Preliminary

Should investors follow the theoretical asset allocation strategies which are derived from capital asset pricing model? Canner et al. (1997) found that the advice on investment portfolio by the fund manager was inconsistent with the mutual fund separation theory which is called the asset allocation puzzle. In the theoretical asset allocation strategy, the investor should hold more proportion of wealth in the riskless asset, but the composition of risky asset is the same for all investors. However, the financial advisors suggest that the more risk-averse investor tend to hold higher ratio of bonds to stocks.

Bajeux-Besnainou et al. (2001) said that this contradiction is caused by that the maturity of investor is longer than that of cash which means there is no riskless asset for the investor. People have to rebalance their investment proportion to obtain the riskless zero coupon bond and then the compositions of risky assets will be adjusted with their risk preferences. On the other hand, Brinson et al. (1991) show that around 90% of the performance of pension funds are determined by the asset allocation decision, which means that the importance is how to decide the weights of underlying assets, not how to choose the underlying assets. Therefore, how to construct the investment strategy for long term investors is very important. In this discourse, we study three essays of the asset allocation problems for long term investors.

Recently, the inflation issue is getting more and more important. For example, in 2008, Zimbabwe face the serious inflation crisis that the habitant has paid 0.1 billion Zimbabwean dollars to buy only five bananas. Moreover, according to Table

Table 1: Consumer Price Index

Country	2005	2006	2007	Country	2005	2006	2007
Australia	116.1	120.2	123.0	Slovak Republic	132.7	138.7	142.5
Belgium	111.0	113.0	115.1	Spain	117.2	121.3	124.7
Canada	112.2	114.4	116.9	Sweden	107.5	109.0	111.4
Denmark	110.2	112.3	114.2	Turkey	319.8	350.4	381.1
France	109.9	111.7	113.4	United Kingdom	107.5	110.0	112.5
Germany	107.9	109.6	112.1	United State	113.4	117.1	120.4
Hungary	132.9	138.1	149.1	OECD - Europe	117.9	120.7	123.8
Italy	112.7	115.1	117.2	Brazil	151.4	157.8	163.5
Japan	97.8	98.1	98.1	India	121.5	128.6	136.8
Korea	117.8	120.5	123.5	Indonesia	156.0	176.5	187.8
Mexico	127.2	131.8	137.0	Russian Federation	199.8	219.2	238.9
Netherlands	113.1	114.4	116.3	South Africa	128.1	134.0	143.5

1[1] which is the statistics data of OECD, we find that the consumer price index (CPI) is larger and larger with time which means that most countries face inflation problems, especially in the developing countries. Therefore, inflation issue is the critical topic of this research.

In this discourse,we investigate three topics of optimal asset allocation problem for long term investors. Firstly, we study the investment problem for the defined contribution (DC) pension schemes. Because pension fund is a long term plan, we should incorporate the uncertainty of inflation rate. The performance of DC pension fund is very important for the contributors, thus, in order to evaluate properly the financial impact of incentive structures on fund management, a performance-oriented arrangement induced by bonus fees and downside penalty is introduced. The fund managers are rewarded with bonus fees for their superior performance,

[1]Data source: http://stats.oecd.org/wbos/Index.aspx?querytype=view&queryname=221. The basic year is 2000.

while downside penalty is also imposed on them once their performance is below the specified benchmark.

In Chapter 1, we investigate the portfolio selection problem with incentive mechanism (i.e., bonus fees and downside penalty) incorporated into defined contribution (DC) pension schemes. We modify the previous work of Battocchio and Menoncin (2004) that allowed the state variables (i.e., the risks from the financial market) and a set of stochastic processes to describe the inflation and labor income uncertainties. Moreover, the fund managers are rewarded with bonus fees for their superior performance, while downside penalty is also imposed on them once their performance is below the specified benchmark. We find that:

1. A five-fund separation theorem is derived to characterize the investment strategy. The five funds are the myopic market portfolio, the hedge portfolio for the state variables, the hedge portfolio for the inflation risk, the hedge portfolio for the labor income uncertainties, and cash. Except cash, all funds are dependent on the incentive setup. When performance-oriented arrangement is taken into account, the fund managers tend to increase the holding in risky asset.

2. When incentive mechanisms are incorporated, the settlement of delegated management contract is vital since the setup could affect significantly the fund dynamics. Our finding is consistent with the conclusion put forward by Raghu et al. (2003). Our numerical results show that performance-oriented arrangements dominate the investment discretion in fund management. Hence, an incentive program has to be carefully implemented in order to balance the risk and reward in DC pension fund management.

On the other hand, many wealth management companies provide product with guarantee return rate to attract long term investors because they think this mechanism is attractive and useful in hedging the inflation. Therefore, in Chapter 2, we revisit the portfolio selection problem for long-term investors incorporating minimum guarantee requirement. Through reviewing the framework in Deelstra et al. (2003), we show that:

1. The optimal investment decision can be decomposed into an optimal constant relative risk aversion CRRA(γ,T) mutual fund and the dynamic hedging demand induced by the guarantee requirement. Moreover, the dynamic hedging demand at time t is related with the ratio of the value of wealth at time t to the value of guarantee fund at time t, ant it can be replicated by the traded assets in the market including cash, stock index and rolling bond.

2. When the coefficient of relative risk aversion γ of the investor decreases, which means the investor is more risk averse, the proportions of wealth invested in stock and bond decrease. The optimal weight in bond decreases within the CRRA(γ,T) mutual fund, but increases in the hedging demands. The investment behavior of the investors is significantly influenced by the time to maturity. Based on the results, the investors are becoming more cautious in rebalancing their portfolio as time approach maturity.

3. The hedge ratio defined as the ratio of the market value of the accumulated wealth to the present value of the minimum guarantee is employed in balancing the speculating and hedging objective. Hedge ratio closes to 1, the investor is recommended to reduce the stock holding through generating a short position in his hedging component. While, hedge ratio closes to 0, the investors should hold the position of the stock index to profit from the capital gain.

However, not only is how to design the product to hedge inflation risk important, but also how to precisely predict the inflation process. Thus, in Chapter 3, we focus on how to predict the dynamics of the inflation rate. In the past, researchers often assumed that the inflation rate followed a fix process; however, in this assumption, the parameters is uncertainty. Therefore, in this chapter, we examine the optimal portfolio selection problem of a long-term investor who has learning capability about the inflation rate process and can invest only in nominal assets. Assuming that the inflation rate process is not directly observable, we first employ the optimal linear filtering equations to estimate the latent process and then use the Bayesian approach to project inflation rates. This learning about inflation rates extends the studies of Campbell and Viceira (2001) and Brennan and Xia (2002). In contrast to Brennan (1998) and Xia (2001) on learning about the mean return or the predictability of the mean return, we analyze how learning about the inflation rate process may affect the composition of an optimal portfolio. We construct the optimal portfolio strategy through a Martingale formulation based on wealth constraints. Our results are given in closed-form solutions as well as numerical illustrations that demonstrate the importance of learning about inflation in the portfolio selection problem. The results are:

1. The optimal investment portfolio contains the stock index fund, the fix income portfolio and cash. The weight of stock index fund does not change with the time. However, the weight of the fix income portfolio, which includes the long-term and short-term nominal rolling bonds, will change with time in order to hedge the interest risk and the inflation risk.

2. Investment horizon, risk attitude, and the standard deviation of the inflation rate will affect the effect of learning process on the terminal wealth and its

expected utility value. The effects are more significant with increasing investment horizon, risk-averse attitude, and the standard deviation of the inflation rate.

3. The learning process also enhances the expected utility value under different values of the standard deviation of CPI and the variance of the estimation error. However, the improvement ratio decreases with increasing standard deviation of CPI and the variance of the estimation error.

We study the above three essays by expanding three lines of researches. Therefore, models of these three chapters are different and their results could not be compared directly.

The tactical asset allocation strategy based on the classical static, single-period framework introduced by Markowitz (1959) is a popular way to construct investment portfolios across broad asset classes such as bonds, stocks, and cash. Sharpe (1991) describes the mean-variance approach as a highly parsimonious characterization of investor goals, employing a myopic view and focusing on only two aspects of the probability distribution of possible returns over that period. The main drawback in the method which discussed in Sharpe (1991) is that the aggregation of single-period optimal decisions across periods might not be optimal for multiple periods as a whole. Thus, the intertemporal investment strategy with the continuous-time framework has been constructed explicitly through stochastic dynamic programming methodology and martingale methodology. Merton (1969, 1971) has initiated the pioneering works by using stochastic dynamic programming method in constructing the optimal investment portfolio. An alternative approach employs the martingale methodology as proposed by Pliska (1986), Cox and Huang (1989) and Karatzas et al. (1987) to study intertemporal consumption and portfolio policies when markets are complete, which was also the case in the earlier dynamic programming literature.

In this discourse, we use both of stochastic dynamic programming and martingale methodology to solve the problems.

The outline of this dissertation is as follows. In Preliminary, we introduce the finding and contribution of this dissertation. We then investigate the labor income and inflation uncertainties of DC pension schemes in Chapter 1. Next, the non-myopic portfolio choice problem with minimum guarantees is solved in Chapter 2. In Chapter 3, the dynamic asset allocation problem is explicitly investigated the learning mechanism in predicting the inflation rate.