行政院國家科學委員會專題研究計畫成果報告

非均質波松過程的後驗常態性 (2/2)
Posterior Normality for Nonhomogeneous Poisson Processes (2/2)

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一、　中文摘要

過去許多研究指出,某些隨機過程的未知參數之後驗分配近似於常態.相關之研究如 Heyde and Johnstone (1979,以下簡稱 H-J), Basawa and Rao (1980). 其中, H-J 提出一組條件,並證明對於一般隨機過程,只要滿足這些條件,則其參數之後驗分配近似於常態. 不幸的,某些 reliability 上常見的非均質波松過程 (nonhomogeneous Poisson processes,以下簡稱 NHPPs)並不符合這組條件. 有鑑於此, Sweeting and Adekola (1987)把 H-J 的條件加以放寬與修改, 並證明這組新的條件涵蓋更多的模型,包括若干出生過程 (birth processes)及上述之 NHPPs 模型.

以上的研究多半以機率密度函數的展開來看後驗分配的問題.在這個計畫中,我修改 Stein’s Identity 來討論未知參數之後驗分配的問題. 這裡提到的 Stein’s Identity 乃由 Woodroofe (1989)所提出. 這個 Identity 可以把後驗期望值作展開. 此展開式的首項與標準常態的結果相同, 而餘項在樣本愈大時愈近似零. 之前的結果只局限於均質的過程.我把這個 Identity 修改後,就可以用於非均質過程. 我們給予一般性證明之後,也分別討論其應用於上述之 NHPPs 模型以及 conditional exponential family 之情形. 我們的證明尚稱簡潔, 所要求的條件也適中.

關鍵詞：非均質波松過程, 隨機過程, 後驗分配

Abstract

We propose a new method to derive the posterior normality of stochastic processes. For a suitable parameter transformation Zt, the likelihood function is converted into a form close to a standard normal density. Then we propose a modified version of Stein's Identity and apply it to obtain an expression for the posterior expectation of h(Zt). From this, posterior normality of Zt can be established. Applications of this method are illustrated by the conditional exponential family and a nonhomogeneous Poisson process.

Keywords: posterior distributions; posterior normality; Stein's identity; stochastic processes.

二、Introduction

Asymptotic posterior normality has been studied since the time of Laplace. It also attracted many researchers’ attention over the past decades. Walker [4] presented a straightforward approach to posterior normality for independent and identically distributed (i.i.d.) observations. This work was improved by Dawid [1] to other i.i.d.

cases where the range of the observations depends on the parameters of interest. For stochastic processes, Heyde and Johnstone [2] simplified Walker's conditions and showed that asymptotic posterior normality holds under weaker conditions than those required for asymptotic normality of maximum likelihood. However, there condition excludes certain processes of practical interest; for example, it fails for some nonhomogeneous Poisson processes which are of interest in reliability. See Sweeting and Adekola [3]. To attack this problem, Sweeting and Adekola [3] adapted Dawid's [1] method to a more flexible continuity condition on information function where a shrinking neighborhood is used. But to generalize Dawid's approach, they needed a sequence to measure the order of the information function, and a condition such as their A3 seemed essential for the proof. It then appeared that the weakening of the continuity condition, in order to cover a broader range of applications, necessitates the introduction of other conditions, which also guarantee the asymptotic normality of the maximum likelihood estimator.

In this paper, we first review the old version of Stein’s Identity. Next, we derive a modified version of Stein’s Identity and present a novel approach to posterior normality of stochastic processes (possibly nonhomogeneous) based on this modified Identity. Finally, two applications, conditional exponential family and nonhomogeneous Poisson processes are considered.

三、Main results

Let Zt be a suitable parameter transformation and h be a measurable function. We start with a version of Stein's Identity by Woodroofe [5]. Then we obtain a more general version of the Identity. The old version can be considered as a special case of the new version. The major difference between the new version and the old one is that the new version allows jump discontinuities at both end points. From this modified version, we can write posterior expectations of h(Zt) in a form from which posterior normality for more general stochastic processes can be easily established. We consider two situations separately: one in which a fixed neighborhood can be used for the continuity condition of information, and the other in which a shrinking one is required. The conditions for the former case are fairly simple. We use the conditional exponential family as its application. The latter case is of particular interest as it can cover a wide class of nonhomogeneous processes. We find that a condition such as A3 of Sweeting and Adekola [3] can be avoided here. In addition, our choice of shrinking rate is more flexible. The advantage of using our shrinkage will be discussed later. The cost we pay for using a modified Stein's Identity is to impose a slightly stronger condition on the prior density.

The approach in this paper is relevant to Woodroofe and Coad [7] and Wend and Woodroofe [6], who applied Stein's Identity to obtain posterior expectations and employed a martingale structure to derive integrable posterior expansions. Their parameter transformations are based on the maximum likelihood estimator. The models they considered included linear models with i.i.d. normal errors and stationary autoregressive processes. Although they were from a frequentist perspective, their results implied posterior normality. From this point of view, the present paper can be viewed as an extension along this line to a general stochastic process (possibly to nonstationary processes).

四、Conclusions

In this report we have provided literature review and summarized our contribution on solving the problem of posterior normality. We found that the proposed approach based on the modified Stein’s Identity considerably simplifies the proof. In addition, the conditions required are quite general. So it also covers a wide range of models. It would be interesting to further explore other applications using this Identity.

五、Bibliography

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