially Weighted Moving Average Schemes," *Journal of Quality Technology* **21**, 155-162.

Kanazuka, T. (1986), "The Effects of Measurement Error on the Power of $\overline{X}-R$ charts," *Journal of Quality Technology* **18**, 91-95.

Linna, K. W. and Woodall, W. H. (2001), "Effect of Measurement Error on Shewhart Control Charts," *Journal of Quality Technology* **33**, 213-222.

Mittag, H.-J. (1993), *Qualitatsregelkarten* (Munich, Hanser).

Mittag, H.-J. (1995), "Measurement Error Effect on Control Chart Performance," *Annual Proceedings of the American Society for Quality Control* **49**, 66-73.

Mittag, H.-J. and Stemann, D (1993), "Effekte Stochastistcher Meβfehler bei der Anwendung von Shewhart-Regelkarten zur Streuungsuberwachung," *Allgemeines Statistisches Archiv* **77**, 240-259.

Mittag, H.-J. and Stemann, D (1998), "Gauge Imprecision Effect on the Performance of the $\overline{X}-S$ Control Chart," *Journal of Applied Statistics* **25**, No.3, 307-317.

Mizuno, S. (1961), "Problems of Measurement Errors in Process Control," *Bulletin of the International Statistical Institute* **38,** 405-415.

Montgomery, D. (2002), *Statistical Quality Control*, 5th edition, John Wiley & Sons, Inc.

Rabinovich, S. G. (2000), *Measurement Errors and Uncertainties*, 2nd edition, Springer-Verlag New York, Inc.

Rahim, A. (1985), "Economic Model of $\overline{X}$ Charts Under Non-normality and Measurement

Error," *Computers and Operations Research* **12**, No. 3, 291-299.

Tricker, A., Coates, E., and Okell, E. (1998), "The Effect on the R Chart of Precision Measurement," *Journal of Quality Technology* **30**, 232-239.

Wade, R. and Woodall, W., "A Review and Analysis of Cause-Selecting Control Charts," *Journal of Quality Technology*, **25**,.161-169 (1993).

Yang, S. (2002), "The Effects of Imprecise Measurement on the Economic Asymmetric $\overline{X}$ and *S* Control Charts," *The Asian Journal on Quality* **3**, No. 2, 46-55.

Yang, S. and Chen, Y. (2003), "Processes Control for Two Failure Mechanisms," *Journal of the Chinese Institute of Industrial Engineers*. (To Appear)

Zhang, G., "A New Type of Control Charts and a Theory of Diagnosis with Control Charts," *World Quality Congress Transactions, American Society for Quality Control, Milwaukee, WI*, 75-85 (1984).