Chapter 1

Investment Decision in Defined Contribution Pension Schemes Incorporating Incentive Mechanism

1.1 Introduction

The investment strategy of pension funds has a profound effect on the global capital markets. They affect the development of financial innovation, the behavior of security prices and rates of return. In recent years, with an increase in the percentage of population that comes under pension age worldwide, pension-related topics have taken on new significance and much attention has been focused on the implementation of a better investment program for the aging society. In 1990, the U.S. government began to implement the defined contribution (hereafter DC) pension plans. Over the last two decades, DC pension plans, such as 401(K) plan plans, have been the primary engine of growth in the U.S. private pension market (see Lachance et al. 2003). In view of the improved mortality rates, other countries such as Germany, UK, Australia, and India have also started to implement DC pension plans.

On July 1, 2005, the Labor Pension Act (hereafter LPA) was enacted in Taiwan, establishing a new, portable, defined contribution scheme for employees. The Taiwan government replaced the defined benefit (hereafter DB) pension plans with DC pension plans, while all employees in Taiwan were given the option to enroll in the LPA or remain with the old DB pension system under the Labor Standards Law. Under the old pension system, an employee receives a lump sum pension at the end of their employment term. Employees are eligible to apply for retirement after having been employed at a company for 25 years. Alternatively, an employee can also retire at age 55 as long as he/she has worked for the same company for at least 15 years. Companies generally pay $2 - 15$ percent of an employee's monthly wages for each month the employee served, capped at 45 months. However, many workers in Taiwan are not eligible to receive a pension since they do not always remain with the same company for at least 15 years.

While the benefit design and contribution arrangement of the DC plans vary between countries, the newly enacted DC labor pension schemes adopt the delegated management schemes. The new LPA creates a labor pension fund, which is made up of individual pension accounts for each employee who enrolls. Employers were required to apply for enrollment in LPA by July 15, 2005. Employer enrollment required an application for labor pension contributions, the report of his/her labor pension contributions and a copy of the company's registration license. All employers are required to contribute at least 6 percent of an employee's monthly salary towards the personal pension account. Employees can also contribute up to 6 percent; the amount contributed will be deducted from the employee's taxable annual income. Once eligible to receive a pension under LPA, an employee will receive their pension funds on a quarterly basis.

As a stimulus for the fund manager to act in the best interest of the plan participants, the prudent man rule is usually adopted in fund management. According to the definition in Wikipedia, the prudent man rule means to observe how men of prudence, discretion and intelligence manage their own affairs, not in regard to speculation, but in regard to the permanent disposition of their funds, considering the probable income, as well as the probable safety of the capital to be invested. The fund investment mandate is often modified with certain incentive mechanism consisting of bonus fees and downside penalty. Since the performance of fund growth affects heavily the pension wealth of the plan participants, it is obligatory for the fund to implement certain downside protection mechanism. For example, in Taiwan, the return of pension fund cannot be less than the interest rate of a 2-year fixed deposit. Incorporating bonus fees and downside penalty in investment mandate may also cause fund managers to deviate from their discretionary behaviors. Therefore, in order to quantify the impact of given incentive mechanism on pension fund dynamics, explicit solution and numerical results are explored.

Previous studies focusing on DB areas can be found in Bowers et al. (1982), McKenna (1982), Shapiro (1977, 1985), O'Brien (1986, 1987), Racinello (1988), Dufresne (1988, 1989), Haberman (1992, 1993, 1994), Haberman and Sung (1994), Janssen and Manca (1997), Haberman and Wong (1997), Chang and Cheng (2002) and amongst others. On the other hand, investment risks including interest rate risk and market risk that had been assumed by the plan sponsor under DB promise is gradually transferred to the worker in DC plans due to the severe longevity risk in aging society (Bodie 1990). Thus, the investment decision is critical for the DC scheme. Moreover, the DC scheme is accumulated through the annual salary-related contributions; and hence, the long-term financial strategy will significantly affect the fund performance. Therefore, both the uncertainties of labor income and the inflation rate, also known as background risks, proposed by Menoncin (2002) are employed in our model. Brinson et al. (1991) have shown convincingly that allocation of investment funds to asset categories is far more important than the selection of individual securities within each asset category. Hence, in this study, the background risks generated within the pension scheme are incorporated to explore the optimal investment strategy for the DC plan.

The rest of this chapter is organized as follows. In Section 1.2, the literature related to inflation risk, uncertainty of labor income and incentive mechanism is reviewed. In Section 1.3, the general framework and financial market structure are introduced. Then, the stochastic optimal control problem is formulated. The optimization algorithm is employed to derive the explicit solution in Section 1.4. In Section 1.5, how bonus fees and downside penalty influence the investment discretions of the fund manager is explicitly discussed. Finally, we conclude and summarize this chapter.

1.2 Literature Review

1.2.1 Uncertainties of Inflation and Salary

When the labor income uncertainty is incorporated into the investment decision, it could significantly influence the holding position of risky assets due to the attained age of the plan member. Tradeoff between the capital gain in the financial market and the expected discounted value of future labor income, i.e., human resource, becomes crucial in life-style investment decision. Hence, by diversifying among stocks and bonds, a more stable and efficient portfolio can be created. Campbell and Viceira (2002) suggest that investors own tradable financial asset as part of their total wealth portfolio, but they also own a valuable asset that is not readily tradable, which is labor income. Imrohoroglu et al. (1995) and Huang et al. (1997) investigate the impact of salary under certain rates of return. Then, Campbell et al. (2001) consider the long-run pattern of lifetime savings and portfolio allocation in the presence of income and rate of return uncertainty and with various pension arrangements. Under no circumstances do they consider the impact of varying degrees of imperfection in annuity markets. On the contrary, they do consider fixed costs of entering the equity market.

Campbell and Viceira (2002) find that the existence of other income prospects tends to substitute for bonds in the investor portfolio. Hence, a relatively young investor with extensive future earnings prospects will tend to possess a higher proportion of stocks than an investor at a later stage of his working life. However, this effect is reduced if the income prospects are uncertain. In line with the literature on background risks, the investor becomes in effect more risk-averse to market risks and, hence, buys fewer stocks. Viceira (2001) optimizes the inter-temporal investment-consumption policy of an investor who has uncertain salary. In his model, labor income follows a geometric process and any savings out of labor income are invested in the portfolio. The single risky asset also follows a possibly correlated geometric process. Viceira finds that the ratio of portfolio wealth to labor income is stationary, and using a log-linear approximation, he derives an optimal portfolio policy that has a constant stock proportion. Moreover, he also finds that when salary risk is independent of the asset return risk, employed investors hold a larger fraction of their savings in the risky asset than retired investors. Koo (1998) and Heaton and Lucas (1997) also derive optimal consumption and portfolio policies with stochastic wage. Koo uses a continuous-time model and shows that the optimal level of risk-taking is lower in the presence of an uninsurable labor income risk. Heaton and Lucas, in an infinite horizon model, do not find any significant effect of labor income risk on portfolio composition.

As for inflation risk, when a longer time horizon is considered, this risk becomes significant. Since the pension fund is a long-term plan, the managers should manage the inflation risk and establish the optimal strategy to resist inflation uncertainties. Modigliani and Cohn (1979), Madsen (2002) and Ritter and Warr (2002) have shown that stock market investors suffer from inflation illusion. Menoncin (2002) considers both the salary uncertainty and also the inflation risk to analyze the portfolio problem of an investor maximizing the expected exponential utility of his terminal real wealth. In his model, the investor must cope with both a set of stochastic investment opportunities and a set of background risks. Given that the market is complete, an explicit solution can be obtained. When the market is incomplete, an approximated solution is recommended. Contrary to other exact solutions obtained in the literature, all the related results are obtained allowing the stochastic inflation risk and without specifying any particular functional form for the variables in our problem. Moreover, in Battocchic and Menoncin (2004), an optimal investment strategy is derived according to the uncertainties of salary and inflation risk. However, these works did not reflect the actual delegated management plan with incentive mechanism in DC pension schemes.

1.2.2 Incentive Mechanism

Most of the literature in pension research focuses on implementing a better benefit scheme, while studies on the financial impact on incentive mechanism are scarce. The original motivation of performance-oriented arrangement in the fund investment mandate is to control the fund manager behaviors within certain risk tolerance. The forms of bonus structure can be varied, such as fixed-dollar fees, asset-based fees and incentive fees (Eugene et al. 1987). Under fixed-dollar fees, the money manager would receive a fix amount of management fees regardless of the performance of the managed fund. For asset-based fees, the manager's fees vary with the value of fund. Incentives fees are contingent upon the performance of the managed fund. Generally speaking, the incentive mechanisms for the fund manager include the penalty for underperformance and bonus for outstanding performance. In our model, when the fund growth shows superior performance to the benchmark portfolio, the fund manager is rewarded with bonus fees, while he is also facing certain downside penalty if the fund shows underperformance results.

Richard and Andrew (1987) suggest that incentive fees offer a way of improving the relationship between money managers and plan sponsors. However, the incentive fee contracts have to be set properly and setting the parameters is important. Mark (1987) use call option to price the incentive fees and find that the value of this option depends on: (1) the spread between the standard deviations of the fund portfolio and benchmark portfolio, (2) the correlation between them, (3) the value of managed fund, (4) the manager's percentage participation in incremental return, and (5) the measurement period. Because the manager could control factors (1) and (2), the setting of incentive fees contract would influence the investment decisions of fund managers. Lawrence and Stephen (1987) claim that it is important to choose the parameters especially for the benchmark portfolio, and Richard and Andrew (1987) propose that this portfolio should be able to represent the manager's typical investment style. In model setting, we assume that the benchmark rate is a positive constant but the performance mechanism is related to the value of management asset. Thus, our model is a time-dependent benchmark portfolio.

Raghu et al. (2003) simulate the delegated investment decisions under five types of incentive mechanisms. They show the efficacy of the incentive contracts in improving the welfare of investors. Edwin et al. (2003) investigate the investment behavior of mutual fund managers under incentive fees. Roy and William (2007) perform a similar study for hedge fund managers. Both studies find that the managers would increase the risk of portfolio when the return rate is below the benchmark rate because they consider the limited liability incentive forms. In this paper, we use the combined form of asset-based and target-based incentive fee mechanisms. The target-based form means that when the performance exceeds the target, the manager would receive the incentive fees. On the other hand, the manager has to make up for the shortage when the performance is below the benchmark; therefore, this is a unlimited liability incentive form. Moreover, the amounts of bonus fees and downside guarantees are related to the value of fund, so it is a kind of asset-based incentive fees. In our study, the financial influence of different bonus fees and downside penalty set is fully explored.

1.3 Proposed Model

First, a time-varying opportunity set in the financial market is introduced and the fund wealth process of DC pension scheme is formulated. Our research broadened the attention from the risks in the financial markets to those outside the financial markets that are referred to as background risks. Background variables can be the investor's wage process and the contributions to and withdrawals from a pension fund. Menoncin (2002) models the background risks as a set of stochastic variables in analyzing the portfolio problem. By inserting the inflation risk that affects the growth rate of an investor's wealth, Menoncin also derive an exact solution to the optimal portfolio problem when the financial market is complete. Menoncin also suggests an approximated general solution if the market is incomplete.

1.3.1 Financial Market and Fund Dynamics

We assume that the financial market is arbitrage free, incomplete and continuously open over the investment time horizon $[0, T]$, where T denotes the terminal date of management contract. The independent Wiener processes $z_r(t)$ and $z_m(t)$ represents the interest rate risk and market risk, respectively. They are defined on a probability space (Ω, F, P), in which P is the real-world probability and $F = \{F(t)\}_{t\in[0,T]}$ is the filtration that represents the information structure assumed to be generated by Brownian motion and satisfying the usual conditions.

Let $r(t)$ be the interest rate at time t. Actually, we can simulate the value of $r(t)$ by calibrating the trading information of the fixed income securities. However, due to the limited trading volume in Taiwan Treasury bond in the fixed income market, model calibration merits further investigation; and hence, a one-factor spot interest rate model is employed. We assume directly that $r(t)$ follows the Vasicek model (1977). Under the real-world probability measure P, the process $r(t)$ satisfies the dynamics:

$$dr(t) = a(b - r(t))dt + \sigma_r dz_r(t), \tag{1.1}$$

where a, b, and σ_r are positive constants. The short rate $r(t)$ is mean reverting, implying that for t going to infinity, the expected interest rate would be close to the value b. Moreover, the strength of this attraction is measured by a.

There are three investment vehicles in the financial market. The first underlying asset is cash, $S_0(t)$, which pays the instantaneous interest rate without any default risk and the price process is expressed as the following stochastic differential equation:

$$\frac{dS_0(t)}{S_0(t)} = r(t)dt, \tag{1.2}$$

Then, the stochastic process of the rolling bond fund $B_K(t)$ (Rutkowski 1999) is as follows:

$$\begin{aligned} \frac{dB_K(t)}{B_K(t)} &= [r(t) + \sigma_B^K \lambda_r]dt - \sigma_B^K dz_r(t), \qquad (1.3) \\ \sigma_B^K &= \frac{1 - e^{-aK}}{a} \sigma_r. \end{aligned}$$

where σ_B^K denotes the volatility measuring how the interest rate volatility affects the bond and λ_r represents the risk premium of interest rate risk. The duration of $B_K(t)$ is fixed with K, so it is easy for application. Moreover, in asset management, manager could use cash and zero coupon bond to replicate the rolling bond fund. (see **Appendix 1**)

The other risky asset is the stock index fund, $S(t)$, whose dynamic process is given by:

$$\frac{dS(t)}{S(t)} = [r(t) + \sigma_{Sr}\lambda_r + \sigma_{Sm}\lambda_m]\, dt + \sigma_{Sr} dz_r(t) + \sigma_{Sm} dz_m(t), \tag{1.4}$$

where σ_{Sr} and σ_{Sm} are positive, indicating that the volatility scale factors are affected by interest rate risk and financial market risk, respectively. λ_m represents the risk premium of financial market risk in addition to interest rate risk.

1.3.2 Background Risks

We first express the dynamic processes of the background risks within the plan scheme. The dynamic evolution of the aggregated labor incomes from contributions is formulated since the employee must contribute a proportion of his labor income to the fund.

$$\frac{dL(t)}{L(t)} = \mu_L(t)dt + \sigma_{Lr} dz_r(t) + \sigma_{Lm} dz_m(t) + \sigma_L dz_L(t), \tag{1.5}$$

where σ_{Lr} and σ_{Lm} are the volatility factors that are affected by interest rate and market risk, respectively. Moreover, $\sigma_L \neq 0$ is a non-hedgable volatility whose risk source does not belong to $z_r(t)$ and $z_m(t)$. This non-hedgable risk is called $z_L(t)$ that is independent of $z_r(t)$ and $z_m(t)$. Moreover, $\mu_L(t)$ is the drift term of labor income process, and we assume it to be constant to simplify the derivation. Next, we assume that each employee contributes a constant proportion, γ, of his labor income into his personal account.

Then, we introduce the other background risk, the inflation risk. We use the consumption price index (CPI) to represent the inflation rate. Hence, we present the stochastic partial differential equation describing the evolution of CPI (C).

$$\frac{dC(t)}{C(t)} = \mu_\pi dt + \sigma_{\pi r} dz_r(t) + \sigma_{\pi m} dz_m(t) + \sigma_\pi dz_L(t), \tag{1.6}$$

Similarly, CPI process is affected by $z_r(t)$, $z_m(t)$ and $z_L(t)$. In particular, we call W_N the nominal fund and W the real fund. According to the Fisher equation (1930), we can write (Battocchio and Menoncin 2002):

$$dW = dW_N - W_N \frac{dC}{C}. \tag{1.7}$$

In the above conversion equation, when we want to convert nominal fund into real fund wealth, we need to incorporate the difference that is caused by change in inflation. Note that the difference is in the form of $\frac{dC}{C}$, so the difference is related only to the increasing rate of inflation. For simplicity, in Eq.(1.6) we assume that the increasing rate of inflation is just a constant. Therefore, C is not a state variable when we derive the optimal solution.

1.3.3 Fund Dynamics

We assume that the fund manager invest $w(t) = \begin{bmatrix} 1 - w_S - w_B & w_S & w_B \end{bmatrix}$ proportions of wealth into cash, stock index fund and rolling bond fund at time t. Then the accumulated nominal wealth process at any time $t \in [0, T]$ must satisfy:

$$\begin{aligned} dW_N &= W_N \left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right] + \gamma dL \\ &\quad - e_1 \max(W_N \left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right], 0) \\ &\quad - e_2 \min(W_N \left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right], 0), \end{aligned} \tag{1.8}$$

where γdL represents the contribution to the pension fund, e_1 denotes the incentive fee ratio when the fund return is positive and e_2 denotes the partial floor protections when the fund return is negative. In the above equation, we can see that the fund manager must charge management incentive fees or face loss compensation, which correlates with the fund's performance. In other words, if the fund performance is good, the fund manager should charge higher management incentive fees. Eq.(1.8) can be viewed as an option problem; however, it is too complicated and difficult to solve. In order to simplify the problem, we assume that $e_1 = e_2 = e$. Simultaneously, we substitute Eq.(1.7) into Eq.(1.8), the accumulated real fund wealth process at any time $t \in [0, T]$ can be written in a reduced form as follows:

$$dW = W_N(1-e)\left[\left((1-w_S-w_B)\frac{dS_0}{S_0} + w_S\frac{dS}{S} + w_B\frac{dB_K}{B_K}\right)\right] + \gamma dL - W_N\frac{dC}{C} \quad (1.9)$$

which can be also written as:

$$dW = W_N\left[(1-e)\left((1-w_S-w_B)\frac{dS_0}{S_0} + w_S\frac{dS}{S} + w_B\frac{dB_K}{B_K}\right) - \frac{dC}{C}\right] + \gamma dL \quad (1.10)$$

After substituting the dynamics of underlying assets, inflation rate and labor income into Eq.(1.10), we obtain:

$$\begin{aligned} dW &= \left\{W_N\left[(1-e)w_G'M + (r - re - \mu_\pi)\right] + \gamma L\mu_L^i\right\} dt \qquad (1.11) \\ &\quad + \left\{W_N\left[\Phi' + (1-e)w_G\Gamma\right] + \gamma L\Lambda\right\} dZ, \end{aligned}$$

where

$$\begin{aligned} w_G &= \begin{bmatrix} w_S & w_B \end{bmatrix}', \; M = \begin{bmatrix} \sigma_{Sr}\lambda_r + \sigma_{Sm}\lambda_m & \sigma_B^K\lambda_r \end{bmatrix}', \\ \Phi &= \begin{bmatrix} -\sigma_{\pi r} & -\sigma_{\pi m} & -\sigma_\pi \end{bmatrix}', \; \Lambda = \begin{bmatrix} \sigma_{Lr} & \sigma_{Lm} & \sigma_L \end{bmatrix}', \\ Z &= \begin{bmatrix} z_r & z_m & z_L \end{bmatrix}', \; \Gamma = \begin{bmatrix} \sigma_{Sr} & \sigma_{Sm} & 0 \\ -\sigma_B^K & 0 & 0 \end{bmatrix}. \end{aligned}$$

1.4 Asset Allocation for Restricted Form

Since the fund manager's attitude to risk varies, utility function is employed to measure investor's satisfaction with wealth accumulation. The goal of the fund manager is to construct an optimal investment strategy to maximize the expected utility value of the terminal wealth.

1.4.1 Stochastic Optimal Control

The stochastic optimal control problem is written as follows:

$$\underset{w}{Max} E_0 \left[K(W(T)) \mid W(0) = W_0, \nu(0) = \nu_0 \right],$$

$$\begin{aligned} d\begin{bmatrix} \nu \\ W \end{bmatrix} &= \begin{bmatrix} \mu_\nu \\ W_N \left[(1-e) w_G' M + (r - re - \mu_\pi) \right] + \gamma L \mu_L^i \end{bmatrix} dt \\ &+ \begin{bmatrix} \Omega' \\ W_N \left[\Phi' + (1-e) w_G \Gamma \right] + \gamma L \Lambda' \end{bmatrix} dZ, \end{aligned}$$

where

$$\begin{aligned} \nu &= \begin{bmatrix} r & L \end{bmatrix}'; \\ \mu_\nu &= \begin{bmatrix} a(b-r) & L\mu_L^i \end{bmatrix}'; \\ \Omega' &= \begin{bmatrix} \sigma_r & 0 & 0 \\ L\sigma_{Lr} & L\sigma_{Lm} & L\sigma_L \end{bmatrix}. \end{aligned}$$

$K(W)$ is the utility function.The scale variables W and ν represent the two state variables, while the elements w_G of represent the two control variables. Let $J(t, W_0, \nu_0)$ denote the value function of our optimal control problem, then it follows that:

$$J(t; W_0, \nu_0) = E_t \left[K(W(T)) \mid W(0) = W_0, \nu(0) = \nu_0 \right].$$

Then, we can get the Hamiltonian equation:

$$
\begin{aligned}
H &= \mu_\nu' J_\nu + J_W \left[W_N((1-e)w_G' M + r - re - \mu_\pi) + \gamma L \mu_L^i\right] \\
&+\frac{1}{2} tr(\Omega' \Omega J_{\nu\nu}) + (W_N \Phi' + W_N(1-e) w_G \Gamma + \gamma L \Lambda')\Omega J_{\nu W} \\
&+\frac{1}{2} J_{WW}[W_N^2 (1-e)^2 w_G \Gamma \Gamma' w_G + W_N^2 \Phi' \Phi + \gamma^2 L^2 \Lambda' \Lambda \\
&+2W_N^2 (1-e) w_G \Gamma \Phi + 2\gamma L W_N (1-e) w_G \Gamma \Lambda + 2\gamma L W_N \Lambda' \Phi)],
\end{aligned}
$$

where we denote $J_\nu \equiv \frac{\partial J}{\partial \nu}$, $J_W \equiv \frac{\partial J}{\partial W}$, $J_{\nu\nu} \equiv \frac{\partial^2 J}{\partial \nu^2}$, $J_{WW} \equiv \frac{\partial^2 J}{\partial W^2}$, and $J_{\nu W} \equiv J_{W\nu} \equiv \frac{\partial^2 J}{\partial \nu \partial W}$.

Next, we apply the first-order condition to the Hamiltonian equation and obtain the optimal weights w_G^*.

$$
\begin{aligned}
w_G^* &= -\frac{J_W}{J_{WW}} \frac{1}{W_N(1-e)} (\Gamma\Gamma')^{-1} M - \frac{1}{J_{WW}} \frac{1}{W_N(1-e)} (\Gamma\Gamma')^{-1} \Gamma \Omega J_{\nu W} \quad (1.12) \\
&-\frac{1}{1-e} (\Gamma\Gamma')^{-1} \Gamma \Phi - \frac{1}{W_N(1-e)} \gamma L (\Gamma\Gamma')^{-1} \Gamma \Lambda.
\end{aligned}
$$

In order to illustrate the optimal behavior, we adopt the results of Markus and William (2004) and rewrite Eq.(1.12) as follows:

$$
\begin{aligned}
w_G^{*\prime} &= -A \frac{J_W}{W_N(1-e) \cdot J_{WW}} w_M' - B \frac{1}{W_N(1-e) \cdot J_{WW}} J_{\nu W}' w_Y' \\
&-C \frac{1}{(1-e)} w_P' - D \frac{\gamma L}{W_N(1-e)} w_L',
\end{aligned}
$$

where

$$
\begin{aligned}
w_M &= \frac{M'(\Gamma\Gamma')^{-1}}{I'M'(\Gamma\Gamma')^{-1}},\ w_Y = \frac{\Omega\Gamma'(\Gamma\Gamma')^{-1}}{I'\Omega\Gamma'(\Gamma\Gamma')^{-1}},\ w_P = \frac{\Phi\Gamma'(\Gamma\Gamma')^{-1}}{I'\Phi\Gamma'(\Gamma\Gamma')^{-1}},\ w_L = \frac{\Lambda\Gamma'(\Gamma\Gamma')^{-1}}{I'\Lambda\Gamma'(\Gamma\Gamma')^{-1}} \\
A &= I'M'(\Gamma\Gamma')^{-1},\quad B = I'\Omega\Gamma'(\Gamma\Gamma')^{-1},\quad C = I'\Phi\Gamma'(\Gamma\Gamma')^{-1},\quad D = I'\Lambda\Gamma'(\Gamma\Gamma')^{-1}.
\end{aligned}
$$

The vectors w_M', w_P' and w_L' are two-dimensional with elements that sum to 1, and w_Y' is of dimension 2×2 with row elements that sum to 1; A, B, C and D are real constants. The optimal portfolio consists of five single portfolios: the market portfolio $w_M{'}$, the hedge portfolio for the state variables $w_Y{'}$, the hedge portfolio for the inflation risk $w_P{'}$, the hedge portfolio for the salary uncertainty $w_L{'}$, and cash. Thus, we can state the following results.

1. The first term denotes a market portfolio, and its investment weight is equal to $-A\frac{J_W}{W_N(1-e)\cdot J_{WW}}$. We should note that this is a speculative component proportional to both the portfolio Sharpe ratio and the inverse of the Arrow-Pratt risk aversion index. In other words, this portfolio's investment weight will be influenced by the fund manager's risk aversion index.

2. The second term denotes a state variable (i.e., the interest rate and market risk) hedge portfolio. This component provides us a detailed mutual fund in the capital market to hedge the uncertainties.

3. The third and fourth components are enthralling. For the background risks (labor income uncertainty and inflation risk), there exist no perfect hedging instruments in the financial markets.

However, the third and fourth portfolios show how background risks can be hedged in the capital market, and these components are preference-free component depending only on the diffusion terms of assets and background variables.

According to our five separated mutual funds, a pension fund manager who plan to hedge market risk, interest rate risk, inflation rate risk and the labor income uncertainty should invest the wealth into the following five funds:

1. The market portfolio $w_M{'}$ with level $-A\frac{J_W}{W_N(1-e)\cdot J_{WW}}$.

2. The state variable hedge portfolio $w_Y{'}$ with level $-B\frac{1}{W_N(1-e)\cdot J_{WW}}J_{\nu W}{'}$.

3. The inflation hedge portfolio $w_P{'}$ with level $-C\frac{1}{(1-e)}$.

4. The salary uncertainty hedge portfolio $w_L{'}$ with level $-D\frac{\gamma L}{W_N(1-e)}$.

5. Finally, cash with level $1 + A\frac{J_W}{W_N(1-e)\cdot J_{WW}} + B\frac{1}{W_N(1-e)\cdot J_{WW}} J_{\nu W}{'} + C\frac{1}{(1-e)} + D\frac{\gamma L}{W_N(1-e)}$.

Note that e is commonly between zero and one. Therefore, $\frac{1}{1-e}$ is greater than one. Thus, when we take the incentive fees or loss compensations into account, we find that the weights in risky assets increase. In other words, the levels invested in the market portfolio, the state variable hedge portfolio, the inflation hedge portfolio and the salary uncertainty hedge portfolio increase; while the level invested in cash decreases. This seems rational and reasonable. Since the fund manager charges the management incentive fee from the pension account and the management fee ratio is positively correlated with the pension fund's preference (i.e. the fund's return), it is necessary to pay extra money in the hedging components.

1.4.2 An Exact Solution

We substitute the optimal weights w_G^* into the Hamiltonian equation to obtain H^*, then:

$$
\begin{aligned}
H^* \; = \; & \mu_\nu{'}J_\nu + J_W\left[W_N(r - re - \mu_\pi) + \gamma L\mu_L^i - (\mathbb{A}{'} + \mathbb{B}{'})\Gamma{'}(\Gamma\Gamma{'})^{-1}M\right] \qquad (1.13) \\
& + \frac{1}{2}tr(\Omega{'}\Omega J_{\nu\nu}) - \frac{1}{2}\frac{(J_W)^2}{J_{WW}}M{'}(\Gamma\Gamma{'})^{-1}M - \frac{J_W}{J_{WW}}M{'}(\Gamma\Gamma{'})^{-1}\Gamma\Omega J_{\nu W} \\
& + (\mathbb{A}{'} + \mathbb{B}{'})(I - \Gamma{'}(\Gamma\Gamma{'})^{-1}\Gamma)\Omega J_{\nu W} - \frac{1}{2}\frac{1}{J_{WW}}J_{\nu W}{'}\Omega{'}\Gamma{'}(\Gamma\Gamma{'})^{-1}\Gamma\Omega J_{\nu W} \\
& + \frac{1}{2}J_{WW}(\mathbb{A}{'} + \mathbb{B}{'})(I - \Gamma{'}(\Gamma\Gamma{'})^{-1}\Gamma)(\mathbb{A} + \mathbb{B}),
\end{aligned}
$$

where we denote $\mathbb{A} = W_N\Phi$, $\mathbb{B} = \gamma L\Lambda$, and that I is the identity matrix.

In the financial literature, researchers commonly use separability condition to solve this PDE. Accordingly, following the previous works in Battocchio and

Menoncin (2004), our value function is assumed to be given by the product of two terms: an increasing and concave function of the wealth W, and an exponential function depending on time and interest rates. Then, the value function J and utility function can be written as follows:

$$\begin{aligned} J(t;W,\nu) &= U(W)e^{h(\nu,t)} \\ U(W) &= \beta_1 e^{\beta_2 W}, \end{aligned} \tag{1.14}$$

Then, substituting Eq.(1.14) into Eq.(1.13), we obtain $J(t,W,\nu)h_t + H^* = 0$ and $h(T,\nu(T)) = 0$ after dividing by J, we can write the Eq.(1.13) in the following way:

$$\begin{aligned} 0 \;=\; & h_t + \mu_\nu \acute{h}_\nu + \frac{U_W}{U}\left[W_N(r - re - \mu_\pi) + \gamma L\mu_L^i - (\acute{\mathbb{A}} + \acute{\mathbb{B}})\acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}M\right] \\ & + \frac{1}{2}tr(\acute{\Omega}\Omega(h_{\nu\nu} + h_\nu \acute{h}_\nu)) - \frac{1}{2}\frac{(U_W)^2}{U_{WW}U}\acute{M}(\Gamma\acute{\Gamma})^{-1}M - \frac{(U_W)^2}{U_{WW}U}\acute{M}(\Gamma\acute{\Gamma})^{-1}\Gamma\Omega h_\nu \\ & + \frac{U_W}{U}(\acute{\mathbb{A}} + \acute{\mathbb{B}})(I - \acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}\Gamma)\Omega h_\nu \\ & - \frac{1}{2}\frac{(U_W)^2}{U_{WW}U}h_\nu \acute{\Omega}\acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}\Gamma\Omega h_\nu \\ & + \frac{1}{2}\frac{U_{WW}}{U}(\acute{\mathbb{A}} + \acute{\mathbb{B}})(I - \acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}\Gamma)(\mathbb{A} + \mathbb{B}). \end{aligned}$$

We have that $\frac{U_W}{U}$ is β_2 and $\frac{(U_W)^2}{U_{WW}U}$ is 1, then substitute these two values into the above equation. Therefore, the HJB equation can be written as follows:

$$\begin{aligned} 0 \;=\; & h_t + [\acute{\mu_\nu} - \acute{M}(\Gamma\acute{\Gamma})^{-1}\Gamma\Omega + \beta_2(\acute{\mathbb{A}} + \acute{\mathbb{B}})(I - \acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}\Gamma)\Omega]h_\nu + \frac{1}{2}tr(\acute{\Omega}\Omega h_{\nu\nu}) \\ & - \frac{1}{2}h_\nu \acute{\Omega}\acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}\Gamma\Omega h_\nu + \beta_2\left[W_N(r - re - \mu_\pi) + \gamma L\mu_L^i - (\acute{\mathbb{A}} + \acute{\mathbb{B}})\acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}M\right] \\ & - \frac{1}{2}\acute{M}(\Gamma\acute{\Gamma})^{-1}M + \frac{1}{2}\beta_2^2(\acute{\mathbb{A}} + \acute{\mathbb{B}})(I - \acute{\Gamma}(\Gamma\acute{\Gamma})^{-1}\Gamma)(\mathbb{A} + \mathbb{B}). \end{aligned}$$

This kind of partial differential equation can be solved using the $Feynman - K\breve{a}c$ theorem, and so we can find the functional form of $h(\nu;t)$, which is given by:

$$h(\nu;t) = E_t\left[\int_t^T g(\widetilde{\nu}(s), s)ds\right],$$

where

$$\begin{aligned}
d\widetilde{\nu}(s) &= [\mu_\nu{}' - M'(\Gamma\Gamma')^{-1}\Gamma\Omega + \beta_2(A' + B')(I - \Gamma'(\Gamma\Gamma')^{-1}\Gamma)\Omega]'ds + \Omega(\widetilde{\nu}(s), s)'dW, \\
\widetilde{\nu}(s) &= \nu(s), \\
g(\widetilde{\nu}(t), t) &= \left[F_N(r - re - \mu_\pi) + \gamma L\mu_L^i - (A' + B')\Gamma'(\Gamma\Gamma')^{-1}M\right] - \frac{1}{2}\frac{1}{\beta_2}M'(\Gamma\Gamma')^{-1}M \\
&\quad + \frac{1}{2}\beta_2(A' + B')(I - \Gamma'(\Gamma\Gamma')^{-1}\Gamma)(A + B).
\end{aligned}$$

Finally, the optimal portfolio is written as follows:

$$\begin{aligned}
w_G^* &= -\frac{1}{\beta_2}\frac{1}{W_N(1-e)}(\Gamma\Gamma')^{-1}M \\
&\quad -\frac{1}{\beta_2}\frac{1}{W_N(1-e)}(\Gamma\Gamma')^{-1}\Gamma\Omega \cdot \int_t^T \frac{\partial}{\partial\nu}E_t\left[g(\widetilde{\nu}(s), s)\right] ds \\
&\quad -\frac{1}{1-e}(\Gamma\Gamma')^{-1}\Gamma\Phi - \frac{1}{W_N(1-e)}\gamma L(\Gamma\Gamma')^{-1}\Gamma\Lambda.
\end{aligned} \tag{1.15}$$

Note that the derivation of the second term of Eq.(1.15) is shown in **Appendix 2**.

1.4.3 Numerical Illustrations

In our numerical illustrations, numerical simulation is employed to demonstrate the dynamic behaviors of both the optimal portfolio strategy and the optimal separation portfolio strategy, which were derived in Section 1.4.2. Table 1.1 reports the set of parameters describing the financial market and background risks. Note that some parameters are consistent with the numerical analysis presented by Battocchio and Menoncin (2004).

Figure 1.1 plots the optimal portfolio holdings of cash, stocks and nominal bonds as a function of investment horizon, i.e., 30 years. We find that the optimal investment decision is short-selling cash and buying stocks and bonds during the early period, and then decreasing the holdings of risky assets when the terminal date draws near.

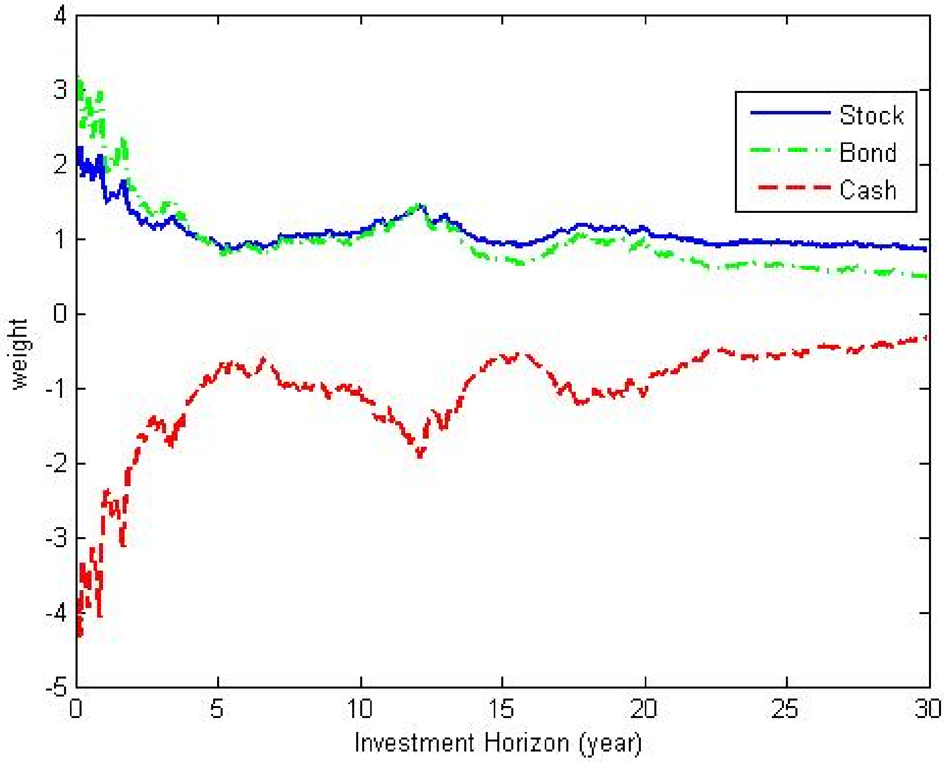

Figure 1.1: Optimal portfolio holdings of cash, stocks and nominal bonds given time horizon $T = 30$ years. (blue solid line: the weight of stock index fund, green dashed-dot line: the weight of rolling bond fund, and red dashed line: the weight of cash.)

Table 1.1: Parameter values used in the numerical analyses

Notation	Values	Notation	Values
Interest rate process		**Rolling bond process**	
Mean reversion a	0.2	Maturity K	10
Mean rate b	0.05	Market price of interest rate risk λ_r	0.15
Volatility σ_r	0.02	**Defined contribution process**	
Initial rate r_0	0.03	Drift term μ_L	0.046
Stock index process		Volatility σ_{Lr}	0.014
Market price of market risk λ_m	0.31	Volatility σ_{Lm}	0.153
Volatility σ_{Sr}	0.06	Volatility σ_L	0.01
Volatility σ_{Sm}	0.17	Initial salary L_0	100
Inflation process		Contribution rate γ	0.06
Drift term μ_π	0.015	Investment horizon T	10
Volatility $\sigma_{\pi r}$	0.018	Incentive mechanism e	0.01
Volatility $\sigma_{\pi m}$	0.136	Risk averse parameter β_2	-20
Volatility σ_π	0.015		

Figure 1.2 confirms the separation of four fund effects in optimal portfolio selections and their behaviors in each component over time. The market portfolio has shown a decreasing trend for stock index and bond fund holdings due to the utility maximization principle. In contrast, the state variable hedge portfolio shows a steady pattern for the optimal weight for bond fund and stock index holdings. To hedge the risk from state variables, the investment strategy needs to hold a fixed proportion of bond fund and also reduce the holding of the stock index. In the inflation hedge portfolio, the investors are required to hold a high proportion of the stock index, up to 80%, to hedge the inflation risks, while only a small proportion of bond fund is sufficient in the hedge portfolio. However, in the labor income hedge portfolio, the investor should short-sell his stock index and the bond portfolio in order to preserve the salary uncertainty over his investment horizon.

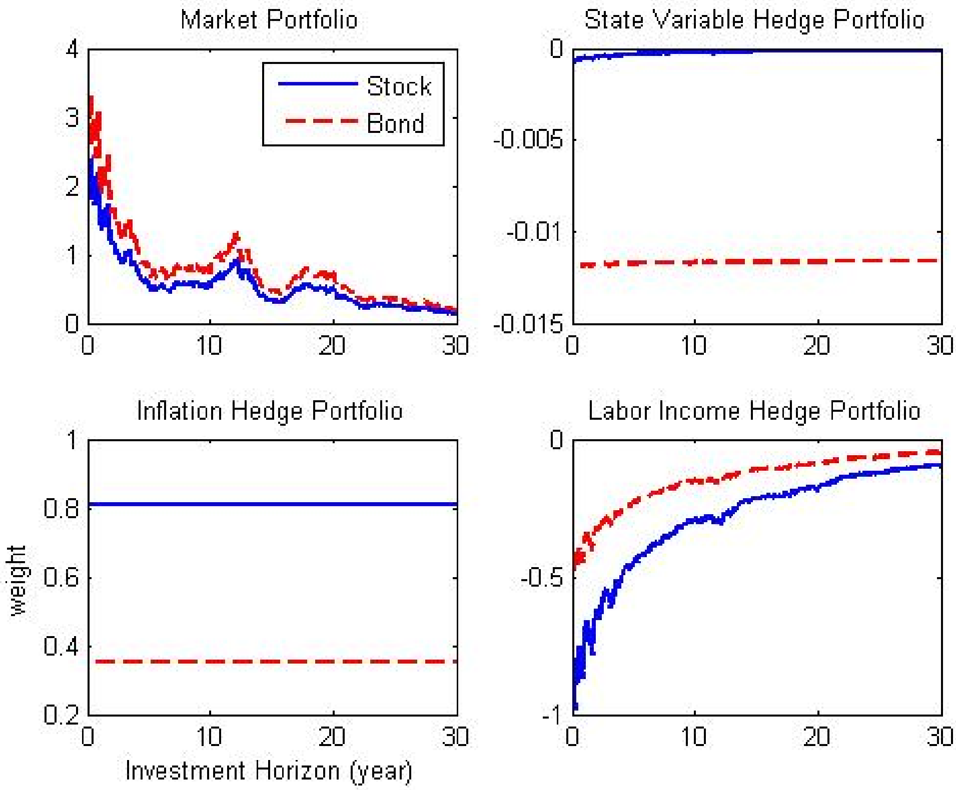

Figure 1.2: Separation of four fund effects in optimal portfolio selections and their behaviors in each component over time. (blue solid line: the weight of stock index fund, and red dashed line: the weight of rolling bond fund.)

In Figure 1.3, the weights of the stock index and bond in the entire optimal portfolios and the weights for the separated mutual funds are shown for illustration. As can be seen, the inflation hedge portfolio constitutes the overwhelming proportion (75%) of the optimal portfolios. On the other hand, when the time is close to the end of the investment horizon, the state variable hedge portfolio, market myopic portfolio and labor income hedge portfolio play only minor parts in the optimal portfolio selection. As for bond fund, these results indicate that the inflation hedge portfolio (around 35%) constitutes the largest proportion of all long-term financial portfolios. In the beginning of the investment period, the myopic portfolio is the main proportion of bond fund. However, the market myopic portfolio and labor income hedge portfolio play only minor roles in the optimal portfolio selection.

However, the optimal weights (Eq. (1.15)) are relative to the fund wealth W_N and labor income L. Therefore, in each simulation (different market condition), the investment strategy is diverse. We perform $50,000$ simulations to find the trends of optimal investment portfolios. Figure 1.4 displays the largest, medium and lowest weights of cash, stock index fund and rolling bond fund. We find that the fund manager has to short-sell cash in every market situation and during the earlier period ($T = 0$ to $T = 10$), the optimal weights are more volatile. The optimal decision suggests that investors should hold risky assets and decline the weights with time. Similarly, the optimal weights of stock index fund and rolling bond fund are more volatile during the earlier period.

In Figure 1.5, we present the investment trends of stock index fund and rolling bond fund in the market portfolio, state variable hedge portfolio and salary uncertainty hedge portfolio. The investment proportions of the inflation hedge portfolio are not presented here because the weights are constant and the values are the same as those in Figure 1.2. From these plots, we find that investors have to adjust the optimal weights of underlying assets according to different financial market situa-

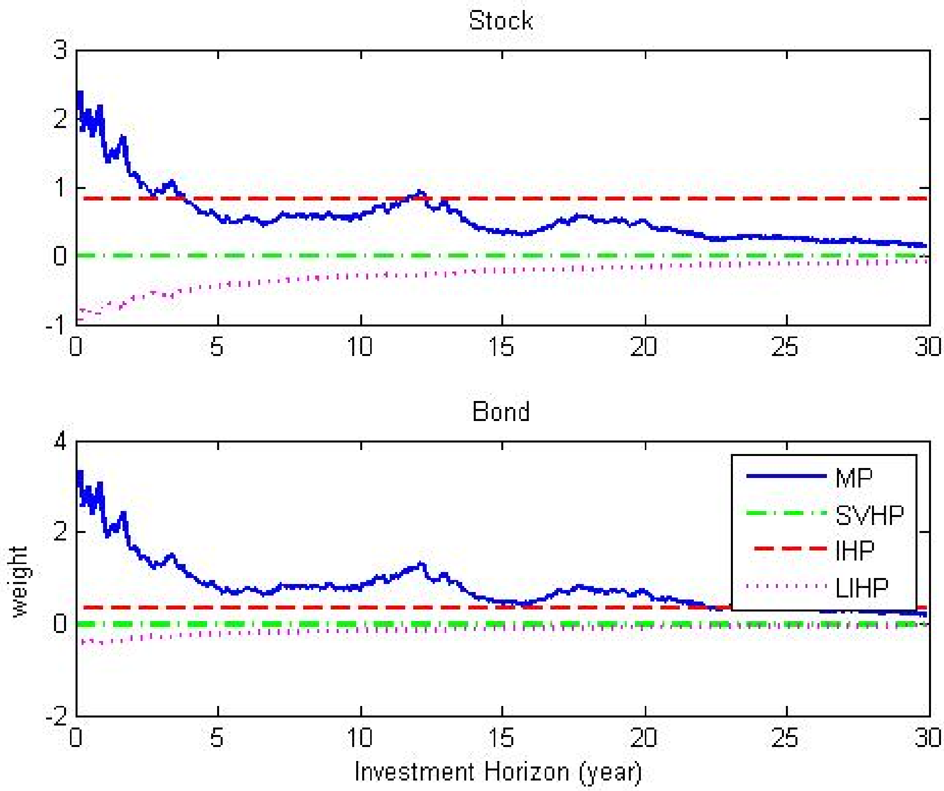

Figure 1.3: Weights of stock index and bond for separated mutual funds. (blue solid line: market portfolio, green dashed-dot line: state variable hedge portfolio, red dashed line: inflation hedge portfolio, and purple dotted line: labor income hedge portfolio.)

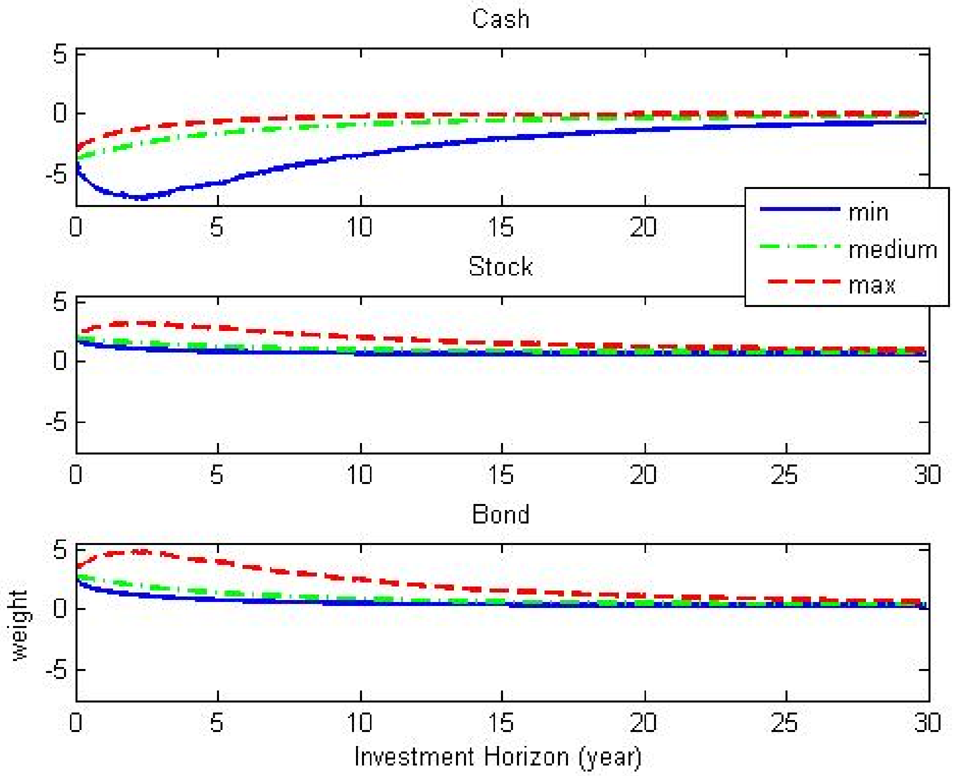

Figure 1.4: Optimal portfolio holding distributions of cash, stocks and nominal bonds given time horizon $T = 30$ years. (blue solid line: minimum weights, green dashed-dot line: medium weights, and red dashed line: maximum weights.)

tions. For both the stock index fund and rolling bond fund, the volatility of market portfolio is larger than that of state variable hedge portfolio and labor income hedge portfolio. However, these volatilities will further decrease when the time horizon is close to the terminal investment date. A reasonable explanation is that the objective of market portfolio is to seek the highest return. Hence, its results show much volatility related to the financial variation. However, the other hedge portfolios have shown relatively stable property.

As seen in Figure 1.5, the optimal investment weights are important and fund managers should not make the same allocation decision in every financial market condition. Investors could adjust the proportions of underlying assets to optimize the expected terminal wealth according to our closed-form solution.

1.5 Asset Allocation for General Form

In this section, we seek to obtain the solution for the investment problem in general form, i.e., $e_1 \neq e_2$, in order to investigate the financial impact due to the bonus fee and downside penalty in performance-oriented arrangement. Moreover, when $e_1 = e_2$ and $p_1 = p_2 = 0$, it means that the fund sponsors reward and penalize the managers at equal standard. However, the distribution of market return is not symmetrical; thus, the performance contract has to set different benchmark rates and participated rates of bonus fees as well as downside penalty.

1.5.1 Optimal Investment Decision

The optimal problem is reset as follows:

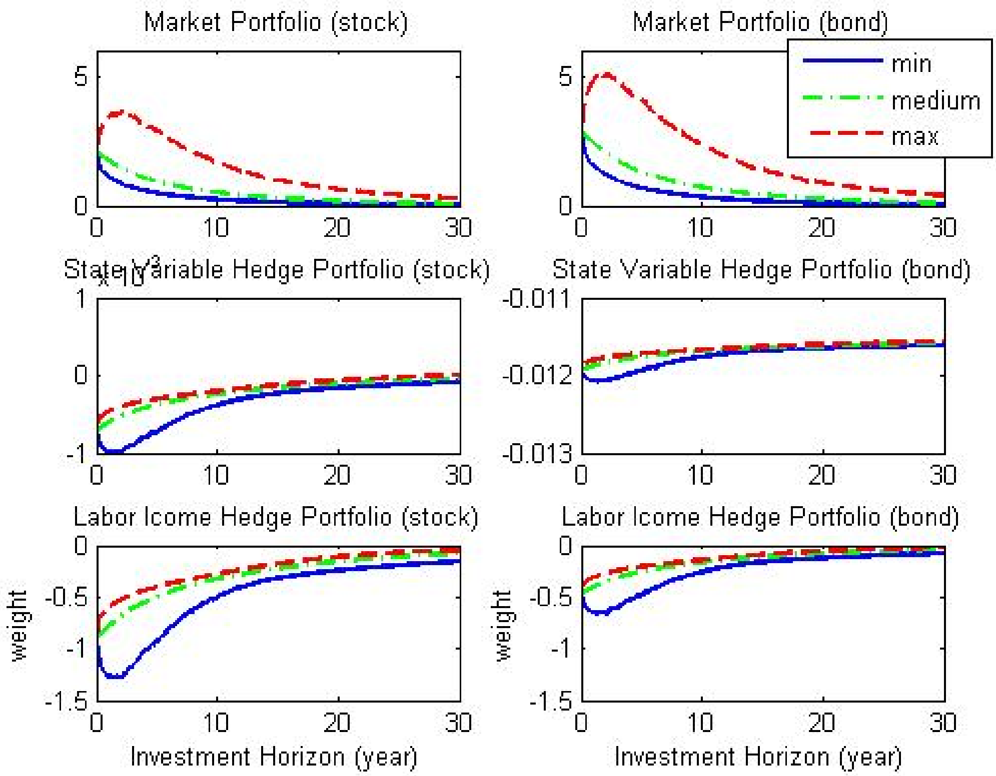

Figure 1.5: Weight distributions of stock index and bond for separated mutual funds. (blue solid line: minimum weights, green dashed-dot line: medium weights, red dashed line: maximum weights. Left: the weights of stock index fund, and Right: the weights of rolling bond fund.)

$$
\begin{aligned}
dW_N &= W_N \left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right] + \gamma dL \qquad (1.16) \\
& -e_1 W_N \max \left(\left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right], p_1 \right) \\
& -e_2 W_N \min \left(\left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right], p_2 \right)
\end{aligned}
$$

Note that it is assumed that there exist in the investment mandate two benchmarks p_1 and p_2 in triggering the bonus fees and downside penalty. This kind of structural setup is intended to be similar to the contract requirement, i.e., according to the Taiwan labor pension fund management regulation, the return rate cannot be less than the interest rate of 2-year fixed deposit. In this regard, when the fund performance is better than p_1, the fund manager is entitled to get the bonus fee; otherwise, the manager is required to reduce the management fee due to downside penalty. In our performance mechanism, the optimal problem contains two kinds of financial options and the explicit solution is sometimes hard to find. Thus, in this section, the optimization method is employed to solve the problem. In case (1.16), we find that the fund manager takes greater risks in seeking higher return since the fund manager is required to guarantee the minimum return rate p_2, which shows that the fund sponsor need not worry about the downside risk of the investment. Thus, the optimal asset allocation is investigated from the view of pension fund manager. The optimal investment problem becomes:

$$
\begin{aligned}
dV &= e_1 W_N \max \left(\left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right], p_1 \right) \\
& +e_2 W_N \min \left(\left[(1 - w_S - w_B) \frac{dS_0}{S_0} + w_S \frac{dS}{S} + w_B \frac{dB_K}{B_K} \right], p_2 \right) \qquad (1.17)
\end{aligned}
$$

where V denotes the fund surplus, i.e., the fund manager is rewarded through obtaining the bonus fee when the fund performance is better than p_1. On the other hand, the management fee of the fund manager has to be reduced due to downside penalty when his performance is worse than the investment benchmark p_2. This is a combined structure of asset-based and target-based incentive mechanisms. In computation, the MATLAB program is written to apply the proposed optimization method in computing the optimal investment weights in Eq. (1.17). In each scenario, $50,000$ realizations are simulated and the short-selling restriction is also employed. The tradeoff parameters e_2 and the performance benchmark p_2 are assumed to be 1 and 2%, respectively. The investment time horizon in our illustration is set to be 10 years.

In Figure 1.6, the optimal multi-period investment strategies are illustrated given different e_1 and p_1. As seen in plots 1.6.1, 1.6.2 and 1.6.3, the optimal weights with increasing p_1 under e_1 is 0.7%. In the plots 1.6.4, 1.6.5 and 1.6.6, p_1 is 0.5% and is rising from 3% to 5%. The blue solid line represents the optimal investment weights of the stock index fund. The red dashed line denotes the proportion of fund in cash. The third line illustrates the weights of rolling bond fund.

First, Figure 1.6 shows that the fund manager will increase the holding of stock index fund as the investment horizon approaches maturity. A probable explanation is that under the optimal investment decision, the performance of fund will be better than the benchmark; hence, the manager increases the risk of portfolio in order to seek higher bonus fees. Comparing Figure 1.1 with Figure 1.4 shows that the optimal investment strategy is to hold more risky assets since the weights in these figures exceed 100%. However, in Figure 1.6, we set the short-selling constrain; thus the holding weight of stock index fund is close to 1 at maturity date. Secondly, the weights of cash are small because its return rate is low. Fund managers would not prefer to hold cash because they have to meet the requirement of minimum guarantees p_2.

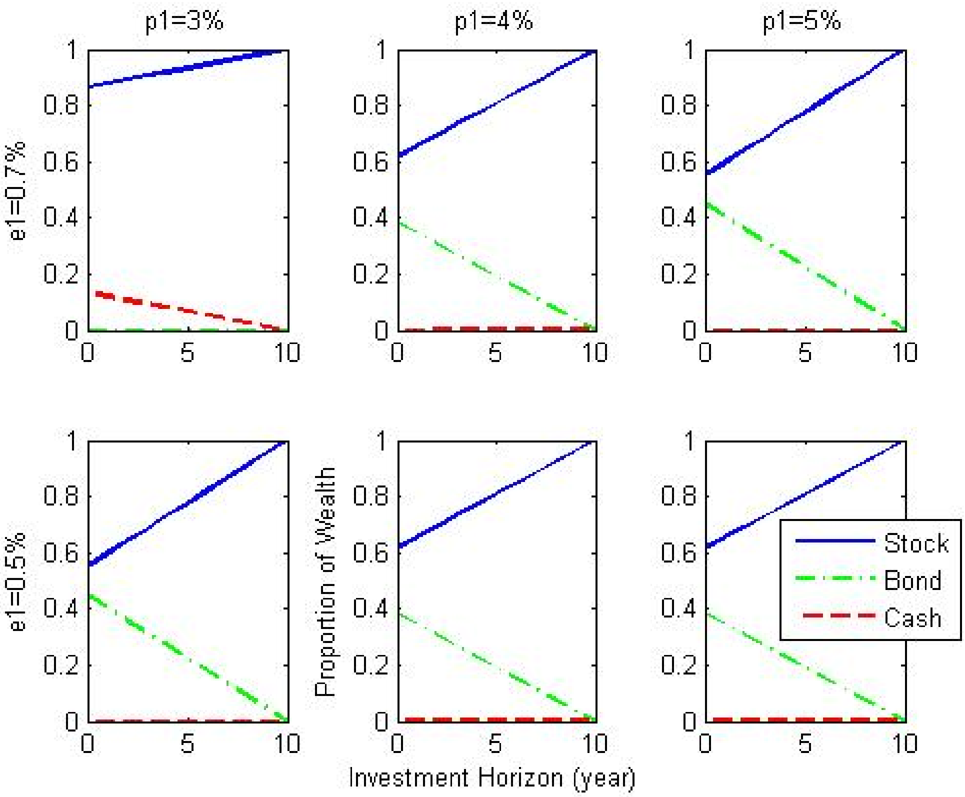

Figure 1.6: Optimal portfolio holdings of cash, stocks and nominal bonds given time horizon T = 10 years under incentive programs. (blue solid line: weights of stock index fund, green dashed-dot line: weights of rolling bond fund, red dashed line: weights of cash. Up left: and , Up centre: and , Up right: and , Bottom left: and , Bottom centre: and , and Bottom right: and)

However, Figure 1.6 shows certain diverse characteristics of the investment behaviors. In plots 1.6.1, 1.6.2 and 1.6.3, the weights of stock index fund (blue solid line) in the beginning are decreasing from 0.88 to 0.58 when p_1 increases. On the other hand, the allocation in plots 1.6.4, 1.6.5 and 1.6.6 shows that when p_1 decreases to 0.5%, the fund manager would hold less stock index fund from 0.48 to 0.62 in the beginning with increasing p_1. This is interesting that the settlement of bonus fees would affect the investment behaviors of fund managers. In other words, this phenomenon implies that the settlement of incentive mechanism contract is important on the delegated management contract because the parameters would affect the investment behaviors of fund managers.

1.5.2 Financial Implication

In Section 1.4.3, the optimal investment decisions are simulated under certain constrains ($e_1 = e_2 = e$ and $p_1 = p_2 = 0$). The fund manager will increase the holding in risky asset. Under the downside protection arrangement, a fund holder will tend to increase the risk profile of fund portfolio. In Section 1.5.1, more realistic performance mechanisms including $e_1 \neq e_2$ and $p_1 \neq p_2$ are investigated. Moreover, Raghu et al. (2003) conclude that there exists the agency problem between the fund managers and the plan participants. In Section 1.5.1, we try to investigate the financial influence of performance mechanisms on the optimal investment decisions. The asset allocation problem in Section 1.4.3 is simplified to derive the explicit solution. The optimal solution varies according to the various scenarios of the financial market. In order to explore the realistic impact of the incentive mechanism, the optimization program is implemented to approximate the optimal investment weights through simulations.

For the DC pension fund management, the setup in Section 1.5.1 is more practical and vital. The optimal investment decisions of fund managers under performance mechanisms are investigated. Our model extends the previous research through implementing the unlimited liability downside protection. The unlimited liability is incorporated since the limited mechanisms would motivate the fund manager to increase the risk profile of portfolio after a period of poor performance (Edwin et al. 2003). Moreover, the labor pension plan implemented in Taiwan includes also this kind of guarantee arrangement. We find that under this performance contract, the benchmark rate (p_1) and the participated rate (e_1) of bonus fees would change the investment trend of the fund manager. This is consistent with the conclusion made by Raghu et al. (2003), that the performance would be influenced by the commission rate. That is, the participated rate of bonus fees would affect the investment behaviors. However, Raghu et al. (2003) propose that the efficacy of limited incentive is better than unlimited contract.

Moreover, the optimal investment decision is analyzed annually. The performance of fund is measured and the fund managers are also rebalancing their asset group every year. Mark (1987) discovers that length of the time horizon is very crucial. During the shorter period, the performance contract would not identify whether the success of the fund performance is the true investment ability or pure luck of the fund manager. Lawrence and Stephen (1987) also suggest that the proper performance index should employ the moving 3-year time period.

1.6 Discussions

In this Chapter, we investigate the asset allocation issue for DC labor pension fund that considers not only the market risk and interest risk, but also the uncertainties from labor incomes, the inflation risk and the incentive scheme. We find that if the fund manager would like to maximize the expected exponential utility of his terminal wealth, he can adopt the mutual fund separation theorem through five components in its optimal asset allocation. Hence, the optimal investment behaviors of the pension fund managers are characterized by the relative weights among the separated mutual funds according to their preference, the financial market and the influential factors.

With both the financial and background risks incorporated, pension fund managers are recommended to consider the short-term fund performance and the hedge requirements simultaneously. Because background risks cannot be controlled by the fund managers, a comprehensive dynamic framework is formulated to describe the decision-making process. As the results show, the dynamic portfolio of the restricted form that maximizes the expected utility of the plan participant consists of five components: the market portfolio, the state variables hedge portfolio, the inflation hedge portfolio, the salary uncertainty hedge portfolio and cash. By solving explicitly the optimal portfolio problem, the numerical results indicate that the inflation hedge portfolio constitutes the overwhelming proportion of stocks in the optimal portfolios. In addition, the inflation hedge portfolio and the state variable hedge portfolio constitute the overwhelming proportions of bond holdings. This shows that long-term investors should hedge inflation rate risk by holding the stock index. In addition, these investors should respond to the inter-temporal hedging demands in the financial markets by increasing the average allocation to their bond fund.

To understand the roles of these components, it is necessary to explore the economic interpretations by solving the dynamic optimization problems. With respect to the most common approach used in the literature, the incorporation of the labor income and inflation risks allows us to characterize the general pattern of the optimal strategy. The results indicate that the inflation hedge portfolio constitutes the main proportion of the optimal stock portfolios, while in the earlier stage the market portfolio makes up the larger part of the stock index fund. However, in the labor income hedge portfolio, the investor should short-sell his stock index and the bond portfolio in order to preserve the salary uncertainty over his investment horizon.

Finally, the optimal asset allocation strategy is solved for the general incentive mechanism. The optimal behaviors of the fund managers alter according to various parameter settings within the incentive mechanism. Our results are also consistent with the findings of Richard and Andrew (1987) and Lawrence and Stephen (1987), who confirm that the incentive setting is essential in the delegated management contract (Richard and Andrew, 1987 and Lawrence and Stephen, 1987).

Now, we work further solve the non-simplified problem, and in the future, the main goal is to find the efficient and adequate algorithm to solve the optimal asset allocation strategy.