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A study of the economic efficiencies in East
European countries using semiparametric approaches

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博士論文

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探討半參數隨機邊界模型的技術與配置效率之
一致性估計方法

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中華民國九十九年六月十日

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摘要

傳統參數隨機成本邊界模型需事先假設其函數型態，但真正的函數型態未知，若是假設錯誤的函數型態可能存在模型設定誤差，另外過去估計成本函數時，大多著重於技術效率的衡量，而忽略配置效率，如此一來，將導致模型參數估計產生偏誤，影響後來效率的計算。基於上述的問題，本研究將應用半參數隨機成本邊界模型且同時考量技術效率與配置效率，不但函數設定具有彈性且能正確的衡量效率值，然而在考量配置效率的衡量後，增加模型估計的困難度，使得估計收斂不易，因此本研究提出一個五階段的估計步驟，應用蒙地卡羅模擬進行分析，該估計步驟不但能簡化估計且能得到技術與配置效率的一致性估計。最後則將本研究提出的估計方法應用在實證研究上，探討 14 個東歐國家在轉型期間其技術與配置效率的衡量，使用不平衡縱橫資料，共 340 家商業銀行進行實證分析。

關鍵字：半參數成本邊界、核估計式、影子價格、技術效率、配置效率

Consistent Estimation of Technical and Allocative Efficiencies for a Semiparametric Stochastic Cost Frontier with Shadow Input Prices

Abstract

Conventional parametric stochastic cost frontier models are likely to suffer from biased inferences due to misspecification and the ignorance of allocative efficiency (AE). To fill up the gap in the literature, this article proposes a semiparametric stochastic cost frontier with shadow input prices that combines a parametric portion with a nonparametric portion and that allows for the presence of both technical efficiency (TE) and AE. The introduction of AE and the nonparametric function into the cost function complicates substantially the estimation procedure. We develop a new estimation procedure that leads to consistent estimators and valid TE and AE measures, which are proved by conducting Monte Carlo simulations. An empirical study using unbalanced panel data on 340 commercial banks from 14 East European countries over the period 1993-2004 is performed to help shed some light on the usefulness of our procedure.

Keywords: semiparametric cost frontier; kernel estimation; shadow prices; technical efficiency; allocative efficiency.

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1. Introduction

A parametric linear or nonlinear regression model requires setting a specific functional form prior to estimation in order to describe the true but unknown relationship between the dependent and the independent variables. Consequently, potential specification errors are likely to occur, leading to an inconsistent estimation. Although some economic models do explicitly suggest relationships among economic variables, most implications of economic theory are nonparametric. Therefore, if one has reservations about a particular parametric form, then a nonparametric function can be an alternative candidate. Nonparametric regression models permit the functional relationship to be unknown and nevertheless fit the data quite well without imposing restrictions beyond some degree of smoothness. They deliver estimators and inference procedures that are less reliant on the imposition of specific functional forms. Inclusion of the nonparametric element may circumvent an inconsistent estimation arising from invalid parameterization. However, the inherent critical element of the “curse of dimensionality” limits the unknown function of a nonparametric model to contain a small number of variables to lessen the approximation error to the unknown function.

A researcher in some cases may be confident about a particular parametric form for one portion of the regression function, but less sure about the shape of another portion. Such prior beliefs justify the necessity for linking parametric with nonparametric techniques to formulate semiparametric regression models. The added value of semiparametric techniques consists in their competence to largely mitigate the curse of dimensionality distress, and the respective estimators of the parametric

and nonparametric components have their conventional rates of convergence.¹ See, for example, Härdle (1990), Wand and Jones (1995), Fan et al. (1996), and Yatchew (1998, 2003).

Fan et al. (1996) first extended the traditional stochastic production frontier model, dated back to Aigner et al. (1977) and Meeusen and Van Den Broeck (1977), to a semiparametric frontier model in the context of cross section. They proposed pseudo-likelihood estimators and proved by Monte Carlo experiments that the finite-sample performance of their estimators is satisfactory. Deng and Huang (2008) further generalized it to a panel data setting and allowed for time-variant technical efficiency (TE) in the form of Battese and Coelli (1992). Nevertheless, almost all of the related works that use a semiparametric frontier model focus on the study of technical efficiency (TE). Kumbhakar and Wang (2006a) found that the assumption of fully allocative efficiency (AE) in a cost function tends to bias parameter estimates of the cost function and subsequent measures using these estimates.

To obtain both TE and AE measures, one is suggested to estimate the shadow cost system, consisting of an expenditure (cost) equation and the corresponding share equations, simultaneously using the maximum likelihood. Unfortunately, the highly nonlinear nature of the simultaneous equations makes the estimation almost untractable. Kumbhakar and Lovell (2000) proposed a two-step procedure with an eye to simplify somewhat the estimation problem of a pure parametric shadow cost system. The share equations are estimated in the first step by the method of nonlinear iterative seemingly unrelated regression (NISUR) to acquire the shadow price

¹ Robinson (1988) showed that the parametric estimators are consistent at the parametric rate of $N^{-1/2}$, while the nonparametric estimators converge at a slower rate than $N^{-1/2}$.

parameter estimates of interest. These estimates are treated as given in the second step, where the maximum likelihood technique is exploited to estimate the stochastic cost frontier alone after appropriately transforming the original expenditure equation using the first step estimates. This procedure is less efficient but computationally simpler. However, Kumbhakar and Lovell (2000) did not address the properties of their proposed estimators. In addition, they repeat estimating the parameters in the second step and do not specify in which step the estimates should be used to calculate technical and allocative efficiencies respectively. Thereby, the main problem is: which estimates should we choose? Do the estimates in the second step behave more efficient than those in the first step? And is it possible to take all of the first step estimates as given and then estimate the remaining parameters only? In this paper we propose three models in order to solve the aforementioned questions.

The purpose of the current work is four-fold. First, we relax the parametric restriction on a cost function representing technology in order to at least diminish the possible specification error. Second, the semiparametric stochastic shadow cost frontier offered by this paper differs from the standard semiparametric regression model and from the stochastic production frontier of Fan et al. (1996). Specifically, our model accommodates both TE and AE to avoid biased estimates of the technology parameters. To the best of our knowledge, no work has been done to introduce both efficiency measures into a semiparametric stochastic shadow cost frontier under the framework of panel data. It is hoped that this research will bridge the existing gap and to better characterize a firm's optimization behavior. Third, a distinct five-step procedure from the one suggested by Kumbhakar and Lovell (2000) is proposed to facilitate the estimation. We argue for the new procedure due to the fact that its estimators of interest are shown to converge to the true values as the sample size

increases by applying Monte Carlo simulations. Finally, an empirical study using unbalanced panel data of commercial banks from 14 East European countries spanning 1993-2004 is carried out to illustrate the superiority of our semiparametric stochastic shadow cost frontier model.

The rest of this paper is organized as follows. Section 2 briefly reviews the relevant literature. Section 3 first presents the semiparametric stochastic cost frontier with shadow input prices and then proposes the estimation procedure. Section 4 introduces the design of Monte Carlo experiments to be conducted in the next section. Section 5 provides and discusses the results of the experiments, which are intended to detect a suitable estimation procedure leading to consistent estimators. Section 6 illustrates the recommended estimation procedure with an empirical study, while the last section concludes the paper.

2. Literature Review

The TE score of a firm can be estimated by two main approaches, i.e., data envelopment analysis (DEA) and stochastic frontier approach (SFA). The former involves mathematical programming without the need for specifying an explicit functional form, while the latter employs the econometric methods to deal with the composed random disturbances. These approaches have their own advantages and weaknesses. Fan et al. (1996) elegantly extended the standard parametric SFA to a semiparametric model in the context of cross section, where the functional form of the production frontier needs not to be specified a priori. Their method makes use of nonparametric regression techniques to avoid the requirement of specifying a particular production function, associating a firm's output with inputs. Therefore, the possible problem of misspecification is no longer a key issue as opposed to the conventional parametric approach, even though a translog functional form is utilized. Deng and Huang (2008) generalized the semiparametric model of Fan et al. (1996) to a panel data setting and allowed for time-varying TE. Their empirical evidence finds that the standard parametric translog production function tends to underestimate the TE score due to the possible specification error and its lack of flexibility in describing firms' production characteristics.

Wheelock and Wilson (2001) estimated and compared the measures of scale and scope economies for U.S. commercial banks, derived from estimating parametric and nonparametric cost equations, without regard to TE and AE. In an expenditure equation modeling both technical inefficiency (TI) and allocative inefficiency (AI), it is difficult for researchers to appropriately relate the two-sided disturbances in the input share equations to the nonnegative AI term in the expenditure equation. This is known as the Greene problem (Bauer, 1990). Berger et al. (1993), Atkinson and

Cornwell (1994), Kumbhakar (1996a), Huang (2000), and Huang and Wang (2004), to mention a few, utilized shadow prices to account for AI in addition to TI. Kumbhakar (1996b, 1997) gave a complete treatment on how to model TI and AI concurrently. Kumbhakar and Wang (2006b) demonstrated an alternative primal system, consisting of a production function and the first-order conditions of cost minimization. However, the cost function associated with the translog production function cannot be analytically derived. The shadow price technique does not need to specify an ad hoc relationship between the AI term of the expenditure equation and the disturbance terms of the share equations. In addition, this technique can be applied to any parametric cost function as well as some semiparametric cost functions. We therefore adopt the technique throughout the paper.

The impact of deregulation on bank performance in East European countries has recently been studied by several researchers, e.g., Kraft and Tirtiroglu (1998), Jemric and Vujcic (2002), Nikiel and Opiela (2002), Hasan and Marton (2003), Bonin et al. (2005a, 2005b), Fries and Taci (2005), and Yildirim and Philippatos (2007). The foregoing works fail to take the potential AI into account. As the input or the output prices may be somewhat under the control of the governments of the transition nations, these prices are likely to respond to market conditions tardily. Allocative distortion may play a crucial role in allocating financial resources in these countries. This justifies the requirement of evaluating bank efficiencies on the basis of both TE and AE.

3. Semiparametric Stochastic Shadow Cost Frontiers

Let the j th shadow input prices, W_j^* , be defined as

$$W_j^* = H_j W_j, \quad j = 1, \dots, J \quad (3-1)$$

, where $H_j (>0)$ denotes the allocative parameter of input j , measuring the extent to which the shadow and actual input prices (W_j) differ. It thus reflects the degree of allocative inefficiency arising from, e.g., regulation or slow adjustment to changes in input prices. Here, a firm's decision is assumed to be grounded on shadow input prices. Following Atkinson and Cornwell (1994), Kumbhakar (1996b, 1997), and Huang and Wang (2004), the minimized efficiency adjusted shadow cost, C^{**} , for a firm employing input vector X to produce output vector Y can be expressed as:

$$\begin{aligned} C^{**}(Y, \frac{W^*}{b}) &= \min \left[\frac{W^*}{b}(bX) \mid F(bX, Y) = 0 \right] \\ &= \frac{1}{b} C^*(Y, W^*) \end{aligned} \quad (3-2)$$

, where $b (0 < b \leq 1)$ represents the degree of input-oriented TI, C^* is referred to as the shadow cost function independent of the TI parameter of b , and Y is an m -vector of output quantities. A firm is said to be technically efficient if it has a value of $b = 1$, while a firm operating beneath the efficiency frontier has a value of $b < 1$. The larger the value of b is, the more technically efficient the firm will be. Function $F(\cdot, \cdot)$ represents the production transformation function.

Since a cost function must satisfy the homogeneity restriction of degree one in input prices, we can only measure $J-1$ relative allocative parameters H_j/H_k , $j, k = 1, \dots, J$, and $j \neq k$. A value of H_j/H_k less (greater) than unity means that the

j th input tends to be overused (underused) relative to input k . Either overuse or underuse reflects the presence of AI. Using Shephard's Lemma, the shadow cost share equation of input j is written as:

$$S_j^*(W^*, Y) \equiv \frac{\partial \ln C^*}{\partial \ln W_j^*} = \frac{bW_j^* X_j}{C^*}. \quad (3-3)$$

After some manipulations and taking a natural logarithm, a firm's actual expenditure (E) can be associated with C^{**} (C^*) and S_j^* as follows:

$$\ln E = \ln C^{**} + \ln \sum_j H_j^{-1} S_j^* = \ln C^*(Y, W^*) + \ln \sum_j H_j^{-1} S_j^* + U \quad (3-4)$$

, where $U = -\ln b$ represents the additional (log) expenditure incurred by TI and is specified as a one-sided error term later, $\ln \sum_j H_j^{-1} S_j^*$ captures a partial extra cost entailed by AI, and the remaining extra cost of AI is embedded in $\ln C^*(Y, W^*)$ due to $W^* \neq W$.

Equation (3-4) becomes a regression equation after appending a two-sided random disturbance v to it, where v is assumed to be distributed as $N(0, \sigma_v^2)$. Term $U + v$ forms the composed error term. This equation associates TI with AI systematically. To identify the allocative parameters, one has to count on the share equations. It can be shown that the actual share equation of input j (S_j) is formulated as

$$S_j = \frac{H_j^{-1} S_j^*}{\sum_j H_j^{-1} S_j^*}, \quad j = 1, \dots, J. \quad (3-5)$$

After appending random disturbances to these share equations, they can be used to help estimate parameters H_j . When panel data are available, it is more ambitious to assume TI term U to be time-varying. The time-variant TE model of Battese and

Coelli (1992) is adopted with $U_{nt} = u_n \exp[-\gamma(t-T)]$, $n = 1, \dots, N$, $t = 1, \dots, T$, where u_n is a firm-specific TI random variable distributed as $|N(0, \sigma_u^2)|$ independent of v_{nt} , and $g(t) = \exp[-\gamma(t-T)]$ contains an extra parameter γ to be estimated.²

We now turn to the functional form of $\ln C^*(Y, W^*)$ in (3-4). It is conventionally specified as a translog form, or as a Fuss functional form like Berger et al. (1993), or as a Fourier flexible function such as Altunbas et al. (2001) and Huang and Wang (2004). In this paper $\ln C^*(Y, W^*)$ is formulated as a semiparametric form:

$$\ln C^*(Y_{nt}, W_{nt}^*) = X_{nt} \beta + M(\ln Y_{nt}) \quad (3-6)$$

, where X_{nt} consists of the linear and quadratic terms of $\ln W_{jnt}^*$ ($j = 1, \dots, J$), the cross product terms among $\ln W_{jnt}^*$, and the cross product terms of $\ln W_{jnt}^*$ with $\ln Y_{int}$ ($i = 1, \dots, m$), β is the corresponding unknown parameter vector, $\ln Y_{nt}$ is a $m \times 1$ random vector of (log) outputs with support, and $M(\cdot)$ is assumed to be a smooth function with unknown form.

We rewrite our cost function system as:

$$\ln E_{nt} = X_{nt} \beta + \ln G_{nt} + M(\ln Y_{nt}) + \varepsilon_{nt} \quad (3-7)$$

$$S_{jnt} = \frac{H_j^{-1} S_{jnt}^*}{\sum_j H_j^{-1} S_{jnt}^*} + \eta_{jnt}, \quad j = 1, \dots, J \quad (3-8)$$

, where $\ln G_{nt} = \ln \sum_j H_j^{-1} S_j^*$, $\varepsilon_{nt} = u_n \exp[-\gamma(t-T)] + v_{nt}$, and $\eta_{nt} = (\eta_{1nt}, \dots, \eta_{Jnt})'$ is a random vector with mean zero and constant covariance matrix. v_{nt} and η_{nt} represent the usual statistical noise and are assumed to be distributed independently of each other. It can be shown that the n th firm's probability density function of the composed disturbance $\varepsilon_n = (\varepsilon_{n1}, \dots, \varepsilon_{nT})'$ is equal to:

² Term $g(t)$ decreases at an increasing rate if $\gamma > 0$, increases at an increasing rate if $\gamma < 0$, or stays constant if $\gamma = 0$.

$$h(\varepsilon_n) = \frac{2}{\sigma_v^{T-1} \sigma} [1 - \Phi(A_n)] \left[\prod_{t=1}^T \phi\left(\frac{\varepsilon_{nt}}{\sigma_v}\right) \right] \exp\left(\frac{1}{2}(-A_n)^2\right) \quad (3-9)$$

, where $A_n = -\lambda \sum_t \varepsilon_{nt} g(t) / \sigma$, $g(t) = e^{-\gamma(t-T)}$, $\lambda = \sigma_u / \sigma_v$, $\sigma^2 = \sigma_v^2 + \sigma_u^2 \sum_{t=1}^T g^2(t)$, and $\phi(\bullet)$ and $\Phi(\bullet)$ are the standard normal density and standard normal cumulative distribution functions, respectively. The log-likelihood function of expenditure equation (3-7) alone can be easily derived by first multiplying (3-9) over firms and then taking the natural logarithm. Combining (3-9) with the joint probability density function of the $(J-1)$ random disturbances of the share equations (η_{nt}), the cost function system can be simultaneously estimated by the maximum likelihood if M has a known form.³ Readers are suggested to refer to, e.g., Ferrier and Lovell (1990) and Kumbhakar (1991), for details.

Three difficulties deserve specific mention. First, since the log-likelihood function of the above cost function system is highly nonlinear, getting maximum likelihood estimators is computationally difficult, even though not infeasible. Second, M has an unknown functional form, hindering the log-likelihood function of the expenditure equation from being maximized with respect to M in particular. One alternative relies on the use of some nonparametric approaches to estimate M . However, M cannot be estimated directly by existing nonparametric regression methods, because M is not the conditional expectation of $\ln E_{nt} - X_{nt}\beta - \ln G_{nt}$ given $\ln Y_{nt}$. This is caused by the nonzero mean of one-sided error U_{nt} , i.e.:

$$E(\ln E_{nt} - X_{nt}\beta - \ln G_{nt} | \ln Y_{nt}) = M(\ln Y_{nt}) + \mu_t(\sigma^2, \lambda, \gamma) \neq M(\ln Y_{nt}) \quad (3-10)$$

, where

$$\mu_t(\sigma^2, \lambda, \gamma) = E(U_{nt} | \ln Y_{nt}) = g(t) \frac{\sqrt{2}}{\sqrt{\pi}} \sigma_u = g(t) \frac{\sqrt{2}}{\sqrt{\pi}} \left(\frac{\lambda \sigma}{\sqrt{1 + \lambda^2 \sum_t g^2(t)}} \right) \quad (3-11)$$

One cannot separate $M(\ln Y_{nt})$ from $E(\ln E_{nt} - X_{nt}\beta - \ln G_{nt} | \ln Y_{nt})$ of (3-10) by employing a nonparametric estimation. This problem can be solved by substituting

³ Note that random vector η_{nt} must now be assumed to be distributed as a multivariate normal with mean vector zero and constant covariance matrix.

$E(\ln E_{nt} - X_{nt}\beta - \ln G_{nt} | \ln Y_{nt}) - \mu_t$ for $M(\ln Y_{nt})$ into the log-likelihood function. $E(\ln E_{nt} - X_{nt}\beta - \ln G_{nt} | \ln Y_{nt})$ can now be consistently estimated by the nonparametric approach. For details, please see, e.g., Fan et al. (1996). Finally, term $\ln G_{nt} = \ln \sum_j H_j^{-1} S_j^*$ is obviously a nonlinear function of unknown parameters, leading the kernel estimation procedure for a standard semiparametric regression model, as proposed by Robinson (1988), to be not applicable. We shall discuss possible ways of getting rid of this difficulty in Subsection 4.1, which influence the consistency of the parameter estimates and are the core of this study.

We adopt the kernel estimation technique to estimate the conditional expectations, such as $E(\ln E_{nt} | \ln Y_{nt})$, since it is one of the popular nonparametric estimation methods. Specifically, the Nadaraya-Watson kernel estimator (Nadaraya, 1964; Watson, 1964) for a scalar $\ln Y_{nt}$ is given by:

$$\hat{E}(\ln E | \ln Y_{nt}) = \frac{\sum_{i=1}^N \sum_{t=1}^T \ln E_{it} K\left(\frac{\ln Y_{nt} - \ln Y_{it}}{h}\right)}{\sum_{i=1}^N \sum_{t=1}^T K\left(\frac{\ln Y_{nt} - \ln Y_{it}}{h}\right)} \quad (3-12)$$

, where $K(\cdot)$ is the kernel function and h is the smoothing parameter. Equation (3-12) can be easily extended to a higher dimensional case of $\ln Y_{nt}$. The rest of the conditional expectations can be estimated analogously.

We now outline the estimation procedure of the semiparametric shadow cost frontier in the following five steps.

Step 1. Simultaneously estimate the $J-1$ input share equations of (3-8) by the NISUR to obtain the $J-1$ estimates of relative allocative parameters H_j/H_k ($j = 1, \dots, J$ and $j \neq k$) and a part of the parameters involving the input prices of expenditure equation (3-7).⁴ These estimates can be shown to be consistent and are

⁴ Terms involving solely (log) outputs do not emerge in the share equations after taking the partial derivatives of the expenditure equation with respect to (log) input prices.

used to calculate $\ln G_{nt}$, denoted by $\ln \hat{G}_{nt}$.

Step 2. Apply formula (3-12) to obtain the kernel estimates of $E(\ln E_{nt} | \ln Y_{nt})$, $E(X_{nt} | \ln Y_{nt})$, and $E(\ln \hat{G}_{nt} | \ln Y_{nt})$, denoted by $\hat{E}(\ln E | \ln Y_{nt})$, $\hat{E}(X | \ln Y_{nt})$, and $\hat{E}(\ln \hat{G} | \ln Y_{nt})$, respectively.

Step 3. Equation (3-7) subtracts its own conditional expectations on $\ln Y_{nt}$ to yield

$$\ln E_{nt} - E(\ln E_{nt} | \ln Y_{nt}) = [X_{nt} - E(X_{nt} | \ln Y_{nt})]\beta + \ln G_{nt} - E(\ln \hat{G}_{nt} | \ln Y_{nt}) + \varepsilon'_{nt} \quad (3-13)$$

After substituting the kernel estimates derived in Step 2 for those conditional expectations in (3-13), parameters β can be consistently estimated by the nonlinear least squares method, since the new error component ε'_{nt} ($= v_{nt} + U_{nt} - \mu_t$) has zero mean asymptotically. The nonlinear least squares is required due to the nonlinearity of $\ln G_{nt}$. This distinguishes the current paper from Robinson (1988), where the ordinary least squares apply.

Step 4. Let

$$\hat{\varepsilon}_{nt} = \ln E_{nt} - \hat{E}(\ln E | \ln Y_{nt}) - [X_{nt} - \hat{E}(X | \ln Y_{nt})]\hat{\beta} - \ln \hat{G}_{nt} + \hat{E}(\ln \hat{G} | \ln Y_{nt}) + \mu_t \quad (3-14)$$

Maximizing the log-likelihood function derived from (3-9) with ε_{nt} replaced by

$\hat{\varepsilon}_{nt}$ over σ^2 and λ , one obtains the solution to σ after tedious manipulation as

$$\hat{\sigma} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (3-15)$$

, where $a = 1 - 2\lambda^2 \sum_t g_t^2 / (\pi TT)$, $TT = T + \lambda^2 \sum_t g_t^2$,

$$b = -2^{3/2} \lambda \sqrt{(1 + \lambda^2 \sum_t g_t^2) / \pi} \sum_i \sum_t e_{it} g_t / (nTT),$$

$$e_{it} = \ln E_{nt} - \hat{E}(\ln E | \ln Y_{nt}) - [X_{nt} - \hat{E}(X | \ln Y_{nt})]\hat{\beta} - [\ln \hat{G}_{nt} - \hat{E}(\ln G | \ln Y_{nt})],$$

and

$$c = -(1 + \lambda^2 \sum_t g_t^2) \sum_i \sum_t e_{it}^2 / (nTT).$$

In (3-15) notation “ \wedge ” is added on σ since the kernel and NISUR estimators of $\hat{E}(\cdot | x_{it})$ and $\hat{\beta}$ are used to replace their respective true counterparts. For details, please see Deng and Huang (2008) for a panel data setting with time variant TI. Because $\hat{\sigma}$ is a function of λ , γ , and data, it can be concentrated out of the log-likelihood function to reduce the number of unknown parameters.

Step 5. Maximize the concentrated log-likelihood function of the expenditure equation over the remaining two unknown parameters of λ and γ , where ε_{nt} is replaced by $\hat{\varepsilon}_{nt}$ in Step 4. The resulting pseudolikelihood estimates are denoted by $\hat{\lambda}$ and $\hat{\gamma}$. Substituting them into (3-15), we get the estimate of σ and still signify it by $\hat{\sigma}$. Plugging the three estimates into (3-11) yields the estimate of μ_t , denoted by $\hat{\mu}_t$. Finally, the nonparametric function $M(\ln Y_{nt})$ can be consistently estimated by

$$\hat{M}(\ln Y_{nt}) = \hat{E}(\ln E | \ln Y_{nt}) - \hat{E}(X | \ln Y_{nt}) \hat{\beta} - \hat{E}(\ln G | \ln Y_{nt}) - \hat{\mu}_t \quad (3-16)$$

where $\hat{\beta}$ comes from the estimates of Step 3.

It is well known that the maximum likelihood estimator of λ and γ must be asymptotically unbiased and efficient if the regularity conditions hold. Although the individual kernel regression estimators of Step 2 have pointwise convergence rates slower than root- NT ($NT^{-1/2}$), where NT signifies the sample size, the average quantities of the elements in (3-15) have an order of $O_p(NT^{-1/2})$ under very weak conditions. See, for example, Härdle and Stoker (1989) and Fan and Li (1992). Fan et al. (1996) claimed that $\hat{\sigma}^2 - \sigma^2 = O_p(NT^{-1/2})$ under quite weak conditions. As

estimator $\hat{M}(\ln Y_{nt})$ of (3-16) is a function of several kernel regression estimators, having slower convergence rates than $NT^{-1/2}$, it consequently converges to $M(\ln Y_{nt})$ for each nt at a slower rate than $NT^{-1/2}$.

The foregoing five steps complete the entire estimation procedure and the resulting estimates can be further utilized to evaluate, e.g., measures of AE and TE. In particular, the formula proposed by Battese and Coelli (1992) is adopted to gauge each firm's TE score. Based on (3-4), the (log) cost of AI, denoted by u_{nt}^{AI} , is defined as the difference between the (log) shadow expenditure ($\ln C^*(Y, W^*) + \ln \sum_j H_j^{-1} S_j^*$) and the (log) optimized cost ($\ln C(Y, W)$) that achieves AE, i.e.:

$$u_{nt}^{AI} = \ln C^*(Y_{nt}, W_{nt}^*) + \ln G(Y_{nt}, W_{nt}^*) - \ln C(Y_{nt}, W_{nt}), \quad (3-17)$$

which is a non-negative value by definition. The measure of AE is then obtained by taking the natural exponent of minus u_{nt}^{AI} , which ranges from zero to unity.

There are three attributes worth noting. First, the consistent estimates of $J-1$ relative allocative parameters H_j/H_k ($j = 1, \dots, J$ and $j \neq k$) yielded in Step 1 are treated as given throughout the remaining four steps. This avoids estimating the whole cost system simultaneously by the maximum likelihood and the difficulty in achieving convergence, on the one hand. The number of parameters to be estimated in later steps is largely decreased, on the other hand. Second, despite the fact that $\ln \hat{G}_{nt}$ can be computed in Step 1 and is used to obtain kernel estimate $\hat{E}(\ln \hat{G} | \ln Y_{nt})$ in Step 2, parameters included in $\ln G_{nt}$ of (3-13) need to be estimated again along with β , even though they have already been estimated in Step 1. Conversely, Kumbhakar and Lovell (2000) suggested subtracting $\ln \hat{G}_{nt}$ directly from the dependent variable of (3-13), which may give rise to undesired estimation results. We will come back to this shortly. Third, since Step 5 aims to estimate merely λ and γ , the log-likelihood function is usually not very difficult to converge.

4. Monte Carlo Simulations

This section first proposes three models to be used to compare the performance of their estimators. The next subsection specifies an expenditure equation and addresses the data generation processes for all variables involved.

4.1 Design of Experiments

We plan to perform Monte Carlo simulations using three models and evaluate the properties of their estimates in terms of bias and mean square errors (MSE). Model A follows the five steps addressed by the previous section. Models B and C are adapted from Model A for the purpose of making comparisons among the three models. At the outset, all of the three models have to estimate the input share equations using the NISUR, i.e., carrying out the first step. The $J - 1$ allocative parameter estimates are next exploited to estimate $\ln G_{nt}$ and AE, denoted by G1 and AE1, respectively, while the subsequent steps of the three models differ from one another. Note that the $J - 1$ allocative parameters are treated as given thereafter. We now introduce them in details.

(i) Model A

This preferred model follows exactly the above five steps. Using the kernel estimates of $\hat{E}(\ln E | \ln Y_{nt})$, $\hat{E}(X | \ln Y_{nt})$, and $\hat{E}(\ln \hat{G} | \ln Y_{nt})$ from Step 2, we estimate equation (3-13) by the NISUR in Step 3 to obtain the estimates of β . At the same time, nonlinear function $\ln G_{nt}$ is assumed to be unknown, i.e., all of the parameters shown in the parametric part of the cost function are jointly estimated, but exclude the parameters associated with the distribution of v and U . Estimates $\hat{\beta}$ together with the $J - 1$ allocative parameter estimates are employed to calculate new estimates of $\ln G_{nt}$ and AE, denoted by G2A and AE2A. The remaining parameters embedded in the distributions of v and U are estimated in Steps 4 and 5.

(ii) Model B

This model is similar to Model A except that function $\ln G_{nt}$ is treated in a different way. Specifically, the estimated $\ln G_{nt}$, $\ln \hat{G}_{nt}$, derived from Step 1 is viewed as fixed so that it can be subtracted from the dependent variable. The new transformed equation becomes

$$\ln E_{nt} - \ln \hat{G}_{nt} - E(\ln E_{nt} - \ln \hat{G}_{nt} | \ln Y_{nt}) = [X_{nt} - E(X_{nt} | \ln Y_{nt})] \beta + \hat{\varepsilon}'_{nt} \quad (4-1)$$

, where the notations are similarly defined to (3-13). After substituting the kernel estimates of $\hat{E}(\ln E - \ln \hat{G} | \ln Y_{nt})$ and $\hat{E}(X | \ln Y_{nt})$ for the corresponding conditional means in (4-1), β is estimated simply by ordinary least squares (OLS). This procedure is analogous to the one proposed by Kumbhakar and Lovell (2000, p.295-296) in spirit, while their underlying model is parametric. Estimates $\hat{\beta}$ are next used to compute $\ln G_{nt}$ and AE, denoted by G2B and AE2B. Finally, Steps 4 and 5 are executed.

(iii) Model C

This model is further adapted from Model B and is similar to the one suggested by Kumbhakar and Lovell (2000, p.165) in essence. Again, their underlying model is parametric. Since the input share equations include vector β , their consistent estimate $\hat{\beta}$ from Step 1 can be treated as fixed. In this manner, the new dependent variable turns out to be $\ln E_{nt} - \ln \hat{G}_{nt} - X_{nt} \hat{\beta}$ with corresponding kernel estimate $\hat{E}(\ln E - \ln \hat{G} - X \hat{\beta} | \ln Y_{nt})$ obtained by Step 2. Step 3 is no longer needed and Equation (3-14) of Step 4 is modified accordingly as:

$$\hat{\varepsilon}_{nt} = \ln E_{nt} - \ln \hat{G}_{nt} - X_{nt} \hat{\beta} - \hat{E}(\ln E - \ln \hat{G} - X \hat{\beta} | \ln Y_{nt}) + \mu_t \quad (4-2)$$

After concentrating out σ^2 , we execute Step 5. This completes the entire procedure.

It is seen that the major differences among the three models stem from distinct treatments on $\ln \hat{G}_{nt}$ and $\hat{\beta}$. As a result, we can compare the performance of the resulting estimates among Models A to C, including the distribution parameters of v and U .

4.2 Model Specifications

This subsection specifies the expenditure equation and the data generation processes for all variables involved that will be used to carry out Monte Carlo simulations to investigate the finite-sample performance of the proposed estimators in the last subsection. Since we are also interested in the effects of the number of firms (N) and time periods (T) on the parameter estimates, we consider several (N, T) combinations. Specifically, we choose $N = 50, 100, 200$ with $T = 6, 10, 20$. Following Olson et al. (1980) and Fan et al. (1996), we consider three sets of variances and variance ratios, viz. $(\sigma^2, \lambda) = (1.88, 1.66), (1.35, 0.83), (1.63, 1.24)$. Finally, $\gamma = 0.025$ and -0.025 are arbitrarily chosen.

The semiparametric cost frontier incorporating a single output and three inputs is formulated as:

$$\begin{aligned} \ln E &= M(\ln Y) + X\beta + \ln G + u + v \\ &= 2\ln(1 + y_1) + b_2 \ln(W_2^*) + b_3 \ln(W_3^*) + \frac{1}{2}d_{22}[\ln(W_2^*)]^2 + \frac{1}{2}d_{33}[\ln(W_3^*)]^2 + \\ &\quad d_{23} \ln(W_2^*) \ln(W_3^*) + e_{12} \ln y_1 \ln(W_2^*) + e_{13} \ln y_1 \ln(W_3^*) + \ln G + u + v \end{aligned} \quad (4-3)$$

Here, smooth function $M(\bullet)$ is arbitrarily assumed to be equal to $2\ln(1 + y_1)$. Recall that a cost function is required to be linearly homogeneous in input prices and symmetrical by the microeconomic theory. We randomly pick W_1 to normalize

dependent variable E and the other two input prices to satisfy this requirement. The symmetry restriction is already imposed on (4-3). To understand whether the performance of the estimates is robust to changes in the functional form of $M(\bullet)$, we specify an alternative form of $M(\bullet) = 0.2y_1$. We also extend (4-3) to a two-output and three-input case, assuming either $M(\bullet) = 2\ln(1 + y_1) + \ln y_2$ or $M(\bullet) = 2\ln y_2 + \sqrt{y_1 y_2}$ ⁵. Note that in this extended case, the parametric part of (4-3) has to contain extra terms involving the cross products of $\ln y_2$ and (log) normalized input prices.

Input prices W_1 , W_2 , and W_3 are randomly drawn from dissimilar uniform distributions $U(0,1)$, $U(0.5,0.5)$, and $U(0.5,1)$, respectively. The three-input and two-output quantities of x_1 , x_2 , x_3 , y_1 , and y_2 are independently generated from normal distributions $N(5,0.5)$, $N(3,0.1)$, $N(5,0.5)$, $N(31,10.1)$, and $N(20,0.8)$, respectively. Two-sided error v is drawn from $N(0, \sigma_v^2)$ and one-sided error u from a half-normal $N^+(0, \sigma_u^2)$. The simulations are executed 1000 times for each model and the bias and the MSE are computed based on the 1000 replications. We set $H_2/H_1 = 0.8$ and $H_3/H_1 = 1.2$. The true values of the coefficients are as follows: $b_2 = 0.3$, $b_3 = 0.7$, $d_{22} = -0.05$, $d_{33} = -0.02$, $d_{23} = 0.5$, $e_{12} = 0.7$, $e_{13} = 0.9$, $e_{22} = 0.3$, and $e_{23} = 0.5$.

The corresponding input share equations can be readily deduced by taking the first partial derivatives of $\ln E$ with respect to $\ln W_i$, $i = 1, 2, 3$. Although the functional form of an expenditure equation is not unique, we recommend using those such as (4-3). A prominent feature of (4-3) consists in its smooth function being specified as a

⁵ Some of the nonparametric settings follow Fan et. al (1996) and Deng and Huang (2008).

function of (log) outputs only, i.e., the (shadow) input prices must be excluded. Otherwise, one is confronted with a problem on how to properly disentangle the allocative parameters contained in $M(\bullet)$. More importantly, the share equations are unable to be explicitly derived by taking partial derivatives due to the unknown smooth function dependent of shadow prices. This impedes a researcher from consequently identifying the allocative parameters.

5. Simulation Results

This section compares the performance of the estimators discussed in the previous two sections. To compare three models we consider the properties of the estimators as the sample size gets very large. We would like the estimators to get close to the true values as the sample size increases. It is natural to consider the objective that the mean square error (MSE) of the estimators should approach zero as the sample size gets very large. The MSE criterion implies that the estimator is unbiased asymptotically and that its variance goes to zero as the sample size increases. Accordingly, the model can be regarded as a good one as its estimators satisfy the large sample properties. Table 1 summarizes the simulation outcomes of the empirical moments, i.e., bias and MSE, from the estimators for the nine (N, T) bundles. We first look at the performance of the allocative parameters, estimated in the first step. One thing that is immediately noticeable is that H_2/H_1 and H_3/H_1 are well estimated even for the case of the smallest sample size, i.e., $(N, T) = (50, 6)$. Another desirable feature is that the bias and the MSE fall when either N or T increases, aside from the bias of H_2/H_1 when $N = 200$. Even in those exceptional cases the biases are negligible.

Table 1. The performance of the allocative parameter estimates setting $M(\cdot) = 2\ln(1 + y_1)$

(N, T)	H_2/H_1		H_3/H_1	
	Bias	MSE	Bias	MSE
(50, 6)	0.0006	0.0013	0.0048	0.0019
(50, 10)	0.0004	0.0007	0.0030	0.0010
(50, 20)	0.0001	0.0004	0.0026	0.0005
(100, 6)	0.0002	0.0006	0.0029	0.0009
(100, 10)	0.0001	0.0004	0.0026	0.0005
(100, 20)	-0.0001	0.0002	0.0010	0.0002
(200, 6)	-1.74E-06	0.0003	0.0021	0.0004
(200, 10)	-0.0001	0.0002	0.0010	0.0002
(200, 20)	0.0003	0.0001	0.0004	0.0001

Table 2 reveals that in general the MSEs of the parameter estimates of the parametric portion fall quickly when either N or T increases. For instance, when fixing $N = 50$, the MSE of the coefficient of $\ln(w_3/w_1)$ shrinks swiftly from 0.2227 to 0.0579 as T increases from 6 to 20. The figure continues to fall to 0.0142 when $(N, T) = (200, 20)$. In addition, the bias measures exhibit a similar pattern, although the biases of some coefficients are a little large in the case of $(N, T) = (50, 6)$ relative to their true values. In summary, the estimators in the first step perform quite well as expected in terms of their biases and MSEs, which improve when either N or T increases.

Table 2. The performance of the parameter estimates in Step 1 setting $M(\cdot) = 2\ln(1 + y_1)$

(N, T)	$(50, 6)$		$(50, 10)$		$(50, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	-0.0722	0.2227	-0.0280	0.1134	-0.0055	0.0579
$[\ln(w_3/w_1)]^2$	0.0114	0.0056	0.0044	0.0030	0.0025	0.0014
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0088	0.0121	0.0028	0.0062	-0.0007	0.0036
$\ln y_1 \ln(w_3/w_1)$	0.0247	0.0649	0.0082	0.0330	-0.0002	0.0178
$\ln(w_2/w_1)$	-0.0475	0.1462	-0.0222	0.0762	-0.0049	0.0427
$[\ln(w_2/w_1)]^2$	0.0043	0.0027	0.0015	0.0014	0.0010	0.0007
$\ln y_1 \ln(w_2/w_1)$	0.0128	0.0360	0.0044	0.0184	-0.0016	0.0105

(N, T)	$(100, 6)$		$(100, 10)$		$(100, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	-0.0179	0.0935	-0.0055	0.0579	-0.0082	0.0284
$[\ln(w_3/w_1)]^2$	0.0036	0.0025	0.0025	0.0014	0.0014	0.0007
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0015	0.0052	-0.0007	0.0036	0.0008	0.0016
$\ln y_1 \ln(w_3/w_1)$	0.0046	0.0276	-0.0002	0.0178	0.0026	0.0083
$\ln(w_2/w_1)$	-0.0171	0.0679	-0.0049	0.0427	-0.0075	0.0211
$[\ln(w_2/w_1)]^2$	0.0011	0.0012	0.0010	0.0007	0.0006	0.0003
$\ln y_1 \ln(w_2/w_1)$	0.0023	0.0157	-0.0016	0.0105	0.0015	0.0050

(N, T)	$(200, 6)$		$(200, 10)$		$(200, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.0084	0.0489	-0.0082	0.0284	-0.0008	0.0142
$[\ln(w_3 / w_1)]^2$	0.0026	0.0012	0.0014	0.0007	-0.0005	0.0003
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.0004	0.0029	0.0008	0.0016	0.0003	0.0008
$\ln y_1 \ln(w_3 / w_1)$	0.0014	0.0148	0.0026	0.0083	-0.0009	0.0042
$\ln(w_2 / w_1)$	-0.0033	0.0354	-0.0075	0.0211	-0.0022	0.0105
$[\ln(w_2 / w_1)]^2$	0.0011	0.0006	0.0006	0.0003	-0.0001	0.0002
$\ln y_1 \ln(w_2 / w_1)$	-0.0013	0.0087	0.0015	0.0050	-0.0003	0.0025

Table 3 presents the biases and MSEs of the parametric part for Models A and B obtained from Step 3. Generally speaking, these estimators perform poorly. Their biases and MSEs are much larger than those derived from the first-stage estimation. In addition, the biases and MSEs of Model A decrease to some extent as the sample size increases, while the biases of Model B are hardly altered with an the increase in the sample size. This leads us to conclude that the computationally simple first-stage estimators of the parametric part outperform the third-step estimators of Models A and B. Does this imply that Step 3 is redundant? The answer is no. It is necessary for the estimation of the distribution parameters. Please see below.

The distribution parameters of v and U are estimated in Step 5 by the maximum likelihood, and Table 4 presents the results. The estimators of Model C have larger biases and MSEs in comparison with those of Models A and B in most cases. We therefore drop Model C from now on whenever not necessary and focus our analysis only on Models A and B. For the case of $(\sigma^2, \lambda) = (1.88, 1.66)$, despite the fact that Model B's estimator of γ has lower biases and MSEs than Model A does in almost all cases, though the differences are quite small. Model B's estimator of σ^2 performs slightly better than Model A's, while the reverse is true for the estimator of

λ . It is a caveat that Model A's estimator of σ^2 tends to have a larger variation when the sample size is small. As far as the estimator of smooth function $M(\cdot)$ is concerned, Model A is found to be superior to Model B since Model A yields much smaller biases and MSEs than Model B does in most cases. Only for the cases of a large time period ($T = 20$) are Model B's biases a little less than Model A. Turning to the cases of $(\sigma^2, \lambda) = (1.35, 0.83)$ and $(1.63, 1.24)$, the results are rather similar to the preceding case.

Table 3. The performance of the parameter estimates from the third-stage setting $M(\cdot) = 2\ln(1 + y_1)$

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$(N, T) = (50, 6)$												
$\ln(w_3/w_1)$	1.0484	1.2796	-0.4020	0.8330	1.0470	1.3472	-0.3920	1.1417	1.0479	1.2987	-0.3980	0.9200
$[\ln(w_3/w_1)]^2$	-0.0298	0.0821	0.7398	1.1935	-0.0305	0.0986	0.7337	1.4732	-0.0297	0.0870	0.7373	1.2713
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0373	0.0796	-0.5661	0.4531	0.0390	0.0946	-0.5619	0.5130	0.0376	0.0841	-0.5644	0.4700
$\ln y_1 \ln(w_3/w_1)$	-0.3118	0.1117	-0.9281	0.9856	-0.3116	0.1173	-0.9220	1.0320	-0.3118	0.1133	-0.9256	0.9973
$\ln(w_2/w_1)$	-1.0744	1.3335	0.2129	0.1685	-1.0684	1.3968	0.2077	0.2251	-1.0721	1.3505	0.2108	0.1845
$[\ln(w_2/w_1)]^2$	-0.0349	0.0912	0.7478	0.6114	-0.0375	0.1090	0.7458	0.6336	-0.0356	0.0965	0.7470	0.6174
$\ln y_1 \ln(w_2/w_1)$	0.3191	0.1162	0.1947	0.0879	0.3173	0.1211	0.1952	0.1108	0.3184	0.1175	0.1949	0.0944

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$(N, T) = (50, 10)$												
$\ln(w_3/w_1)$	0.8685	0.8114	-0.3865	0.3834	0.8724	0.8423	-0.3911	0.4996	0.8701	0.8204	-0.3884	0.4148
$[\ln(w_3/w_1)]^2$	-0.0098	0.0368	0.7075	0.7116	-0.0106	0.0441	0.7142	0.8225	-0.0099	0.0389	0.7102	0.7419
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0106	0.0367	-0.5500	0.3446	0.0113	0.0430	-0.5482	0.3638	0.0108	0.0385	-0.5493	0.3494
$\ln y_1 \ln(w_3/w_1)$	-0.2588	0.0716	-0.9175	0.8822	-0.2599	0.0740	-0.9161	0.8999	-0.2592	0.0723	-0.9170	0.8865
$\ln(w_2/w_1)$	-0.9118	0.8920	0.1990	0.0788	-0.9134	0.9214	0.1974	0.0979	-0.9125	0.9003	0.1984	0.0837
$[\ln(w_2/w_1)]^2$	-0.0115	0.0405	0.7459	0.5752	-0.0120	0.0475	0.7476	0.5863	-0.0115	0.0425	0.7466	0.5785
$\ln y_1 \ln(w_2/w_1)$	0.2715	0.0786	0.2028	0.0580	0.2719	0.0810	0.2006	0.0647	0.2717	0.0793	0.2020	0.0596

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
$(N, T) = (50, 20)$	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.6812	0.4800	-0.4096	0.2192	0.6831	0.4885	-0.4090	0.2419	0.6819	0.4824	-0.4093	0.2245
$[\ln(w_3 / w_1)]^2$	-0.0011	0.0167	0.7320	0.5860	-0.0012	0.0180	0.7331	0.6097	-0.0011	0.0170	0.7323	0.5917
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0016	0.0169	-0.5532	0.3160	0.0015	0.0182	-0.5507	0.3175	0.0015	0.0172	-0.5522	0.3159
$\ln y_1 \ln(w_3 / w_1)$	-0.2027	0.0423	-0.9217	0.8594	-0.2032	0.0430	-0.9194	0.8592	-0.2028	0.0425	-0.9208	0.8587
$\ln(w_2 / w_1)$	-0.7134	0.5246	0.2025	0.0505	-0.7137	0.5309	0.2001	0.0535	-0.7135	0.5262	0.2016	0.0511
$[\ln(w_2 / w_1)]^2$	-0.0021	0.0180	0.7522	0.5699	-0.0020	0.0196	0.7524	0.5720	-0.0020	0.0184	0.7523	0.5704
$\ln y_1 \ln(w_2 / w_1)$	0.2122	0.0462	0.1966	0.0425	0.2122	0.0467	0.1960	0.0440	0.2122	0.0463	0.1964	0.0429

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
$(N, T) = (100, 6)$	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.8227	0.7571	-0.3908	0.4784	0.8286	0.8027	-0.3999	0.6393	0.8250	0.7712	-0.3944	0.5251
$[\ln(w_3 / w_1)]^2$	-0.0078	0.0404	0.7216	0.8238	-0.0089	0.0500	0.7323	0.9832	-0.0081	0.0433	0.7258	0.8709
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0083	0.0400	-0.5468	0.3606	0.0094	0.0482	-0.5441	0.3881	0.0085	0.0425	-0.5458	0.3682
$\ln y_1 \ln(w_3 / w_1)$	-0.2446	0.0661	-0.9165	0.8992	-0.2462	0.0697	-0.9139	0.9232	-0.2452	0.0672	-0.9155	0.9056
$\ln(w_2 / w_1)$	-0.8548	0.8141	0.1972	0.0970	-0.8587	0.8568	0.1945	0.1242	-0.8563	0.8271	0.1961	0.1048
$[\ln(w_2 / w_1)]^2$	-0.0079	0.0440	0.7469	0.5831	-0.0091	0.0531	0.7501	0.5998	-0.0082	0.0467	0.7482	0.5883
$\ln y_1 \ln(w_2 / w_1)$	0.2536	0.0709	0.1990	0.0630	0.2548	0.0743	0.1953	0.0724	0.2541	0.0719	0.1975	0.0655

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
$(N, T) = (100, 10)$	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.6792	0.4909	-0.3976	0.2700	0.6828	0.5104	-0.3997	0.3295	0.6806	0.4967	-0.3984	0.2861
$[\ln(w_3 / w_1)]^2$	-0.0019	0.0186	0.7234	0.6283	-0.0022	0.0221	0.7275	0.6892	-0.0019	0.0196	0.7250	0.6452
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0028	0.0188	-0.5493	0.3237	0.0029	0.0220	-0.5464	0.3312	0.0028	0.0197	-0.5482	0.3253
$\ln y_1 \ln(w_3 / w_1)$	-0.2019	0.0431	-0.9174	0.8632	-0.2029	0.0447	-0.9147	0.8688	-0.2023	0.0436	-0.9164	0.8642
$\ln(w_2 / w_1)$	-0.7082	0.5319	0.1985	0.0602	-0.7098	0.5489	0.1956	0.0692	-0.7089	0.5368	0.1974	0.0625
$[\ln(w_2 / w_1)]^2$	-0.0035	0.0208	0.7490	0.5698	-0.0036	0.0245	0.7501	0.5763	-0.0034	0.0218	0.7494	0.5718
$\ln y_1 \ln(w_2 / w_1)$	0.2103	0.0466	0.1987	0.0477	0.2108	0.0480	0.1970	0.0515	0.2105	0.0470	0.1980	0.0486

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
$(N, T) = (100, 20)$	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.5231	0.2817	-0.3974	0.1864	0.5242	0.2861	-0.3972	0.1949	0.5235	0.2829	-0.3974	0.1864
$[\ln(w_3 / w_1)]^2$	0.0009	0.0076	0.7203	0.5472	0.0013	0.0081	0.7209	0.5566	0.0011	0.0077	0.7203	0.5472
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.0017	0.0076	-0.5471	0.3047	-0.0020	0.0081	-0.5461	0.3052	-0.0018	0.0077	-0.5471	0.3047
$\ln y_1 \ln(w_3 / w_1)$	-0.1552	0.0247	-0.9181	0.8483	-0.1556	0.0251	-0.9170	0.8477	-0.1554	0.0248	-0.9181	0.8483
$\ln(w_2 / w_1)$	-0.5457	0.3055	0.1975	0.0441	-0.5458	0.3085	0.1964	0.0452	-0.5458	0.3063	0.1975	0.0441
$[\ln(w_2 / w_1)]^2$	0.0021	0.0081	0.7496	0.5642	0.0023	0.0087	0.7498	0.5651	0.0022	0.0083	0.7496	0.5642
$\ln y_1 \ln(w_2 / w_1)$	0.1622	0.0269	0.1997	0.0421	0.1623	0.0272	0.1993	0.0426	0.1623	0.0270	0.1997	0.0421

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
$(N, T) = (200, 6)$	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.6314	0.4391	-0.3838	0.3063	0.6342	0.4613	-0.3847	0.3769	0.6325	0.4458	-0.3842	0.3260
$[\ln(w_3 / w_1)]^2$	-0.0003	0.0184	0.7112	0.6546	0.0004	0.0226	0.7145	0.7277	0.0001	0.0196	0.7125	0.6755
$\ln(w_2 / w_1)\ln(w_3 / w_1)$	0.0003	0.0181	-0.5413	0.3237	-5.45E-06	0.0219	-0.5389	0.3356	0.0001	0.0192	-0.5404	0.3269
$\ln y_1 \ln(w_3 / w_1)$	-0.1877	0.0384	-0.9113	0.8611	-0.1886	0.0402	-0.9080	0.8685	-0.1880	0.0390	-0.9100	0.8626
$\ln(w_2 / w_1)$	-0.6563	0.4722	0.1914	0.0661	-0.6566	0.4904	0.1884	0.0782	-0.6565	0.4774	0.1903	0.0695
$[\ln(w_2 / w_1)]^2$	-0.0002	0.0201	0.7459	0.5689	-0.0005	0.0244	0.7469	0.5762	-0.0002	0.0213	0.7463	0.5712
$\ln y_1 \ln(w_2 / w_1)$	0.1948	0.0412	0.2014	0.0523	0.1951	0.0428	0.1998	0.0571	0.1949	0.0417	0.2008	0.0536

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
$(N, T) = (200, 10)$	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.5261	0.2911	-0.3955	0.2086	0.5274	0.3000	-0.3955	0.2355	0.5265	0.2936	-0.3955	0.2157
$[\ln(w_3 / w_1)]^2$	0.0008	0.0083	0.7221	0.5714	0.0013	0.0099	0.7230	0.6002	0.0010	0.0088	0.7224	0.5792
$\ln(w_2 / w_1)\ln(w_3 / w_1)$	0.0003	0.0084	-0.5484	0.3112	-0.0005	0.0097	-0.5456	0.3135	0.0000	0.0087	-0.5473	0.3114
$\ln y_1 \ln(w_3 / w_1)$	-0.1563	0.0256	-0.9162	0.8497	-0.1566	0.0263	-0.9137	0.8503	-0.1564	0.0258	-0.9153	0.8494
$\ln(w_2 / w_1)$	-0.5441	0.3102	0.1974	0.0488	-0.5444	0.3174	0.1946	0.0528	-0.5442	0.3122	0.1963	0.0498
$[\ln(w_2 / w_1)]^2$	-0.0013	0.0094	0.7488	0.5648	-0.0006	0.0109	0.7493	0.5678	-0.0010	0.0098	0.7490	0.5657
$\ln y_1 \ln(w_2 / w_1)$	0.1615	0.0272	0.1989	0.0435	0.1617	0.0278	0.1982	0.0454	0.1616	0.0274	0.1986	0.0440

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)				(1.63, 1.24)			
$(N, T) = (200, 20)$	Model A		Model B		Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.4047	0.1674	-0.4005	0.1729	0.4054	0.1695	-0.4010	0.1792	0.4049	0.1680	-0.4007	0.1745
$[\ln(w_3 / w_1)]^2$	-0.0007	0.0036	0.7215	0.5323	-0.0011	0.0039	0.7224	0.5395	-0.0008	0.0037	0.7218	0.5343
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-4.81E-05	0.0037	-0.5495	0.3044	0.0004	0.0039	-0.5497	0.3056	0.0001	0.0037	-0.5496	0.3047
$\ln y_1 \ln(w_3 / w_1)$	-0.1199	0.0147	-0.9208	0.8503	-0.1201	0.0148	-0.9209	0.8515	-0.1200	0.0147	-0.9209	0.8506
$\ln(w_2 / w_1)$	-0.4240	0.1835	0.2002	0.0424	-0.4243	0.1852	0.2003	0.0434	-0.4241	0.1840	0.2002	0.0426
$[\ln(w_2 / w_1)]^2$	0.0007	0.0039	0.7501	0.5636	0.0002	0.0042	0.7502	0.5642	0.0005	0.0040	0.7501	0.5638
$\ln y_1 \ln(w_2 / w_1)$	0.1258	0.0161	0.1997	0.0408	0.1258	0.0162	0.1995	0.0412	0.4049	0.1680	-0.4007	0.1745

Although both Models A and B perform reasonably well, the simulation results appear to be in favor of an advantage for Model A in general and for the estimation of TE scores in particular (see Tables 7 and 14 below). Comparing (3-13) with (4-1), one can tell that their disparity originates from how β and $\ln G_{nt}$ are estimated. For Model A, they are estimated by NISUR viewing parameters contained in $\ln G_{nt}$ as unknown, while for Model B $\ln G_{nt}$ is replaced by $\ln \hat{G}_{nt}$ leaving β to be estimated by OLS. The superiority of Model A may be explained by its allowance for the presence of $\ln G_{nt}$ in the expenditure equation.

It is apparent from Table 4 that Model C gives rise to undesirable estimators of $(\gamma, \sigma^2, \lambda)$. This is mainly ascribable to the fact that it overlooks Step 3 and proceeds from Step 2 directly to Steps 4 and 5. By doing so, the residual of (4-2) is indirectly derived using the NISUR estimates of $\ln \hat{G}_{nt}$ and $\hat{\beta}$, which are obtained by simultaneously estimating the $(J-1)$ share equations, instead of the expenditure equation. Conversely, the residuals of (3-13) and (4-1) corresponding to Models A and B, respectively, are directly deduced from estimating the expenditure equation. Step 3 is thus necessary.

We have learned from Tables 2 and 3 that the parameter estimates of the parametric part of the cost function obtained in the first step outperform those obtained in the third step. These estimates are applied to compute $\ln G_{nt}$. We now compare the performance of the estimated $\ln G_{nt}$ to gain further insight into the properties of alternative models. Not surprisingly, G1 has smaller biases and MSEs than G2A and G2B, derived from Models A and B, respectively, in almost all (N, T) and (σ^2, λ) combinations. The outcomes support the use of G1 as the estimate of $\ln G_{nt}$.

Table 4. The performance of the estimators of $(\gamma, \sigma^2, \lambda)$ setting $M(\cdot) = 2 \ln(1 + y_1)$
 $(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$

Model A								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0033	0.0029	-0.0694	3.9557	-0.1374	0.1311	0.0161	0.0407
(50, 10)	0.0013	0.0003	-0.1841	0.2083	-0.1497	0.0782	0.0171	0.0065
(50, 20)	0.0013	2.90E-05	-0.2094	0.1992	-0.2330	0.0918	0.0126	0.0034
(100, 6)	0.0007	0.0006	-0.1253	0.1108	-0.0808	0.0432	0.0147	0.0071
(100, 10)	0.0007	0.0001	-0.1207	0.1032	-0.0975	0.0377	0.0098	0.0037
(100, 20)	0.0012	1.48E-05	-0.1498	0.1007	-0.1758	0.0507	0.0087	0.0024
(200, 6)	0.0009	0.0003	-0.0877	0.0557	-0.0568	0.0224	0.0093	0.0040
(200, 10)	0.0005	0.0001	-0.0845	0.0497	-0.0685	0.0181	0.0066	0.0023
(200, 20)	0.0011	8.00E-06	-0.1233	0.0540	-0.1467	0.0312	0.0077	0.0019

Model B								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0013	0.0016	-0.1866	0.2169	-0.1457	0.0917	0.1607	0.0500
(50, 10)	0.0007	0.0003	-0.1562	0.1949	-0.1595	0.0775	0.1136	0.0252
(50, 20)	0.0009	2.78E-05	-0.1795	0.1849	-0.2515	0.0965	0.0105	0.0059
(100, 6)	0.0003	0.0007	-0.1101	0.1060	-0.0870	0.0433	0.1518	0.0363
(100, 10)	0.0004	0.0001	-0.1027	0.0979	-0.1046	0.0372	0.1073	0.0185
(100, 20)	0.0009	1.37E-05	-0.1307	0.0938	-0.1879	0.0530	0.0077	0.0039
(200, 6)	0.0006	0.0003	-0.0769	0.0535	-0.0598	0.0223	0.1472	0.0289
(200, 10)	0.0004	0.0001	-0.0729	0.0472	-0.0731	0.0181	0.1048	0.0152
(200, 20)	0.0010	7.45E-06	-0.1127	0.0506	-0.1552	0.0329	0.0067	0.0028

Model C								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	-0.7062	36.2508	1.5206	16.9525	-0.8946	1.1997	0.0665	0.5821
(50, 10)	-0.8480	53.4654	0.7297	10.3146	-0.9426	1.3762	0.0797	0.3318
(50, 20)	-0.0534	1.4710	0.3539	1.4141	-1.0981	1.5028	0.0790	0.2108
(100, 6)	-0.4715	27.8117	0.5424	1.8874	-0.7704	0.9213	0.0787	0.2948
(100, 10)	-0.1906	11.8544	0.3473	0.8185	-0.8647	1.0488	0.0727	0.2177
(100, 20)	-0.0586	3.0532	0.1286	0.3725	-0.9295	1.1387	0.0438	0.1089
(200, 6)	-0.1163	6.2152	0.2694	0.4872	-0.6770	0.7413	0.0625	0.1903
(200, 10)	-0.0792	4.6876	0.1382	0.2180	-0.6909	0.7443	0.0329	0.1112
(200, 20)	-0.0365	1.3757	0.0279	0.1321	-0.8049	0.8860	0.0368	0.0573

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

Model A								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0067	0.0097	0.0290	3.8612	-0.0343	0.0844	0.0092	0.0499
(50, 10)	0.0019	0.0012	-0.1009	0.1262	-0.0423	0.0355	0.0131	0.0082
(50, 20)	0.0014	1.07E-04	-0.1249	0.1117	-0.0683	0.0236	0.0089	0.0034
(100, 6)	0.0020	0.0037	-0.0705	0.0730	-0.0245	0.0278	0.0118	0.0100
(100, 10)	0.0009	0.0006	-0.0689	0.0643	-0.0281	0.0176	0.0070	0.0046
(100, 20)	0.0012	5.46E-05	-0.0912	0.0560	-0.0511	0.0127	0.0061	0.0022
(200, 6)	0.0020	0.0018	-0.0501	0.0357	-0.0187	0.0145	0.0067	0.0054
(200, 10)	0.0005	0.0003	-0.0478	0.0306	-0.0194	0.0085	0.0043	0.0027
(200, 20)	0.0012	2.85E-05	-0.0737	0.0286	-0.0436	0.0068	0.0054	0.0016

Model B								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0054	0.0102	-0.0995	0.1409	-0.0388	0.0596	0.1565	0.0563
(50, 10)	0.0011	0.0013	-0.0881	0.1227	-0.0405	0.0353	0.1148	0.0283
(50, 20)	0.0009	1.05E-04	-0.1065	0.1066	-0.0667	0.0225	0.0076	0.0062
(100, 6)	0.0016	0.0037	-0.0647	0.0726	-0.0245	0.0279	0.1524	0.0403
(100, 10)	0.0005	0.0006	-0.0594	0.0629	-0.0270	0.0171	0.1108	0.0208
(100, 20)	0.0009	5.28E-05	-0.0792	0.0531	-0.0500	0.0119	0.0056	0.0039
(200, 6)	0.0016	0.0018	-0.0454	0.0355	-0.0180	0.0146	0.1497	0.0315
(200, 10)	0.0003	0.0003	-0.0413	0.0297	-0.0186	0.0083	0.10947	0.017091
(200, 20)	0.0010	2.77E-05	-0.0668	0.0272	-0.0430	0.0065	0.0046	0.0026

Model C								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	-2.1225	127.4198	1.4582	12.1469	-0.3124	0.3358	0.0579	0.5439
(50, 10)	-0.8325	54.4356	0.6831	2.4049	-0.3217	0.2831	0.0714	0.2908
(50, 20)	-0.4538	24.5411	0.3740	1.2215	-0.4069	0.2858	0.0477	0.1876
(100, 6)	-0.8983	50.7877	0.6066	2.2731	-0.2619	0.2349	0.0408	0.2655
(100, 10)	-0.6136	42.2780	0.4063	0.9710	-0.3193	0.2484	0.0555	0.1863
(100, 20)	-0.1489	10.9556	0.1234	0.2132	-0.3376	0.1997	0.0260	0.0986
(200, 6)	-0.1513	7.1550	0.3134	0.5957	-0.2163	0.1378	0.0387	0.1582
(200, 10)	-0.0430	1.4204	0.1496	0.1566	-0.2203	0.1111	0.0160	0.0917
(200, 20)	0.0023	0.0051	0.0378	0.0537	-0.2761	0.1307	0.0216	0.0494

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.63, 1.24)$$

Model A								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0044	4.57E-03	-0.0149	4.1176	-0.0787	0.1037	0.0148	0.0412
(50, 10)	0.0015	0.0005	-0.1459	0.1651	-0.0871	0.0501	0.0152	0.0069
(50, 20)	0.0014	4.85E-05	-0.1701	0.1537	-0.1366	0.0461	0.0109	0.0033
(100, 6)	0.0010	0.0013	-0.1005	0.0906	-0.0471	0.0321	0.0132	0.0079
(100, 10)	0.0008	0.0002	-0.0972	0.0825	-0.0568	0.0245	0.0085	0.0039
(100, 20)	0.0012	2.48E-05	-0.1220	0.0774	-0.1020	0.0249	0.0075	0.0023
(200, 6)	0.0012	0.0006	-0.0708	0.0450	-0.0343	0.0167	0.0081	0.0689
(200, 10)	0.0006	0.0001	-0.0680	0.0394	-0.0400	0.0117	0.0056	0.0023
(200, 20)	0.0012	1.32E-05	-0.1000	0.0407	-0.0855	0.0143	0.0067	0.0017

Model B								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0021	0.0031	-0.1478	0.1768	-0.0817	0.0672	0.1622	0.0527
(50, 10)	0.0008	0.0005	-0.1244	0.1566	-0.0885	0.0487	0.1160	0.0265
(50, 20)	0.0009	4.71E-05	-0.1450	0.1437	-0.1420	0.0452	0.0092	0.0059
(100, 6)	0.0006	0.0013	-0.0892	0.0881	-0.0486	0.0319	0.1555	0.0386
(100, 10)	0.0004	0.0002	-0.0827	0.0791	-0.0579	0.0235	0.1108	0.2019
(100, 20)	0.0009	2.35E-05	-0.1058	0.0724	-0.1053	0.0242	0.0067	0.0038
(200, 6)	0.0008	0.0006	-0.0624	0.0438	-0.0345	0.0165	0.1518	0.0308
(200, 10)	0.0004	0.0001	-0.0586	0.0377	-0.0407	0.0114	0.10894	0.0163
(200, 20)	0.0010	1.26E-05	-0.0908	0.0383	-0.0880	0.0143	0.0058	0.0026

Model C								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	-1.1126	66.6463	1.1981	12.9044	-1.0296	1.4089	0.0491	0.5125
(50, 10)	-0.7126	40.2184	0.4239	2.2994	-1.0475	1.3598	0.0760	0.3298
(50, 20)	-0.4412	22.4537	0.1219	1.2945	-1.1432	1.4953	0.0562	0.2012
(100, 6)	-1.1345	66.3433	0.3065	1.7909	-0.9243	1.0864	0.0648	0.2771
(100, 10)	-0.4090	27.9652	0.1410	0.9314	-0.9896	1.1737	0.0681	0.2019
(100, 20)	-0.1664	8.8257	-0.1376	0.2607	-1.0234	1.2005	0.0351	0.1006
(200, 6)	-0.1698	7.2282	0.0542	0.6162	-0.8542	0.9065	0.0498	0.1772
(200, 10)	-0.0697	2.3189	-0.1117	0.1607	-0.8494	0.8528	0.0201	0.0978
(200, 20)	0.1329	6.2066	-0.2256	0.1287	-0.9415	1.0193	0.0305	0.0568

Table 5. The performance of estimated function $\ln G$ setting $M(\cdot) = 2\ln(1 + y_1)$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A				Model B	
	G1		G2A		G2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0061	0.0296	-0.0047	0.0356	-0.0023	0.0384
(50, 10)	0.0005	0.0150	-0.0050	0.0165	-0.0026	0.0174
(50, 20)	-0.0026	0.0085	-0.0056	0.0092	-0.0037	0.0086
(100, 6)	-0.0010	0.0125	-0.0056	0.0143	-0.0035	0.0157
(100, 10)	-0.0026	0.0085	-0.0057	0.0093	-0.0044	0.0091
(100, 20)	0.0002	0.0038	-0.0015	0.0043	-0.0003	0.0038
(200, 6)	-0.0017	0.0070	-0.0045	0.0079	-0.0035	0.0082
(200, 10)	0.0002	0.0038	-0.0016	0.0043	-0.0010	0.0041
(200, 20)	-0.0012	0.0019	-0.0021	0.0021	-0.0017	0.0019

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

(N, T)	Model A				Model B	
	G1		G2A		G2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0061	0.0296	-0.0044	0.0365	-0.0024	0.0412
(50, 10)	0.0005	0.0150	-0.0049	0.0169	-0.0026	0.0185
(50, 20)	-0.0026	0.0085	-0.0057	0.0093	-0.0039	0.0088
(100, 6)	-0.0010	0.0125	-0.0054	0.0148	-0.0036	0.0170
(100, 10)	-0.0026	0.0085	-0.0057	0.0095	-0.0046	0.0096
(100, 20)	0.0002	0.0038	-0.0015	0.0043	-0.0004	0.0039
(200, 6)	-0.0017	0.0070	-0.0045	0.0082	-0.0036	0.0088
(200, 10)	0.0002	0.0038	-0.0016	0.0044	-0.0010	0.0043
(200, 20)	-0.0012	0.0019	-0.0021	0.0022	-0.0018	0.0020

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.63, 1.24)$$

(N, T)	Model A				Model B	
	G1		G2A		G2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0061	0.0296	-0.0046	0.0358	-0.0024	0.0392
(50, 10)	0.0005	0.0150	-0.0050	0.0166	-0.0026	0.0177
(50, 20)	-0.0026	0.0085	-0.0057	0.0092	-0.0038	0.0086
(100, 6)	-0.0010	0.0125	-0.0055	0.0145	-0.0035	0.0161
(100, 10)	-0.0026	0.0085	-0.0057	0.0094	-0.0045	0.0092
(100, 20)	0.0002	0.0038	-0.0015	0.0043	-0.0004	0.0039
(200, 6)	-0.0017	0.0070	-0.0045	0.0080	-0.0036	0.0083
(200, 10)	0.0002	0.0038	-0.0016	0.0044	-0.0010	0.0041
(200, 20)	-0.0012	0.0019	-0.0021	0.0021	-0.0017	0.0019

Table 6 shows the performance of the estimated AE measures. Again, the biases of AE1 are found to be smaller than both AE2A and AE2B, derived from Models A and B, respectively, in most (N, T) and (σ^2, λ) bundles, although the biases of both AE2A and AE2B measures are already tiny. In addition, those biases and MSEs are decreasing as the sample size grows. One is led to conclude that AE1 is superior to AE2A and AE2B as an estimate of the AE.

Given that technical efficiency often constitutes one of the primary issues in empirical studies, we thereby investigate the performance of TE measures based on the three models. It can be seen from Table 7 that the biases and MSEs of Model C vary dramatically, implying that the model is apt to yield uncertain and incredible TE measures. However, the biases and MSEs of both Models A and B are much less than those of Model C. The other two cases of (σ^2, λ) give similar implications. Comparing the first two columns of Table 7 with the middle two columns of the same table, we see that the performance of the estimated TE score from Model A surpasses that of Model B in most (N, T) and (σ^2, λ) bundles. Model B may be valid only when T is greater than or equal to 20. We also conduct simulations assuming $\gamma = -0.025$ and the remaining parameter values are held intact. The results are shown in Tables I and II of the Appendix. A negative value of γ implies that the TE score of a firm deteriorates over time. Tables I and II provide similar results to the foregoing. To assess the small sample properties of three models we have repeated the simulation techniques for $N=30$ and $T=6, 10, 20$. The results are shown in Tables III to IX of the Appendix. We find that the first step estimators of the cost share equations perform well and Model A still works well on the whole. Since there is no substantial difference in performance from large-sample size, the foregoing results are still hold even if $N=30$.

Table 6. The performance of estimated AE setting $M(\cdot) = 2 \ln(1 + y_1)$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A				Model B	
	AE1		AE2A		AE2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0031	0.0027	0.0139	0.3993	0.0079	0.0611
(50, 10)	0.0010	0.0015	0.0030	0.0007	0.0019	0.0014
(50, 20)	0.0002	0.0007	0.0010	0.0004	-0.0003	0.0006
(100, 6)	0.0004	0.0012	0.0023	0.0006	0.0017	0.0012
(100, 10)	0.0002	0.0007	0.0009	0.0004	0.0001	0.0007
(100, 20)	2.89E-06	0.0003	0.0008	0.0002	0.0001	0.0003
(200, 6)	0.0001	0.0006	0.0010	0.0003	0.0008	0.0006
(200, 10)	2.89E-06	0.0003	0.0006	0.0002	0.0001	0.0003
(200, 20)	0.0004	0.0002	0.0006	0.0001	0.0002	0.0002

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

(N, T)	Model A				Model B	
	AE1		AE2A		AE2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0031	0.0027	0.0142	0.3993	0.0092	0.0155
(50, 10)	0.0010	0.0015	0.0031	0.0008	0.0024	0.0015
(50, 20)	0.0002	0.0007	0.0010	0.0004	-0.0001	0.0006
(100, 6)	0.0004	0.0012	0.0024	0.0007	0.0024	0.0014
(100, 10)	0.0002	0.0007	0.0010	0.0004	0.0005	0.0007
(100, 20)	2.89E-06	0.0003	0.0008	0.0002	0.0002	0.0003
(200, 6)	0.0001	0.0006	0.0011	0.0003	0.0012	0.0007
(200, 10)	2.89E-06	0.0003	0.0007	0.0002	0.0004	0.0004
(200, 20)	0.0004	0.0002	0.0006	0.0001	0.0002	0.0002

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.63, 1.24)$$

(N, T)	Model A				Model B	
	AE1		AE2A		AE2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0031	0.0027	0.0140	0.3993	0.0082	0.0217
(50, 10)	0.0010	0.0015	0.0030	0.0007	0.0020	0.0014
(50, 20)	0.0002	0.0007	0.0010	0.0004	-0.0003	0.0006
(100, 6)	0.0004	0.0012	0.0023	0.0006	0.0019	0.0013
(100, 10)	0.0002	0.0007	0.0010	0.0004	0.0002	0.0007
(100, 20)	2.89E-06	0.0003	0.0008	0.0002	0.0001	0.0003
(200, 6)	0.0001	0.0006	0.0011	0.0003	0.0009	0.0006
(200, 10)	2.89E-06	0.0003	0.0006	0.0002	0.0002	0.0003
(200, 20)	0.0004	0.0002	0.0006	0.0001	0.0002	0.0002

Table 7. The performance of estimated TE scores setting $M(\cdot) = 2\ln(1 + y_1)$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A		Model B		Model C	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0059	0.0106	0.0762	0.0137	138.2386	2.01E+09
(50, 10)	0.0056	0.0031	0.0638	0.0077	2.3715	1.64E+06
(50, 20)	0.0215	0.0017	0.0216	0.0017	0.0485	0.2112
(100, 6)	0.0022	0.0062	0.0704	0.0118	39.7962	5.79E+08
(100, 10)	0.0029	0.0027	0.0607	0.0069	0.0881	825.1991
(100, 20)	0.0204	0.0016	0.0204	0.0016	0.0373	0.0197
(200, 6)	0.0001	0.0059	0.0679	0.0110	0.0824	693.1477
(200, 10)	0.0012	0.0025	0.0589	0.0064	1.0285	1.94E+06
(200, 20)	0.0200	0.0016	0.0200	0.0015	0.0314	0.0119

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

(N, T)	Model A		Model B		Model C	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0033	0.0262	0.0569	0.0169	13.8431	3.66E+07
(50, 10)	0.0029	0.0064	0.0531	0.0095	0.0531	2.0763
(50, 20)	0.0183	0.0022	0.0186	0.0022	0.0348	0.4297
(100, 6)	-0.0007	0.0121	0.0522	0.0149	9149.6239	3.47E+13
(100, 10)	0.0005	0.0059	0.0510	0.0087	0.0781	674.3316
(100, 20)	0.0175	0.0020	0.0177	0.0020	0.0274	0.0134
(200, 6)	-0.0034	0.0114	0.0498	0.0140	18.5228	3.88E+08
(200, 10)	-0.0012	0.0056	0.0496	0.0083	0.0115	0.0175
(200, 20)	0.0172	0.0019	0.0173	0.0019	0.0223	0.0105

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.63, 1.24)$$

(N, T)	Model A		Model B		Model C	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0063	0.0259	0.0702	0.0150	46.5646	4.11E+08
(50, 10)	0.0043	0.0042	0.0613	0.0085	0.1216	799.6278
(50, 20)	0.0201	0.0018	0.0203	0.0018	0.0404	0.4436
(100, 6)	0.0006	0.0085	0.0654	0.0133	6106.3710	2.03E+13
(100, 10)	0.0019	0.0038	0.0589	0.0077	0.0644	301.2878
(100, 20)	0.0192	0.0017	0.0192	0.0017	0.0303	0.0135
(200, 6)	-0.0015	0.0081	0.0632	0.0125	53.6804	3.33E+09
(200, 10)	0.0002	0.0036	0.0573	0.0073	0.0245	108.7752
(200, 20)	0.0188	0.0017	0.0188	0.0016	0.0270	0.0088

To understand the effects of dimensionality of smooth function $M(\cdot)$ on the performance of the various estimators, we re-specify M as a function of two variables at present, i.e., $\ln y_1$ and $\ln y_2$. The simulation results given in Tables 8 to 14 are comparable with Tables 1 to 7. Raising the dimension of M and extra regressors appears to exert no significant impacts on the performance of the estimators of interest. In particular, the first step parameter estimates of the parametric part still perform well, as shown in Tables 8 and 9 in comparison with Table 10 of the third step estimates. Table 11 displays analogous outcomes to Table 4. We argue for the employment of Model A once more. Table 12 unveils that G1 has the smallest biases and MSEs in most of the (N, T) combinations and hence outperforms G2A and G2B. Moreover, Table 13 shows that AE1 acts reasonably well. The biases and MSEs of both AE2A and AE2B decrease faster than AE1. Finally, Table 14 reveals that the TE scores of Model A outperform those of Model B, especially when the time period is short and they perform equally well when the time period exceeds, say, 20. Generally speaking, the performance of the estimators under consideration seems to be irrespective of the increase in the dimension of M and the explanatory variables in the parametric part. Note that Model C produces quite large and infeasible TE scores with quite large variability.

We intend to check the performance of our proposed estimator in the context of cross sectional data. As different variance ratios give similar results for the three models, we only report the results of $(\sigma^2, \lambda) = (1.88, 1.66)$ in the Appendix to save space. The first step parameter estimates of Table X behave well and better than those in Table XI, where the biases of some estimates remain large even though $N = 1000$. Table XII shows mixed results on the performance of the estimators of (σ^2, λ) and $M(\cdot)$ from Models A and B, while Model C behaves badly. Measures G1 and AE1 in Tables XIII and XIV also perform satisfactorily, despite that the biases and MSEs in a few cases exceed G2 and AE2. Finally, Table XV displays that the biases and MSEs of the TE scores from Models A and B are close to each other and are relatively large, but smaller than those of Model C. It is well known in the area of efficiency and productivity analysis that the TE scores of firms cannot be consistently estimated, since the variance of the conditional mean or the conditional mode for each individual firm does not vanish as the size of the cross section increases (Schmidt and Sickles, 1984). Table XV confirms this, because the biases reduce tardily as the sample size

enlarges. Despite this drawback, which was avoidable in a panel data setting, evidence is found that Model A performs at least as well as Model B using cross sectional data.

We also estimate the translog cost function for the purpose of comparison. The $M(\cdot)$ is re-specified as $\sqrt{y_1} + \ln(1 + y_1)$ and the results are shown in the Appendix. Table XVI reveals that the parameter estimates in Step 1 perform well even though we re-specify $M(\cdot)$ as a complicated functional form. The biases and MSEs of the parameter estimates fall when either N or T increases. Table XVII presents the biases and MSEs of the parameteric part for model A and Translog functional form. Although the estimators of Model A have larger biases and MSEs than those of Translog, the biases and MSEs of Model A decrease as the sample size increase. Table XVIII presents the parameter estimates of the distribution. It is obvious that model A's estimators have lower biases and MSEs than Translog does. In addition, Translog's estimator of γ perform poorly and the other estimators have large bias and MSE for the case of large time period (T=20). Table XIX shows the performance of the estimated $\ln G$. G1 has smaller biases and MSEs than G2A and G2B in most of (N, T) combinations. Table XX reveals the performance of the estimated AE measures. Again, the biases and MSEs of AE1 are found to be smaller than both AE2A and AE2T, respectively, in almost all (N, T) bundles. As to the performance of the estimated TE measures, it can be seen from Table XXI that the biases and MSEs of Model A are much less than those of Translog. On the whole, evidence tends to support the superiority of Model A, implying that using semiparametric regression model is better than using translog functional form when the true functional form is complicated and unknown.

Table 8. The performance of the allocative parameter estimates setting $M(\cdot) = 2 \ln(1 + y_1) + \ln y_2$

(N, T)	η_2/η_1		η_3/η_1	
	Bias	MSE	Bias	MSE
(50, 6)	0.0015	0.0012	0.0073	0.0015
(50, 10)	0.0008	0.0006	0.0042	0.0007
(50, 20)	0.0004	0.0003	0.0031	0.0004
(100, 6)	0.0007	0.0005	0.0039	0.0006
(100, 10)	0.0004	0.0003	0.0031	0.0004
(100, 20)	3.67E-05	0.0001	0.0012	0.0002
(200, 6)	0.0002	0.0002	0.0026	0.0003
(200, 10)	3.67E-05	0.0001	0.0012	0.0002
(200, 20)	0.0004	0.0001	0.0004	0.0001

Table 9. The performance of the parameter estimates in Step 1 setting $M(\cdot) = 2\ln(1+y_1) + \ln y_2$

(N, T)	$(50, 6)$		$(50, 10)$		$(50, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	-0.1526	3.9190	-0.0741	1.8965	-0.0420	0.9191
$[\ln(w_3/w_1)]^2$	0.0194	0.0087	0.0081	0.0043	0.0042	0.0021
$\ln(w_2/w_1)\ln(w_3/w_1)$	1.46E-02	0.0222	5.25E-03	0.0113	-8.61E-06	0.0064
$\ln y_1 \ln(w_3/w_1)$	0.0416	0.1237	0.0155	0.0628	0.0022	0.0345
$\ln y_2 \ln(w_3/w_1)$	0.0103	0.3747	0.0057	0.1854	0.0061	0.0894
$\ln(w_2/w_1)$	0.0037	4.6611	0.0189	2.3634	0.0011	1.1927
$[\ln(w_2/w_1)]^2$	0.0072	0.0027	0.0030	0.0014	0.0018	0.0007
$\ln y_1 \ln(w_2/w_1)$	0.0228	0.0665	0.0085	0.0342	-0.0003	0.0197
$\ln y_2 \ln(w_2/w_1)$	-0.0297	0.4733	-0.0209	0.2431	-0.0066	0.1211

(N, T)	$(100, 6)$		$(100, 10)$		$(100, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	-0.0647	1.5528	-0.0420	0.9191	-0.0444	0.4257
$[\ln(w_3/w_1)]^2$	0.0064	0.0036	0.0042	0.0021	0.0024	0.0010
$\ln(w_2/w_1)\ln(w_3/w_1)$	2.90E-03	0.0094	-8.61E-06	0.0064	1.39E-03	0.0029
$\ln y_1 \ln(w_3/w_1)$	0.0095	0.0530	0.0022	0.0345	0.0044	0.0161
$\ln y_2 \ln(w_3/w_1)$	0.0069	0.1522	0.0061	0.0894	0.0095	0.0414
$\ln(w_2/w_1)$	-0.0011	2.0212	0.0011	1.1927	-0.0151	0.5765
$[\ln(w_2/w_1)]^2$	0.0023	0.0012	0.0018	0.0007	0.0010	0.0003
$\ln y_1 \ln(w_2/w_1)$	0.0050	0.0295	-0.0003	0.0197	0.0026	0.0093
$\ln y_2 \ln(w_2/w_1)$	-0.0117	0.2080	-0.0066	0.1211	0.0006	0.0594

(N, T)	$(200, 6)$		$(200, 10)$		$(200, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	-0.0405	0.7488	-0.0444	0.4257	-0.0001	0.0005
$[\ln(w_3/w_1)]^2$	0.0040	0.0017	0.0024	0.0010	0.0004	0.0001
$\ln(w_2/w_1)\ln(w_3/w_1)$	7.70E-05	0.0052	1.39E-03	0.0029	0.0004	0.0001
$\ln y_1 \ln(w_3/w_1)$	0.0031	0.0284	0.0044	0.0161	0.0016	0.0194
$\ln y_2 \ln(w_3/w_1)$	0.0062	0.0735	0.0095	0.0414	-0.0075	0.2882
$\ln(w_2/w_1)$	-0.0016	1.0180	-0.0151	0.5765	0.0001	0.0002
$[\ln(w_2/w_1)]^2$	0.0017	0.0006	0.0010	0.0003	0.0001	0.0046
$\ln y_1 \ln(w_2/w_1)$	-0.0004	0.0161	0.0026	0.0093	0.0006	0.0295
$\ln y_2 \ln(w_2/w_1)$	-0.0040	0.1042	0.0006	0.0594	-0.0001	0.0005

Table 10. The performance of the parameter estimates from the third-stage setting M
 $(\cdot) = 2 \ln(1 + y_1) + \ln y_2$

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (50, 6)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	1.4913	10.6782	-0.5701	40.0629	1.4752	14.4928	-0.6309	59.1456
$[\ln(w_3/w_1)]^2$	-0.0777	0.0965	0.9459	40.0895	-0.0791	0.1100	0.9438	59.9369
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0881	0.0912	-0.5630	0.4723	0.0901	0.1032	-0.5582	0.5405
$\ln y_1 \ln(w_3/w_1)$	-0.2919	0.0993	-0.9256	1.0022	-0.2905	0.1042	-0.9196	1.0569
$\ln y_2 \ln(w_3/w_1)$	-0.1719	0.9651	0.0100	0.1442	-0.1684	1.3940	0.0046	0.2109
$\ln(w_2/w_1)$	-1.5152	10.0516	0.3878	0.2155	-1.5612	13.3644	0.3823	0.2445
$[\ln(w_2/w_1)]^2$	-0.0827	0.1032	0.9493	0.9644	-0.0851	0.1180	0.9530	1.0031
$\ln y_1 \ln(w_2/w_1)$	0.2936	0.0996	-0.3321	4.4678	0.2905	0.1033	-0.3041	6.5039
$\ln y_2 \ln(w_2/w_1)$	0.1783	0.8762	0.1251	4.3575	0.1972	1.2215	0.1200	6.5374

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (50, 10)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	1.2340	4.4188	-0.5040	13.9787	1.2318	5.7442	-0.4983	20.5904
$[\ln(w_3/w_1)]^2$	-0.0427	0.0441	0.7953	13.6379	-0.0438	0.0506	0.7737	20.1633
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0440	0.0430	-0.5478	0.3486	0.0449	0.0485	-0.5468	0.3724
$\ln y_1 \ln(w_3/w_1)$	-0.2459	0.0651	-0.9171	0.8877	-0.2467	0.0673	-0.9169	0.9111
$\ln y_2 \ln(w_3/w_1)$	-0.1356	0.3330	-0.0026	0.0454	-0.1339	0.4812	-0.0032	0.0687
$\ln(w_2/w_1)$	-1.3175	4.4968	0.3915	0.1766	-1.3323	5.6856	0.3921	0.1879
$[\ln(w_2/w_1)]^2$	-0.0438	0.0470	0.9540	0.9312	-0.0443	0.0530	0.9524	0.9382
$\ln y_1 \ln(w_2/w_1)$	0.2548	0.0696	-0.3555	1.6195	0.2546	0.0714	-0.3578	2.3331
$\ln y_2 \ln(w_2/w_1)$	0.1539	0.3219	0.1692	1.4479	0.1591	0.4470	0.1780	2.1719

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (50, 20)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	0.9773	1.7409	-0.4049	3.2023	0.9713	1.9978	-0.4079	4.5599
$[\ln(w_3/w_1)]^2$	-0.0209	0.0186	0.7599	3.5382	-0.0209	0.0198	0.7517	4.8282
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0216	0.0188	-0.5540	0.3177	0.0216	0.0199	-0.5514	0.3194
$\ln y_1 \ln(w_3/w_1)$	-0.1961	0.0396	-0.9225	0.8617	-0.1965	0.0402	-0.9201	0.8617
$\ln y_2 \ln(w_3/w_1)$	-0.1053	0.0973	0.0033	0.0103	-0.1028	0.1269	0.0007	0.0146
$\ln(w_2/w_1)$	-0.9926	1.7175	0.4011	0.1658	-0.9978	1.9809	0.4010	0.1679
$[\ln(w_2/w_1)]^2$	-0.0227	0.0198	0.9456	0.8988	-0.0226	0.0211	0.9451	0.8999
$\ln y_1 \ln(w_2/w_1)$	0.2030	0.0424	-0.4005	0.4895	0.2029	0.0427	-0.3990	0.6353
$\ln y_2 \ln(w_2/w_1)$	0.1028	0.0900	0.1919	0.3597	0.1046	0.1181	0.1948	0.5041

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (100, 6)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	1.1757	5.4746	-0.4287	20.2334	1.1748	7.3733	-0.4563	29.2564
$[\ln(w_3/w_1)]^2$	-0.0357	0.0432	0.7869	19.7487	-0.0375	0.0522	0.7863	28.8443
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0372	0.0421	-0.5466	0.3676	0.0385	0.0496	-0.5445	0.3997
$\ln y_1 \ln(w_3/w_1)$	-0.2338	0.0606	-0.9162	0.9063	-0.2347	0.0637	-0.9149	0.9367
$\ln y_2 \ln(w_3/w_1)$	-0.1292	0.4715	-0.0032	0.0652	-0.1277	0.6799	-0.0051	0.0974
$\ln(w_2/w_1)$	-1.1959	5.3548	0.3939	0.1846	-1.2217	6.9828	0.3960	0.2006
$[\ln(w_2/w_1)]^2$	-0.0370	0.0460	0.9487	0.9280	-0.0382	0.0541	0.9463	0.9366
$\ln y_1 \ln(w_2/w_1)$	0.2393	0.0630	-0.3837	2.3469	0.2396	0.0658	-0.3763	3.3195
$\ln y_2 \ln(w_2/w_1)$	0.1296	0.4509	0.1781	2.1392	0.1379	0.6220	0.1808	3.1362

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (100, 10)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	0.9429	2.2677	-0.4203	6.9076	0.9412	2.9192	-0.4228	10.2355
$[\ln(w_3/w_1)]^2$	-0.0215	0.0206	0.7311	6.7131	-0.0220	0.0237	0.7258	9.8472
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0224	0.0207	-0.5486	0.3250	0.0227	0.0234	-0.5458	0.3335
$\ln y_1 \ln(w_3/w_1)$	-0.1952	0.0403	-0.9170	0.8644	-0.1961	0.0417	-0.9145	0.8711
$\ln y_2 \ln(w_3/w_1)$	-0.0946	0.1615	-0.0021	0.0228	-0.0930	0.2337	-0.0048	0.0337
$\ln(w_2/w_1)$	-1.0017	2.4235	0.3982	0.1693	-1.0077	3.0681	0.3988	0.1753
$[\ln(w_2/w_1)]^2$	-0.0234	0.0226	0.9479	0.9085	-0.0237	0.0257	0.9465	0.9110
$\ln y_1 \ln(w_2/w_1)$	0.2016	0.0429	-0.3914	0.8894	0.2017	0.0439	-0.3908	1.2500
$\ln y_2 \ln(w_2/w_1)$	0.1071	0.1675	0.1982	0.7187	0.1090	0.2373	0.2011	1.0632

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (100, 20)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	0.7660	0.9884	-0.3828	1.6880	0.7649	1.1309	-0.3956	2.3807
$[\ln(w_3/w_1)]^2$	-0.0087	0.0078	0.7403	2.0456	-0.0084	0.0083	0.7441	2.7093
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0080	0.0079	-0.5476	0.3050	0.0078	0.0084	-0.5458	0.3055
$\ln y_1 \ln(w_3/w_1)$	-0.1519	0.0237	-0.9189	0.8494	-0.1523	0.0240	-0.9168	0.8480
$\ln y_2 \ln(w_3/w_1)$	-0.0842	0.0509	-0.0019	0.0049	-0.0834	0.0667	-0.0038	0.0072
$\ln(w_2/w_1)$	-0.7652	0.9738	0.3990	0.1616	-0.7729	1.1259	0.3993	0.1628
$[\ln(w_2/w_1)]^2$	-0.0078	0.0084	0.9489	0.9027	-0.0077	0.0090	0.9482	0.9024
$\ln y_1 \ln(w_2/w_1)$	0.1573	0.0253	-0.4044	0.3278	0.1573	0.0255	-0.4001	0.3977
$\ln y_2 \ln(w_2/w_1)$	0.0783	0.0480	0.1944	0.1986	0.0809	0.0637	0.1935	0.2697

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (200, 6)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	0.8785	2.7073	-0.3656	9.2288	0.8833	3.6508	-0.0405	0.7488
$[\ln(w_3/w_1)]^2$	-0.0156	0.0182	0.6969	9.0029	-0.0156	0.0221	0.0040	0.0017
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0159	0.0178	-0.5393	0.3232	0.0161	0.0212	0.0026	0.0003
$\ln y_1 \ln(w_3/w_1)$	-0.1822	0.0361	-0.9097	0.8598	-0.1829	0.0377	0.0001	0.0052
$\ln y_2 \ln(w_3/w_1)$	-0.0880	0.2217	-0.0103	0.0312	-0.0888	0.3258	0.0002	0.0002
$\ln(w_2/w_1)$	-0.9141	2.7726	0.3941	0.1694	-0.9268	3.6379	0.0031	0.0284
$[\ln(w_2/w_1)]^2$	-0.0160	0.0193	0.9509	0.9176	-0.0166	0.0232	0.0062	0.0735
$\ln y_1 \ln(w_2/w_1)$	0.1868	0.0378	-0.4037	1.1603	0.1867	0.0389	-0.0016	1.0180
$\ln y_2 \ln(w_2/w_1)$	0.0945	0.2234	0.2049	0.9748	0.0991	0.3164	0.0017	0.0006

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (200, 10)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	0.7438	1.2505	-0.4134	3.2232	0.7474	1.6236	-0.4268	4.9339
$[\ln(w_3/w_1)]^2$	-0.0086	0.0086	0.7292	3.4584	-0.0084	0.0100	0.7383	5.1139
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0096	0.0087	-0.5471	0.3107	0.0091	0.0099	-0.5442	0.3135
$\ln y_1 \ln(w_3/w_1)$	-0.1531	0.0245	-0.9153	0.8489	-0.1535	0.0251	-0.9127	0.8499
$\ln y_2 \ln(w_3/w_1)$	-0.0757	0.0827	-0.0037	0.0108	-0.0764	0.1234	-0.0066	0.0164
$\ln(w_2/w_1)$	-0.7837	1.3069	0.3988	0.1637	-0.7914	1.6652	0.3993	0.1664
$[\ln(w_2/w_1)]^2$	-0.0106	0.0096	0.9475	0.9022	-0.0101	0.0110	0.9467	0.9030
$\ln y_1 \ln(w_2/w_1)$	0.1567	0.0256	-0.3939	0.4877	0.1568	0.0261	-0.3894	0.6674
$\ln y_2 \ln(w_2/w_1)$	0.0848	0.0833	0.1990	0.3596	0.0874	0.1216	0.1964	0.5370

(σ^2, λ)	(1.88, 1.66)				(1.35, 0.83)			
$(N, T) = (200, 20)$	Model A		Model B		Model A		Model B	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3/w_1)$	0.6003	0.5395	-0.3955	0.8435	0.5993	0.6009	-0.3964	1.1368
$[\ln(w_3/w_1)]^2$	-0.0062	0.0037	0.7404	1.1994	-0.0065	0.0039	0.7405	1.4700
$\ln(w_2/w_1)\ln(w_3/w_1)$	0.0056	0.0038	-0.5497	0.3048	0.0060	0.0039	-0.5498	0.3059
$\ln y_1 \ln(w_3/w_1)$	-0.1183	0.0143	-0.9210	0.8507	-0.1184	0.0144	-0.9210	0.8517
$\ln y_2 \ln(w_3/w_1)$	-0.0668	0.0239	0.0004	0.0024	-0.0662	0.0308	0.0003	0.0035
$\ln(w_2/w_1)$	-0.6055	0.5468	0.4004	0.1615	-0.6056	0.6110	0.4006	0.1621
$[\ln(w_2/w_1)]^2$	-0.0051	0.0040	0.9487	0.9010	-0.0056	0.0042	0.9484	0.9010
$\ln y_1 \ln(w_2/w_1)$	0.1233	0.0155	-0.4022	0.2366	0.1234	0.0156	-0.4021	0.2689
$\ln y_2 \ln(w_2/w_1)$	0.0630	0.0237	0.1949	0.1087	0.0629	0.0309	0.1953	0.1387

Table 11. The performance of the estimators of $(\gamma, \sigma^2, \lambda)$ setting $M(\cdot) = 2\ln(1+y_1) + \ln y_2$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

Model A								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0019	0.0033	-0.3856	4.6154	-0.2517	0.1448	0.0563	0.0796
(50, 10)	0.0017	0.0003	-0.4462	0.9560	-0.2561	0.1177	0.0476	0.0213
(50, 20)	0.0015	3.46E-05	-0.4316	0.3057	-0.3218	0.1363	0.0291	0.0071
(100, 6)	0.0008	0.0008	-0.4040	0.2748	-0.1730	0.0646	0.0519	0.0242
(100, 10)	0.0012	0.0001	-0.3538	0.1942	-0.1788	0.0573	0.0337	0.0106
(100, 20)	0.0012	1.67E-05	-0.3202	0.1669	-0.2416	0.0760	0.0206	0.0047
(200, 6)	0.0011	1.13E-03	-0.3055	-0.3055	-0.1251	-0.1251	0.0375	0.0139
(200, 10)	0.0007	6.51E-05	-0.2625	0.1040	-0.1278	0.0288	0.0241	0.0069
(200, 20)	0.0012	8.81E-06	-0.2527	0.0973	-0.1957	0.0473	0.0167	0.0034

Model B								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0012	0.0020	-0.5489	0.4219	-0.2566	0.1309	0.2074	0.0873
(50, 10)	0.0010	0.0003	-0.4609	0.3329	-0.2593	0.1129	0.1455	0.0418
(50, 20)	0.0010	3.38E-05	-0.4046	0.2810	-0.3315	0.1394	0.0231	0.0089
(100, 6)	0.0004	0.0008	-0.4006	0.2304	-0.1766	0.0648	0.1941	0.0639
(100, 10)	0.0007	0.0001	-0.3384	0.1827	-0.1827	0.0569	0.1332	0.0305
(100, 20)	0.0009	1.57E-05	-0.3023	0.1543	-0.2495	0.0781	0.0180	0.0059
(200, 6)	0.0008	3.70E-04	-0.2970	0.1262	-0.1268	0.0338	0.1804	0.0485
(200, 10)	0.0005	6.51E-05	-0.2522	0.0980	-0.1309	0.0290	0.1255	0.0240
(200, 20)	0.0010	8.11E-06	-0.2416	0.0911	-0.2017	0.0489	0.0148	0.0042

Model C								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	-2.2125	1.43E+02	3.5794	88.6171	-1.0450	1.5504	0.1120	1.6625
(50, 10)	-1.3346	84.2538	1.4988	15.6058	-1.1207	1.6422	0.1389	0.9227
(50, 20)	-0.7278	48.6884	0.7277	7.6566	-1.1935	1.7107	0.1080	0.5988
(100, 6)	-1.5417	87.0333	1.4096	14.0000	-0.9706	1.3197	0.1468	0.8457
(100, 10)	-0.5660	34.9410	0.8349	5.9937	-1.0459	1.3925	0.1230	0.5949
(100, 20)	-0.4736	28.7234	0.2183	1.0058	-1.1212	1.5233	0.0641	0.2752
(200, 6)	-0.8007	43.6116	0.6856	3.7512	-0.9161	1.1750	0.1149	0.5348
(200, 10)	-0.2753	11.8120	0.2787	0.9357	-0.9408	1.1834	0.0835	0.3256
(200, 20)	-0.4421	29.5198	0.0799	0.3670	-1.0680	1.4029	0.0789	0.1817

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

Model A								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0056	0.0134	-0.1849	4.4666	-0.0677	0.0838	0.0423	0.0935
(50, 10)	0.0023	0.0014	-0.2814	0.7635	-0.0767	0.0406	0.0382	0.0259
(50, 20)	0.0016	0.0001	-0.2866	0.1560	-0.0957	0.0274	0.0230	0.0080
(100, 6)	0.0026	0.0042	-0.2648	0.1643	-0.0545	0.0310	0.0415	0.0316
(100, 10)	0.0016	0.0007	-0.2353	0.1013	-0.0546	0.0192	0.0267	0.0138
(100, 20)	0.0009	0.0001	-0.2030	0.0797	-0.0680	0.0136	0.0141	0.0067
(200, 6)	0.0024	0.0019	-0.2030	0.0679	-0.0408	0.0159	0.0292	0.0188
(200, 10)	0.0008	0.0003	-0.1741	0.0537	-0.0378	0.0095	0.0186	0.0091
(200, 20)	0.0012	2.99E-05	-0.1671	0.0478	-0.0576	0.0081	0.0130	0.0037

Model B								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0047	0.0125	-0.3589	0.2150	-0.0754	0.0652	0.1850	0.0958
(50, 10)	0.0016	0.0014	-0.3073	0.1761	-0.0744	0.0382	0.1355	0.0462
(50, 20)	0.0010	0.0001	-0.2701	0.1463	-0.0920	0.0261	0.0178	0.0102
(100, 6)	0.0022	0.0043	-0.2684	0.1226	-0.0544	0.0310	0.1790	0.0683
(100, 10)	0.0011	0.0007	-0.2277	0.0979	-0.0529	0.0187	0.1281	0.0336
(100, 20)	0.0009	0.0001	-0.2030	0.0797	-0.0680	0.0136	0.0141	0.0067
(200, 6)	0.0020	0.0019	-0.1995	0.0669	-0.0397	0.0159	0.1708	0.0512
(200, 10)	0.0006	0.0003	-0.1685	0.0515	-0.0367	0.0092	0.1236	0.0265
(200, 20)	0.0010	2.89E-05	-0.1597	0.0451	-0.0562	0.0077	0.0112	0.0046

Model C								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	-2.6917	174.4586	4.0429	97.9785	-0.3596	0.4196	0.0627	1.7475
(50, 10)	-1.2696	75.9287	1.7455	23.0290	-0.4341	0.4123	0.1474	0.8933
(50, 20)	-0.6811	44.0150	1.0510	8.9850	-0.4798	0.3766	0.0980	0.5578
(100, 6)	-1.9241	106.5750	1.5962	18.2757	-0.3309	0.3580	0.1051	0.7505
(100, 10)	-1.0313	66.1306	1.0080	6.6854	-0.3837	0.2920	0.0920	0.5553
(100, 20)	-0.6347	44.9129	0.3496	1.2458	-0.4527	0.3291	0.0571	0.2869
(200, 6)	-1.2416	73.8272	0.7995	4.6920	-0.3222	0.2446	0.0795	0.4970
(200, 10)	-0.8888	57.8543	0.3654	1.5396	-0.3468	0.2384	0.0549	0.2850
(200, 20)	-0.0033	0.0158	0.1167	0.2356	-0.4063	0.2447	0.0486	0.1601

Table 12. The performance of estimated function $\ln G$ setting $M(\cdot) = 2\ln(1 + y_1) + \ln y_2$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A				Model B	
	G1		G2A		G2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0070	0.0523	-0.0072	0.0629	-0.0225	0.0736
(50, 10)	-0.0002	0.0268	-0.0063	0.0281	-0.0136	0.0315
(50, 20)	-0.0041	0.0152	-0.0073	0.0155	-0.0108	0.0153
(100, 6)	-0.0025	0.0224	-0.0074	0.0238	-0.0135	0.0271
(100, 10)	-0.0041	0.0152	-0.0075	0.0157	-0.0115	0.0160
(100, 20)	-0.0001	0.0069	-0.0020	0.0072	-0.0032	0.0066
(200, 6)	-0.0029	0.0124	-0.0059	0.0131	-0.0092	0.0139
(200, 10)	-0.0001	0.0069	-0.0021	0.0072	-0.0039	0.0069
(200, 20)	-0.0015	0.0034	-0.0026	0.0036	-0.0035	0.0033

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

(N, T)	Model A				Model B	
	G1		G2A		G2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0070	0.0523	-0.0067	0.0639	-0.0223	0.0775
(50, 10)	-0.0002	0.0268	-0.0061	0.0285	-0.0134	0.0328
(50, 20)	-0.0041	0.0152	-0.0074	0.0156	-0.0109	0.0156
(100, 6)	-0.0025	0.0224	-0.0071	0.0243	-0.0134	0.0289
(100, 10)	-0.0041	0.0152	-0.0074	0.0159	-0.0116	0.0167
(100, 20)	-0.0001	0.0069	-0.0020	0.0072	-0.0033	0.0067
(200, 6)	-0.0029	0.0124	-0.0058	0.0133	-0.0092	0.0147
(200, 10)	-0.0001	0.0069	-0.0021	0.0073	-0.0039	0.0072
(200, 20)	-0.0015	0.0034	-0.0026	0.0036	-0.0036	0.0033

Table 13. The performance of estimated AE setting $M(\cdot) = 2\ln(1 + y_1) + \ln y_2$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A				Model B	
	AE1		AE2A		AE2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0101	0.0039	0.0129	0.0905	0.0262	0.2526
(50, 10)	0.0047	0.0017	0.0046	0.0050	0.0090	0.0351
(50, 20)	0.0025	0.0007	0.0012	0.0004	0.0014	0.0006
(100, 6)	0.0037	0.0013	0.0033	0.0243	0.0051	0.0015
(100, 10)	0.0025	0.0007	0.0012	0.0004	0.0020	0.0007
(100, 20)	0.0009	0.0003	0.0008	0.0002	0.0007	0.0003
(200, 6)	0.0019	0.0006	0.0011	0.0003	0.0023	0.0006
(200, 10)	0.0009	0.0003	0.0007	0.0002	0.0009	0.0003
(200, 20)	0.0010	0.0002	0.0007	0.0001	0.0007	0.0001

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

(N, T)	Model A				Model B	
	AE1		AE2A		AE2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0101	0.0039	0.0205	0.8222	0.0455	0.9369
(50, 10)	0.0047	0.0017	0.0047	0.0050	0.0104	0.1541
(50, 20)	0.0025	0.0007	0.0012	0.0004	0.0017	0.0007
(100, 6)	0.0037	0.0013	0.0035	0.0243	0.0073	0.0098
(100, 10)	0.0025	0.0007	0.0012	0.0004	0.0024	0.0007
(100, 20)	0.0009	0.0003	0.0008	0.0002	0.0009	0.0003
(200, 6)	0.0019	0.0006	0.0012	0.0003	0.0027	0.0007
(200, 10)	0.0009	0.0003	0.0007	0.0002	0.0012	0.0003
(200, 20)	0.0010	0.0002	0.0007	0.0001	0.0007	0.0002

Table 14. The performance of estimated **TE** scores setting $M(\cdot) = 2 \ln(1 + y_1) + \ln y_2$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A		Model B		Model C	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0282	0.0127	0.1089	0.0210	1.65E+06	5.30E+17
(50, 10)	0.0247	0.0060	0.0871	0.0119	3.9055	1.48E+06
(50, 20)	0.0271	0.0018	0.0274	0.0018	0.2057	1.67E+04
(100, 6)	0.0224	0.0076	0.0962	0.0171	68.5948	2.09E+09
(100, 10)	0.0170	0.0033	0.0786	0.0098	0.1251	353.2717
(100, 20)	0.0246	0.0016	0.0247	0.0016	0.1103	6.65E+03
(200, 6)	0.0153	0.0065	0.0873	0.0146	2.00E+05	3.80E+16
(200, 10)	0.0118	0.0029	0.0724	0.0085	0.0901	402.4579
(200, 20)	0.0231	0.0016	0.0232	0.0016	0.0510	97.9164

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.35, 0.83)$$

(N, T)	Model A		Model B		Model C	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0239	0.0196	0.0828	0.0215	1.84E+09	6.04E+23
(50, 10)	0.0204	0.0091	0.0710	0.0123	4.27E+05	6.20E+16
(50, 20)	0.0233	0.0023	0.0237	0.0023	25.5140	4.99E+08
(100, 6)	0.0180	0.0133	0.0719	0.0181	1.09E+09	5.29E+23
(100, 10)	0.0134	0.0063	0.0647	0.0107	0.1351	1595.8131
(100, 20)	0.0213	0.0021	0.0214	0.0020	0.0426	1.1975
(200, 6)	0.0107	0.0119	0.0645	0.0162	1.80E+04	3.38E+14
(200, 10)	0.0084	0.0058	0.0599	0.0097	5278.5950	5.11E+13
(200, 20)	0.0200	0.0020	0.0201	0.0019	0.0334	0.7069

6. An Empirical Application

This section presents an empirical application using the proposed semiparametric estimation techniques to examine a sample of 340 commercial banks over the period 1993-2004. The sample banks come from 14 East European countries and are compiled from the unconsolidated accounting data of Bankscope database. After precluding all missing and zero observations, this unbalanced panel data contain a total of 1399 bank-year observations. Since 1993, East European countries have started to transit from socialism to capitalism gradually. During the transition period, these countries liberalize the financial institutions by privatizing state-owned banks and allowing for new entry of private and foreign banks. These nations are in the long process of building a stable, reliable, independent, and transparent banking system. The enforcement of the deregulation policy should have intensified the competition in the banking industries. Consequently, an essential issue to regulators, financial institution managers, industry consultants, and potential investors is whether the policy does improve banks' performance.

In the past two decades, the academic research on the performance of financial services has mainly utilized the frontier approaches to evaluate a firm's TE. Measure TE reflects the ability of a firm to employ a minimal input mix to produce a given level of outputs or to produce maximum outputs using a given input combination. It has been widely studied by previous works. The economic efficiency of a firm combines TE with AE, i.e., $EE=TE*AE$. Measure AE reflects the capability of a firm to hire inputs in an optimal proportion, given their respective prices, so as to achieve a minimum cost for a certain level of outputs. In contrast to TE, AE draws relatively less attention of researchers. However, the investigation of allocative efficiency is crucial particularly in transition economies. As the input and/or the output prices are frequently under the control of the governments of the transition countries, these prices may be slowly adjusted in response to market conditions. Possible allocative distortion may play a pivotal role in transition countries. Therefore, we apply a cost function to estimate TE and AE scores simultaneously.

Following the intermediation approach, which regards a bank as an intermediary between depositors and borrowers, we identify two outputs: total loans (y_1) and

investments (y_2). Bank inputs are defined as labor (x_1), physical capital (x_2), and borrowed funds (x_3). The price of labor (w_1) is equal to the total personnel expenses divided by total assets.⁶ The price of physical capital (w_2) is measured as the ratio of non-interest expenses and total depreciation to total fixed assets, and the price of funds (w_3) represents total interest expenses divided by total funds. Note that we also estimate the translog cost function for the purpose of comparison.

In the first step, we simultaneously estimate two input share equations by the NISUR to obtain the estimates of country-specific allocative parameters, i.e., H_{3l} / H_{1l} and H_{2l} / H_{1l} ($l = 1, \dots, 14$). Herein, labor input is arbitrarily chosen as the numeraire. According to the allocative parameter estimates in Table 15, all but three such estimates are less than unity. This implies that banks in Poland are inclined to under-utilize input capital, while banks in Kazakhstan and FYR Macedonia are apt to under-utilize funds, relative to the numeraire labor as their corresponding allocative parameter estimates exceed unity. This may happen because banks in those three countries have lower input prices or use fewer input quantities. It can be seen on Table 16 that banks in Lithuania has the lowest average W_2 / W_1 (11.1825), followed by banks in Poland (16.2563) in ascending order. Although banks in Lithuania has the lowest input prices, they utilize more input quantities (X_2 / X_1) than banks in Poland. That might cause the allocative parameter estimates in Poland to exceed unity, implying that banks in Poland are inclined to under-utilize input capital, relative to the numeraire labor. It also reveals that banks operating in Kazakhstan reach the lowest average W_3 / W_1 (0.7066), followed by banks operating in Estonia (1.2313), Lithuania (1.5370), Latvia (1.5395), and FYR Macedonia (2.0040) in ascending order. Even though banks in Estonia, Lithuania and Latvia have the lower input prices than banks in FYR Macedonia, they employ more input levels (X_3 / X_1) than banks in FYR Macedonia. This could be the reason that banks in Kazakhstan and FYR Macedonia are apt to under-utilize funds, relative to the numeraire labor.

⁶ Since data on the number of employees are missing for many banks, we instead use total assets to calculate the price of labor. Altunbas et al. (2001), Weill (2004), Bos and Schmiedel (2007) and others have utilized similar definitions.

To cut the production costs and eliminate the AI in the three nations, sample banks should decrease the employment of labor and/or increase the employment of the other two inputs. On the contrary, banks of the remaining countries tend to over-utilize both capital and funds relative to labor. Therefore, those banks should lower the employment of both capital and funds, along with hiring more labor.

Table 15. Estimates of the country-specific AE parameters

	η_2 / η_1		η_3 / η_1	
	Parametric Estimate	Standard Error	Parametric Estimate	Standard Error
Croatia	0.1201***	0.0344	0.5763***	0.0482
Czech Republic	0.2614***	0.0772	0.4108***	0.0234
Estonia	0.0807***	0.0240	0.5520***	0.1114
Hungary	0.0974***	0.0301	0.2613***	0.0335
Kazakhstan	0.1085***	0.0327	1.4382***	0.4954
Latvia	0.0612***	0.0176	0.4993***	0.0684
Lithuania	0.0702***	0.0211	0.4926***	0.0785
FYR Macedonia	0.0760***	0.0219	1.0146***	0.1351
Poland	1.1461***	0.3473	0.5376***	0.0319
Romania	0.0772***	0.0240	0.2860***	0.0466
Russia	0.0682***	0.0193	0.4017***	0.0272
Slovak Republic	0.1339***	0.0415	0.7159***	0.0820
Slovenia	0.1426***	0.0436	0.4453***	0.0665
Ukraine	0.0772***	0.0240	0.4281***	0.0810
R-square	0.7095		0.8880	

Note : ***: Significant at the 1% level. **: Significant at the 5% level. *: Significant at the 10% level.

Table 16. average relative input prices and relative input quantities

	W_2 / W_1	X_2 / X_1	W_3 / W_1	X_3 / X_1
Croatia	34.6558	0.0204	2.5953	0.8556
Czech Republic	108.5914	0.0247	17.3226	0.8785
Estonia	157.5801	0.0412	1.2313	0.8050
Hungary	196.5227	0.0226	4.4943	0.8825
Kazakhstan	23.3571	0.0380	0.7066	0.8264
Latvia	47.1168	0.0526	1.5395	0.8147
Lithuania	11.1825	0.0908	1.5370	0.8721
FYR Macedonia	37.0202	0.0527	2.0040	0.7695

Poland	16.2563	0.0251	4.6080	0.8089
Romania	37.1894	0.0477	5.9356	0.8458
Russia	133.5765	0.0374	3.9529	0.8000
Slovak Republic	497.3563	0.0440	8.6207	0.9338
Slovenia	76.7797	0.0324	2.7684	0.8447
Ukraine	78.9257	0.1121	3.4675	0.8156

Table 17 presents both the first and the third step parameter estimates for Models A and B. All of the first step estimates are significantly different from zero and their magnitudes and signs deviate from those of the third step estimates. Recall that the first step estimates are recommended to be exploited to calculate the AE measures shown in Table 19 as AE1.

Table 17. Parameter estimates of the Semiparametric regression

	Model A				Model B	
	First step		Third step		Third step	
	Parametric Estimate	Standard Error	Parametric Estimate	Standard Error	Parametric Estimate	Standard Error
$\ln(w_3 / w_1)$	0.2729***	0.0125	-0.0491	0.0538	-0.0875*	0.0518
$[\ln(w_3 / w_1)]^2$	0.1925***	0.0027	0.0741***	0.0088	0.0503***	0.0102
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.0099***	0.0024	-0.0023	0.0027	0.0191***	0.0057
$\ln y_1 \ln(w_3 / w_1)$	0.0068***	0.0014	0.0393***	0.0068	0.0313***	0.0075
$\ln y_2 \ln(w_3 / w_1)$	0.0076***	0.0013	0.0079	0.0065	0.0139*	0.0071
$\ln(w_2 / w_1)$	0.0873***	0.0191	0.0669***	0.0136	0.2476***	0.0426
$[\ln(w_2 / w_1)]^2$	0.0159***	0.0041	0.0102***	0.0014	0.0199***	0.0048
$\ln y_1 \ln(w_2 / w_1)$	-1.99E-03**	8.75E-04	-3.94E-03**	1.68E-03	-1.62E-02***	5.61E-03
$\ln y_2 \ln(w_2 / w_1)$	-2.35E-03***	8.61E-04	2.01E-03	1.67E-03	-7.72E-03	5.27E-03

Table 18 summarizes the estimates of the distribution parameters for the three models. All of the three parameters are significantly estimated at the 1% level. It is important to note that the estimates of Model A are intimately close to those of Model B as expected, while they substantially deviate from those of Model C. As the estimate of γ is positive, the TE of the sample banks is found to improve over time during the sample period. However, Model C tends to overestimate the rate of

enhancement in production efficiency.

Table 18. Estimates of the distribution parameters of the three models

	Model A		Model B		Model C	
	Parametric Estimate	Standard Error	Parametric Estimate	Standard Error	Parametric Estimate	Standard Error
γ	0.0267***	0.0020	0.0234***	0.0022	0.0573***	0.0021
σ^2	0.4650***	0.0255	0.4067***	0.0232	0.5166***	0.0307
λ	1.2348***	0.0425	1.1563***	0.0431	0.9407***	0.0358
log-likelihood	-2700.90		-2698.70		-2878.38	

We use the parameter estimates to evaluate the TE and AE scores. Table 19 reports the outcomes. The average AE1 is equal to 89.21%, meaning that the potential percent of cost savings for an average bank that achieves AE alone is 10.79%. We ignore measure AE2 since the simulation results fail to support it as a suitable measure. The average TE score of Model A attains 79.39%, which is indistinguishable from the TE measure of Model B. This verifies the finding of the previous section, i.e., Model B is compatible with Model A particularly for panel data with long time periods. Evidence is found that measure TI dominates measure AI for the sample states. Sample banks are advised to elevate their managerial capability, followed by optimizing the input mix.

Table 19. Average TE and AE scores of the three models

	Model A		Model B		Model C	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
AE1 (%)	89.21	0.1013	89.21	0.1013	89.21	0.1013
AE2 (%)	85.85	0.0950	94.11	0.0816	N/A	N/A
TE (%)	79.39	0.1209	79.37	0.1102	77.30	0.1185

Since both Models A and B have similar TE and AE measures, Table 20 merely reports the country-specific efficiency scores of Model A. Banks in Poland reach the highest average AE score (99.25%), followed by Czech Republic (98.66%), Slovak Republic (92.10%), and Slovenia (91.35%). Banks in FYR Macedonia are the most technically efficient (90.47%), followed by Slovenia (84.02%), Croatia (83.78%), and

Poland (83.77%). The correlation coefficient between the TE and AE measures is equal to 0.19. Both measures are somewhat positively correlated, indicating that a more technically efficient bank tends to be more or less allocatively efficient.

Table 20. Country-specific TE and AE measures of Model A

	AE (%)		TE (%)	
	Mean	St. Dev.	Mean	St. Dev.
Croatia	88.11	0.0314	83.78	0.1096
Czech Republic	98.66	0.0162	77.65	0.1050
Estonia	77.91	0.0601	77.72	0.0920
Hungary	90.08	0.0599	83.36	0.0813
Kazakhstan	76.63	0.0399	79.76	0.1264
Latvia	72.07	0.0642	71.97	0.1234
Lithuania	82.00	0.0498	67.46	0.1158
FYR Macedonia	75.96	0.0668	90.47	0.0626
Poland	99.25	0.0045	83.77	0.0856
Romania	90.55	0.0806	78.88	0.0705
Russia	83.60	0.0913	71.61	0.1469
Slovak Republic	92.10	0.0488	73.93	0.0597
Slovenia	91.35	0.0244	84.02	0.0571
Ukraine	83.75	0.0558	72.61	0.1677
<i>Corr(AE,TE)</i>	0.19			

Figure 1 depicts the various efficiency measures over the sample period of 1993 to 2004. The average AE scores vary in a narrow range between 86.92% and 91.59%, while the TE scores of the three models show a gradually increasing trend over the transition period. It is evident that the enforcement of financial liberalization and privatization by the governments of the sample countries is indeed effective in prompting the TE of their financial industries across time. Models A and B provide similar trends on the TE scores, while Model C underestimates the average TE measure for the first half of the sample period.

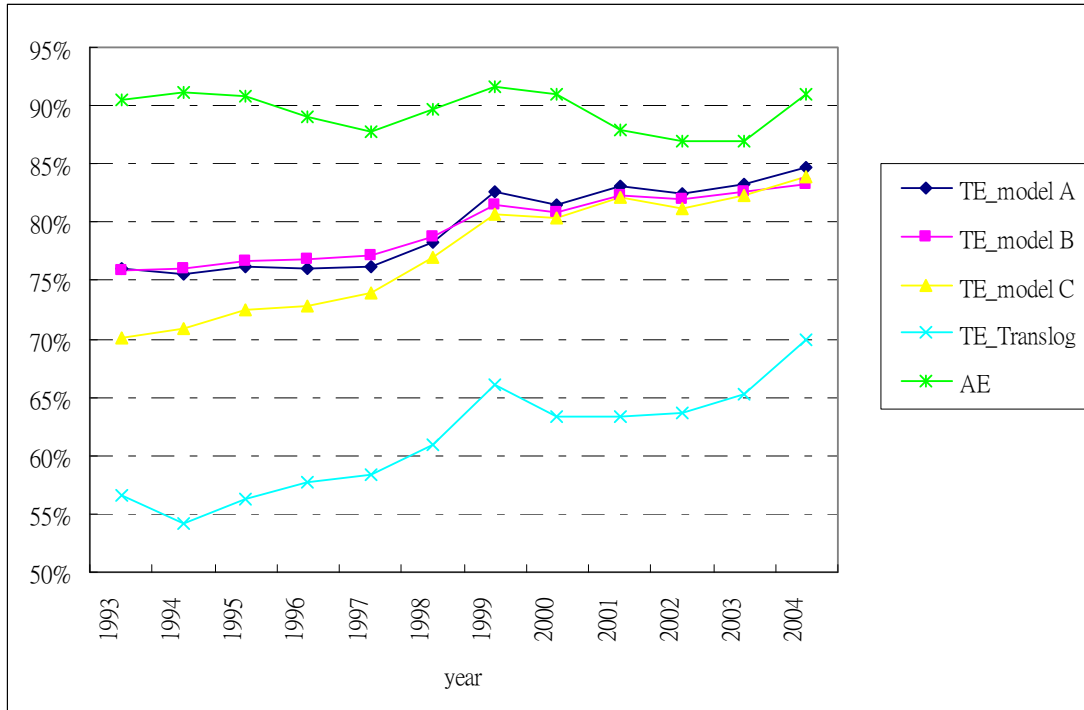


Figure 1. Average efficiency by year

We estimate the translog cost function as well. Its parameter estimates and the TE and AE measures are shown in Appendices XXII and XXIII. A vast majority of the parameters are significantly estimated. As far as the three distribution parameters are concerned, although the translog estimate of γ is fairly near those of Models A and B, the remaining two translog estimates of σ^2 and λ differ considerably from those of Table 18. If we feel comfortable with the semiparametric model, then the inferences based on a purely parametric specification are likely to be doubtful. Consequently, the average TE scores obtained from the translog cost function tend to be undervalued to a large extent, as shown in Figure 1 and Table XXIII. The introduction of a more flexible semiparametric model likely avoids the possibility of an inconsistent estimation arising from incorrect parameterization and potential confounding of specification error with inefficiency.

7. Conclusion

Since most economic relationships predicted by economic theory are unknown, one has to count on a particular parametric form, which may lead to a biased estimation due to invalid parameterization. The importance of nonparametric and semiparametric regression techniques has drawn much attention from the econometricians and applied researchers recently. These techniques allow the functional form to be determined at least partially by the data. Fan et al. (1996) and Deng and Huang (2008) generalized the conventional linear stochastic frontier model to a semiparametric stochastic production frontier model. On the basis of previous works, this article adds to the current literature by considering both TE and AE in the context of a semiparametric stochastic cost frontier model using panel data.

This paper has solved two major problems faced by applied researchers. First, the cost system must be estimated simultaneously, suffering from computational difficulties. Second, the log-likelihood function of the expenditure equation cannot be maximized due to the presence of the nonparametric component. Even worse, the nonparametric function is unable to be estimated by existing nonparametric regression methods. We propose a five-step procedure to cope with these problems. Evidence from a set of Monte Carlo simulations tends to support the superiority of Model A at least for a moderate sample size, while the performance of Model B's estimators is nearly as good as that of Model A's, particularly when the time period of the panel data is long. In other words, Model B is appropriate for long panel data.

The first step estimators of the cost share equations perform reasonably well. We thus advocate using these estimates to compute the AE measure and treat the estimated allocative parameters as given in the following steps. It is noticeable that despite the uselessness of the parameter estimates obtained in the third step, this step is necessary to yield the residuals and to concentrate out variance σ^2 . Otherwise, estimators of Step 5 will perform poorly. Moreover, Models A and B are robust to include additional explanatory variables to both parametric and nonparametric portions of the cost function. When cross sectional data are available, the foregoing conclusions continue to hold in general, except that the bias of the estimated TE measure does not decrease as the sample size increases.

The three models are applied to investigate the TE and AE measures of the financial sectors of 14 East European countries over the period of 1993-2004. As expected, the average TE scores obtained by Models A and B are close to each other. Model C underestimates the average TE scores, but exaggerates the rate of improvement on the TE. Financial deregulation appears to successfully prompt the TE of the sample banks over the transition period. Since the TI dominates the AI, bank managers are suggested to promote their managerial ability in such a way as to reduce the production costs for a given level of outputs, followed by adjusting for their input mix given the ratios of input prices.

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Appendix

Table I. The performance of the estimators of $(\gamma, \sigma^2, \lambda)$ for the case of $(\gamma, \sigma^2, \lambda) = (-0.025, 1.88, 1.66)$ setting $M(\cdot) = 2 \ln(1 + y_1)$

Model A								
(N, T)	γ		σ^2		λ		$m(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0019	0.0039	-0.0727	3.7895	-0.1149	0.1217	0.0137	0.0438
(50, 10)	-0.0005	0.0004	-0.1723	0.2153	-0.1093	0.0711	0.0159	0.0073
(50, 20)	-0.0017	6.68E-05	-0.1870	0.2035	-0.1144	0.0560	0.0108	0.0039
(100, 6)	-0.0005	8.45E-04	-0.1210	0.1146	-0.0669	0.0427	0.0144	0.0077
(100, 10)	-0.0006	2.01E-04	-0.1146	0.1068	-0.0704	0.0345	0.0089	0.0041
(100, 20)	-0.0014	3.37E-05	-0.1355	0.1041	-0.0804	0.0288	0.0078	0.0027
(200, 6)	0.0002	4.25E-04	-0.0843	0.0574	-0.0475	0.0224	0.0090	0.0043
(200, 10)	-0.0004	9.20E-05	-0.0811	0.0515	-0.0498	0.0166	0.0061	0.0025
(200, 20)	-0.0011	1.79E-05	-0.1103	0.0545	-0.0658	0.0156	0.0069	0.0020

Model B								
(N, T)	γ		σ^2		λ		$m(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0011	0.0020	-0.1811	0.2237	-0.1264	0.0913	0.1600	0.0518
(50, 10)	0.0001	0.0004	-0.1486	0.2037	-0.1205	0.0724	0.1079	0.0268
(50, 20)	-0.0011	6.68E-05	-0.1619	0.1938	-0.1280	0.0589	0.0090	0.0067
(100, 6)	0.0001	8.58E-04	-0.1071	0.1104	-0.0741	0.0434	0.1505	0.0369
(100, 10)	-0.0002	2.06E-04	-0.0976	0.1024	-0.0772	0.0347	0.1026	0.0198
(100, 20)	-0.0010	3.34E-05	-0.1189	0.0987	-0.0883	0.0297	0.0076	0.0043
(200, 6)	0.0005	4.31E-04	-0.0751	0.0556	-0.0514	0.0226	0.1455	0.0289
(200, 10)	-0.0001	9.31E-05	-0.0695	0.0492	-0.0538	0.0168	0.1014	0.0164
(200, 20)	-0.0009	1.75E-05	-0.1011	0.0519	-0.0708	0.0162	0.0061	0.0030

Model C								
(N, T)	γ		σ^2		λ		$m(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	-1.6396	94.1362	1.4959	14.2625	-0.8517	1.1932	0.0816	0.5696
(50, 10)	-0.5004	25.6243	0.6345	2.5856	-0.8592	1.1224	0.0623	0.3373
(50, 20)	-0.3321	19.9674	0.3627	1.4705	-0.8649	1.0531	0.0533	0.2124
(100, 6)	-0.5824	38.4079	0.5747	2.0683	-0.7297	0.8760	0.0667	0.2895
(100, 10)	-0.2949	25.9268	0.3743	0.9873	-0.7570	0.8620	0.0571	0.2220
(100, 20)	-0.0069	0.0155	0.1120	0.2748	-0.6895	0.7276	0.0207	0.1049
(200, 6)	-0.1588	5.0006	0.2852	0.5507	-0.6196	0.6629	0.0398	0.1706
(200, 10)	-0.0552	2.5165	0.1307	0.1763	-0.5916	0.5988	0.0306	0.1123
(200, 20)	-0.0373	1.2597	0.0122	0.0824	-0.5449	0.4697	0.0209	0.0549

Table II. The performance of the estimators of $\ln G$, AE and TE for the case of $(\gamma, \sigma^2, \lambda) = (-0.025, 1.88, 1.66)$ setting $M(\cdot) = 2\ln(1 + y_1)$

(N, T)	Model A				Model B	
	G1		G2		G2	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0061	0.0296	-0.0050	0.0357	-0.0027	0.0389
(50, 10)	0.0005	0.0150	-0.0049	0.0167	-0.0027	0.0178
(50, 20)	-0.0026	0.0085	-0.0058	0.0093	-0.0038	0.0088
(100, 6)	-0.0010	0.0125	-0.0056	0.0144	-0.0037	0.0159
(100, 10)	-0.0026	0.0085	-0.0059	0.0094	-0.0047	0.0093
(100, 20)	0.0002	0.0038	-0.0016	0.0043	-0.0004	0.0040
(200, 6)	-0.0017	0.0070	-0.0043	0.0080	-0.0036	0.0083
(200, 10)	0.0002	0.0038	-0.0018	0.0044	-0.0012	0.0042
(200, 20)	-0.0012	0.0019	-0.0022	0.0022	-0.0018	0.0020

(N, T)	Model A				Model B	
	AE1		AE2		AE2	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0031	0.0027	0.0141	0.3993	0.0085	0.0061
(50, 10)	0.0010	0.0015	0.0030	0.0007	0.0020	0.0014
(50, 20)	0.0002	0.0007	0.0012	0.0004	0.0002	0.0006
(100, 6)	0.0004	0.0012	0.0023	0.0006	0.0018	0.0013
(100, 10)	0.0002	0.0007	0.0011	0.0004	0.0005	0.0007
(100, 20)	2.89E-06	0.0003	0.0009	0.0002	0.0004	0.0003
(200, 6)	0.0001	0.0006	0.0009	0.0003	0.0004	0.0006
(200, 10)	2.89E-06	0.0003	0.0008	0.0002	0.0006	0.0003
(200, 20)	0.0004	0.0002	0.0006	0.0001	0.0003	0.0002

TE	Model A		Model B		Model C	
(N, T)	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0196	45.0670	0.0720	0.0143	19.7267	6.31E+07
(50, 10)	0.0047	0.0043	0.0569	0.0080	0.2411	1123.5347
(50, 20)	-0.0172	0.0020	-0.0165	0.0020	0.0054	0.8439
(100, 6)	0.0014	0.0074	0.0663	0.0124	36.5326	4.53E+08
(100, 10)	0.0020	0.0038	0.0539	0.0071	0.0408	0.8003
(100, 20)	-0.0183	0.0019	-0.0180	0.0019	-0.0060	5.4501
(200, 6)	-0.0007	0.0070	0.0639	0.0116	13.4637	2.10E+08
(200, 10)	0.0003	0.0036	0.0521	0.0066	0.0229	0.0251
(200, 20)	-0.0188	0.0018	-0.0185	0.0018	-0.0142	0.0087

Table III. The performance of the allocative parameter estimates setting $M(\cdot) = 2 \ln(1 + y_1)$ as $N=30$

(N, T)	H_2 / H_1		H_3 / H_1	
	Bias	MSE	Bias	MSE
(30, 6)	0.0017	0.0026	0.0100	0.0038
(30, 10)	0.0006	0.0013	0.0048	0.0019
(30, 20)	0.0002	0.0006	0.0029	0.0009

Table IV. The performance of the parameter estimates in Step 1 setting $M(\cdot) = 2 \ln(1 + y_1)$ as $N=30$

(N, T)	(30, 6)		(30, 10)		(30, 20)	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.1092	0.3974	-0.0722	0.2227	-0.0179	0.0935
$[\ln(w_3 / w_1)]^2$	0.0189	0.0102	0.0114	0.0056	0.0036	0.0025
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0107	0.0216	0.0088	0.0121	0.0015	0.0052
$\ln y_1 \ln(w_3 / w_1)$	0.0342	0.1137	0.0247	0.0649	0.0046	0.0276
$\ln(w_2 / w_1)$	-0.0722	0.2648	-0.0475	0.1462	-0.0171	0.0679
$[\ln(w_2 / w_1)]^2$	0.0081	0.0044	0.0043	0.0027	0.0011	0.0012
$\ln y_1 \ln(w_2 / w_1)$	0.0169	0.0658	0.0128	0.0360	0.0023	0.0157

Table V. The performance of the parameter estimates from the third-stage setting $M(\cdot) = 2 \ln(1 + y_1)$ as $N=30$

Model A						
(N, T)	(30, 6)		(30, 10)		(30, 20)	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	1.2346	1.8721	1.0348	1.1897	0.8247	0.7068
$[\ln(w_3 / w_1)]^2$	-0.0452	0.1382	-0.0245	0.0668	-0.0053	0.0276
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0542	0.1270	0.0280	0.0642	0.0050	0.0279
$\ln y_1 \ln(w_3 / w_1)$	-0.3683	0.1640	-0.3080	0.1045	-0.2454	0.0623
$\ln(w_2 / w_1)$	-1.2559	1.9200	-1.0724	1.2705	-0.8620	0.7680
$[\ln(w_2 / w_1)]^2$	-0.0404	0.1467	-0.0223	0.0725	-0.0044	0.0300
$\ln y_1 \ln(w_2 / w_1)$	0.3741	0.1680	0.3190	0.1115	0.2564	0.0677
Model B						
(N, T)	(30, 6)		(30, 10)		(30, 20)	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.4113	1.3673	-0.3811	0.5510	-0.4085	0.2526
$[\ln(w_3 / w_1)]^2$	0.7627	1.7383	0.7096	0.8904	0.7343	0.6251
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.5538	0.5282	-0.5418	0.3702	-0.5514	0.3208
$\ln y_1 \ln(w_3 / w_1)$	-0.9245	1.0667	-0.9130	0.9075	-0.9216	0.8654
$\ln(w_2 / w_1)$	0.2057	0.2486	0.1929	0.1093	0.2018	0.0563
$[\ln(w_2 / w_1)]^2$	0.7514	0.6602	0.7451	0.5873	0.7519	0.5722
$\ln y_1 \ln(w_2 / w_1)$	0.1869	0.1268	0.2011	0.0708	0.1958	0.0450

Table VI. The performance of the estimators of $(\gamma, \sigma^2, \lambda)$ setting $M(\cdot) = 2\ln(1 + y_1)$ as $N=30$

Model A								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(30, 6)	0.0024	0.0045	0.0571	14.8897	-0.1954	0.2037	0.0052	0.1138
(30, 10)	0.0023	0.0005	-0.0519	11.4292	-0.2145	0.1890	0.0131	0.0388
(30, 20)	0.0017	5.67E-05	-0.2783	0.3223	-0.2990	0.1502	0.0161	0.0047

Model B								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(30, 6)	0.0017	0.0033	-0.2848	0.3544	-0.2147	0.1589	0.1691	0.0696
(30, 10)	0.0015	0.0005	-0.2304	0.3146	-0.2290	0.1354	0.1184	0.0328
(30, 20)	0.0010	5.51E-05	-0.2344	0.2994	-0.3187	0.1555	0.0131	0.0084

Model C								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(30, 6)	-1.92E+09	2.40E+21	3.8586	181.8836	-0.9692	1.8655	0.1395	1.2015
(30, 10)	-3.46E+09	9.55E+21	2.3593	139.5833	-0.9619	1.9062	0.0936	0.6653
(30, 20)	-7.44E+08	4.82E+20	2.7976	46.0182	0.5232	2.2982	0.1109	0.2950

Table VII. The performance of estimated function $\ln G$ setting $M(\cdot) = 2\ln(1 + y_1)$ as $N=30$

(N, T)	Model A				Model B	
	G1		G2A		G2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(30, 6)	0.0050	0.0529	-0.0154	0.0706	-0.0083	0.0761
(30, 10)	0.0061	0.0296	-0.0056	0.0347	-0.0024	0.0356
(30, 20)	-0.0010	0.0125	-0.0057	0.0136	-0.0026	0.0135

Table VIII. The performance of estimated AE setting $M(\cdot) = 2\ln(1 + y_1)$ as $N=30$

(N, T)	Model A				Model B	
	AE1		AE2A		AE2B	
	Bias	MSE	Bias	MSE	Bias	MSE
(30, 6)	0.0064	0.0057	0.0412	2.4780	0.0446	16.7190
(30, 10)	0.0031	0.0027	0.0139	0.3991	0.0098	1.4458
(30, 20)	0.0004	0.0012	0.0022	0.0006	0.0006	0.0010

Table IX. The performance of estimated TE scores setting $M(\cdot) = 2\ln(1 + y_1)$ as $N=30$

(N, T)	Model A		Model B		Model C	
	Bias	MSE	Bias	MSE	Bias	MSE
(30, 6)	0.0063	0.0190	0.0821	0.0160	1.26E+08	2.16E+21
(30, 10)	0.0064	0.0072	0.0670	0.0087	3.9368	1.17E+06
(30, 20)	0.0221	0.0019	0.0224	0.0018	445.4975	1.03E+11

Table X. The performance of the first-stage parameter estimates using cross-sectional data assuming $(\sigma^2, \lambda) = (1.88, 1.66)$

	$N = 100$		$N = 300$		$N = 500$		$N = 1000$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.1899	0.8262	-0.0722	0.2227	-0.0280	0.1134	-0.0055	0.0579
$[\ln(w_3 / w_1)]^2$	0.0354	0.0236	0.0114	0.0056	0.0044	0.0030	0.0025	0.0014
η_2 / η_1	0.0245	0.1943	0.0006	0.0013	0.0004	0.0007	0.0001	0.0004
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0133	0.0414	0.0088	0.0121	0.0028	0.0062	-0.0007	0.0036
η_3 / η_1	0.0200	0.0230	0.0048	0.0019	0.0030	0.0010	0.0026	0.0005
$\ln y_1 \ln(w_3 / w_1)$	0.0498	0.2256	0.0247	0.0649	0.0082	0.0330	-0.0002	0.0178
$\ln(w_2 / w_1)$	-0.1506	0.5921	-0.0475	0.1462	-0.0222	0.0762	-0.0049	0.0427
$[\ln(w_2 / w_1)]^2$	0.0184	0.0107	0.0043	0.0027	0.0015	0.0014	0.0010	0.0007
$\ln y_1 \ln(w_2 / w_1)$	0.0404	0.1457	0.0128	0.0360	0.0044	0.0184	-0.0016	0.0105

Table XI. The performance of the third-stage parameter estimates using cross-sectional data assuming $(\sigma^2, \lambda) = (1.88, 1.66)$

$N = 1000$	Model A		Model B	
	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.7115	0.7115	-0.4164	1.0152
$[\ln(w_3 / w_1)]^2$	-0.0109	0.0604	0.7686	1.3797
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0061	0.0567	-0.5260	0.4454
$\ln y_1 \ln(w_3 / w_1)$	-0.2098	0.0608	-0.9062	0.9836
$\ln(w_2 / w_1)$	-0.7172	0.7247	0.1809	0.1912
$[\ln(w_2 / w_1)]^2$	-0.0013	0.0652	0.7571	0.6412
$\ln y_1 \ln(w_2 / w_1)$	0.2121	0.0621	0.1844	0.0973

Table XII. The performance of the estimators of (σ^2, λ) and $M(\cdot)$ using cross-sectional data assuming $(\sigma^2, \lambda) = (1.88, 1.66)$

Model A						
N	λ		σ^2		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE
100	-0.3739	1.8220	-0.0477	1.6832	0.2678	0.8357
300	-0.1212	0.4518	-0.1117	0.5108	0.1037	0.1211
500	-0.0628	0.1446	-0.1151	0.0744	0.0540	0.0428
1000	-0.0318	0.0621	-0.0609	0.0336	0.0231	0.0190

Model B						
N	λ		σ^2		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE
100	-0.2358	1.2522	-0.3163	0.2877	0.2716	0.8130
300	-0.0962	0.2863	-0.1726	0.1248	0.0863	0.1264
500	-0.0624	0.1377	-0.1156	0.0721	0.0477	0.0615
1000	-0.0339	0.0615	-0.0619	0.0335	0.0229	0.0298

Model C						
N	λ		σ^2		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE
100	-1.1114	135.0659	11.9583	532.0402	0.8387	3.0253
300	-0.7597	18.1286	3.7948	50.6329	0.6325	1.4538
500	-0.7224	5.8017	1.8657	13.0069	0.5125	0.9517
1000	-0.5412	3.2962	0.9885	4.4140	0.3775	0.6153

Table XIII. The performance of the estimated lnG using cross-sectional data assuming $(\sigma^2, \lambda)=(1.88, 1.66)$

N	Model A				Model B	
	G1		G2		G2	
	Bias	MSE	Bias	MSE	Bias	MSE
100	0.0043	0.1050	-0.0518	0.3585	0.0797	6.4793
300	0.0061	0.0296	-0.0007	0.0432	-0.0001	0.0629
500	0.0005	0.0150	-0.0020	0.0216	-0.0013	0.0328
1000	-0.0026	0.0085	-0.0049	0.0119	-0.0049	0.0163

Table XIV. The performance of the estimated AE using cross-sectional data assuming $(\sigma^2, \lambda)=(1.88, 1.66)$

N	Model A				Model B	
	AE1		AE2		AE2	
	Bias	MSE	Bias	MSE	Bias	MSE
100	0.0212	0.0288	0.0225	0.0093	0.0355	0.0156
300	0.0031	0.0027	0.0167	0.4033	0.0194	0.0288
500	0.0010	0.0015	0.0045	0.0012	0.0074	0.0039
1000	0.0002	0.0007	0.0019	0.0006	0.0042	0.0014

Table XV. The performance of the estimated TE score using cross-sectional data assuming $(\sigma^2, \lambda)=(1.88, 1.66)$

N	Model A		Model B		Model C	
	Bias	MSE	Bias	MSE	Bias	MSE
100	-0.3892	0.5978	-0.3738	0.5756	-0.4832	0.8100
300	-0.3302	0.5220	-0.3273	0.5177	-0.4724	0.7531
500	-0.3192	0.5073	-0.3194	0.5076	-0.4500	0.7071
1000	-0.3133	0.5005	-0.3138	0.5011	-0.4172	0.6550

Table XVI. The performance of the parameter estimates in Step 1 setting $M(\cdot) = \sqrt{y_1} + \ln(1 + y_1)$

(N, T)	$(50, 6)$		$(50, 10)$		$(50, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.0722	0.2227	-0.0280	0.1134	-0.0055	0.0579
$[\ln(w_3 / w_1)]^2$	0.0114	0.0056	0.0044	0.0030	0.0025	0.0014
H_3 / H_1	0.0048	0.0019	0.0030	0.0010	0.0026	0.0005
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0088	0.0121	0.0028	0.0062	-0.0007	0.0036
H_2 / H_1	0.0006	0.0013	0.0004	0.0007	0.0001	0.0004
$\ln y_1 \ln(w_3 / w_1)$	0.0247	0.0649	0.0082	0.0330	-0.0002	0.0178
$\ln(w_2 / w_1)$	-0.0475	0.1462	-0.0222	0.0762	-0.0049	0.0427
$[\ln(w_2 / w_1)]^2$	0.0043	0.0027	0.0015	0.0014	0.0010	0.0007
$\ln y_1 \ln(w_2 / w_1)$	0.0128	0.0360	0.0044	0.0184	-0.0016	0.0105

(N, T)	$(100, 6)$		$(100, 10)$		$(100, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.0179	0.0935	-0.0055	0.0579	-0.0082	0.0284
$[\ln(w_3 / w_1)]^2$	0.0036	0.0025	0.0025	0.0014	0.0014	0.0007
H_3 / H_1	0.0029	0.0009	0.0026	0.0005	0.0010	0.0002
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0015	0.0052	-0.0007	0.0036	0.0008	0.0016
H_2 / H_1	0.0002	0.0006	0.0001	0.0004	-0.0001	0.0002
$\ln y_1 \ln(w_3 / w_1)$	0.0046	0.0276	-0.0002	0.0178	0.0026	0.0083
$\ln(w_2 / w_1)$	-0.0171	0.0679	-0.0049	0.0427	-0.0075	0.0211
$[\ln(w_2 / w_1)]^2$	0.0011	0.0012	0.0010	0.0007	0.0006	0.0003
$\ln y_1 \ln(w_2 / w_1)$	0.0023	0.0157	-0.0016	0.0105	0.0015	0.0050

(N, T)	$(200, 6)$		$(200, 10)$		$(200, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.0084	0.0489	-0.0082	0.0284	-0.0008	0.0142
$[\ln(w_3 / w_1)]^2$	0.0026	0.0012	0.0014	0.0007	-0.0005	0.0003
H_3 / H_1	0.0021	0.0004	0.0010	0.0002	0.0004	0.0001
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.0004	0.0029	0.0008	0.0016	0.0003	0.0008
H_2 / H_1	-1.74E-06	0.0003	-0.0001	0.0002	0.0003	0.0001
$\ln y_1 \ln(w_3 / w_1)$	0.0014	0.0148	0.0026	0.0083	-0.0009	0.0042
$\ln(w_2 / w_1)$	-0.0033	0.0354	-0.0075	0.0211	-0.0022	0.0105
$[\ln(w_2 / w_1)]^2$	0.0011	0.0006	0.0006	0.0003	-0.0001	0.0002
$\ln y_1 \ln(w_2 / w_1)$	-0.0013	0.0087	0.0015	0.0050	-0.0003	0.0025

Table XVII. The performance of the parameter estimates from the third-stage setting $M(\cdot) = \sqrt{y_1} + \ln(1 + y_1)$

Model A						
(N, T)	$(50, 6)$		$(50, 10)$		$(50, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	1.5709	2.6596	1.3189	1.7974	1.0357	1.0890
$[\ln(w_3 / w_1)]^2$	-0.0286	0.0829	-0.0108	0.0369	-0.0047	0.0168
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0363	0.0804	0.0117	0.0368	0.0051	0.0171
$\ln y_1 \ln(w_3 / w_1)$	-0.4677	0.2344	-0.3930	0.1592	-0.3081	0.0962
$\ln(w_2 / w_1)$	-1.6214	2.8195	-1.3864	1.9833	-1.0866	1.1969
$[\ln(w_2 / w_1)]^2$	-0.0343	0.0921	-0.0127	0.0407	-0.0059	0.0182
$\ln y_1 \ln(w_2 / w_1)$	0.4824	0.2482	0.4131	0.1756	0.3232	0.1057

Model A						
(N, T)	$(100, 6)$		$(100, 10)$		$(100, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	1.2454	1.6297	1.0336	1.0978	0.8009	0.6496
$[\ln(w_3 / w_1)]^2$	-0.0096	0.0406	-0.0054	0.0188	-0.0035	0.0076
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0101	0.0402	0.0063	0.0190	0.0029	0.0077
$\ln y_1 \ln(w_3 / w_1)$	-0.3705	0.1435	-0.3073	0.0968	-0.2377	0.0571
$\ln(w_2 / w_1)$	-1.3008	1.7738	-1.0814	1.1996	-0.8371	0.7085
$[\ln(w_2 / w_1)]^2$	-0.0099	0.0443	-0.0072	0.0210	-0.0026	0.0082
$\ln y_1 \ln(w_2 / w_1)$	0.3866	0.1559	0.3213	0.1056	0.2487	0.0625

Model A						
(N, T)	$(200, 6)$		$(200, 10)$		$(200, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.9643	0.9697	0.8038	0.6603	0.6213	0.3896
$[\ln(w_3 / w_1)]^2$	-0.0042	0.0185	-0.0036	0.0084	-0.0052	0.0037
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0042	0.0183	0.0048	0.0085	0.0045	0.0037
$\ln y_1 \ln(w_3 / w_1)$	-0.2866	0.0853	-0.2387	0.0581	-0.1841	0.0342
$\ln(w_2 / w_1)$	-1.0064	1.0535	-0.8353	0.7118	-0.6514	0.4281
$[\ln(w_2 / w_1)]^2$	-0.0042	0.0203	-0.0060	0.0094	-0.0040	0.0040
$\ln y_1 \ln(w_2 / w_1)$	0.2989	0.0925	0.2479	0.0626	0.1932	0.0376

Translog						
(N, T)	$(50, 6)$		$(50, 10)$		$(50, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	-0.0036	0.3030	-0.0025	0.0931	-0.4843	0.3947
$[\ln(w_3 / w_1)]^2$	0.0041	0.0667	0.0041	0.0190	0.0146	0.0031
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	0.0002	0.0665	-0.0037	0.0186	-0.3456	0.0028
$\ln y_1 \ln(w_3 / w_1)$	-0.0008	0.0242	-0.0002	0.0073	-0.6200	0.0018
$\ln(w_2 / w_1)$	0.0038	0.3292	0.0015	0.1009	-0.2048	0.0354
$[\ln(w_2 / w_1)]^2$	-0.0030	0.0715	0.0037	0.0199	0.0355	0.3916
$\ln y_1 \ln(w_2 / w_1)$	-0.0015	0.0263	-0.0001	0.0081	-0.4830	0.0021

Translog						
(N, T)	$(100, 6)$		$(100, 10)$		$(100, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.0167	0.1510	0.0054	0.0480	-0.3596	0.2957
$[\ln(w_3 / w_1)]^2$	0.0025	0.0308	0.0049	0.0094	0.0107	0.0017
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.0020	0.0300	-0.0049	0.0092	-0.2590	0.0016
$\ln y_1 \ln(w_3 / w_1)$	-0.0056	0.0117	-0.0023	0.0039	-0.4661	0.0004
$\ln(w_2 / w_1)$	-0.0155	0.1609	-0.0026	0.0502	-0.1565	0.0147
$[\ln(w_2 / w_1)]^2$	0.0020	0.0318	0.0050	0.0099	0.0264	0.2936
$\ln y_1 \ln(w_2 / w_1)$	0.0045	0.0126	0.0012	0.0041	-0.3614	0.0005

Translog						
(N, T)	$(200, 6)$		$(200, 10)$		$(200, 20)$	
	Bias	MSE	Bias	MSE	Bias	MSE
$\ln(w_3 / w_1)$	0.0010	0.0768	0.0018	0.0246	-0.0672	0.0600
$[\ln(w_3 / w_1)]^2$	0.0068	0.0154	0.0045	0.0050	0.0012	0.0009
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.0062	0.0152	-0.0045	0.0048	-0.0487	0.0008
$\ln y_1 \ln(w_3 / w_1)$	-0.0015	0.0062	-0.0012	0.0019	-0.0891	0.0004
$\ln(w_2 / w_1)$	0.0014	0.0783	0.0007	0.0247	-0.0308	0.0053
$[\ln(w_2 / w_1)]^2$	0.0054	0.0163	0.0042	0.0051	0.0046	0.0557
$\ln y_1 \ln(w_2 / w_1)$	0.0006	0.0064	0.0005	0.0020	-0.0689	0.0004

Table XVIII. The performance of the estimators of $(\gamma, \sigma^2, \lambda)$ setting $M(\cdot) = \sqrt{y_1} + \ln(1 + y_1)$
 $(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$

Model A								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0026	0.0016	-0.0931	3.7473	-0.1930	0.1357	0.0144	0.0455
(50, 10)	0.0018	0.0003	-0.1947	0.2112	-0.2205	0.1024	0.0174	0.0086
(50, 20)	0.0017	3.43E-05	-0.2248	0.2057	-0.3281	0.1433	0.0130	0.0046
(100, 6)	0.0010	0.0007	-0.1318	0.1129	-0.1205	0.0508	0.0149	0.0090
(100, 10)	0.0010	0.0001	-0.1309	0.1055	-0.1456	0.0490	0.0101	0.0049
(100, 20)	0.0015	1.67E-05	-0.1621	0.1039	-0.2406	0.0770	0.0090	0.0031
(200, 6)	0.0010	0.0003	-0.0931	0.0567	-0.0820	0.0258	0.0094	0.0050
(200, 10)	0.0007	0.0001	-0.0925	0.0512	-0.0996	0.0232	0.0069	0.0030
(200, 20)	0.0013	8.72E-06	-0.1332	0.0562	-0.1887	0.0450	0.0080	0.0253

Translog								
(N, T)	γ		σ^2		λ		$M(\cdot)$	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	-1.79E+05	2.81E+13	-0.0557	0.3743	-0.2204	0.8447	0.0639	0.0612
(50, 10)	-3.92E+09	1.52E+22	0.0258	0.9059	-0.2136	0.9183	0.0448	0.0448
(50, 20)	-1.81E+02	1.83E+07	-1.2521	3.1000	-1.3548	1.1035	-3.7058	20.8629
(100, 6)	-3.92E+09	1.46E+22	-0.0111	0.1739	-0.0878	0.3646	0.0258	0.0294
(100, 10)	-4.89E+06	2.38E+16	0.0154	0.2349	-0.1270	0.5399	0.0255	0.0219
(100, 20)	-24.1321	4.04E+05	-0.8082	3.0078	-1.1570	1.6806	-2.7605	15.7022
(200, 6)	-0.1191	14.2472	-0.0246	0.0542	-0.0211	0.0716	0.0058	0.0058
(200, 10)	-190.0124	3.51E+07	-0.0218	0.0644	-0.0295	0.0935	0.0053	0.0172
(200, 20)	-6723.3554	4.03E+10	0.1172	2.2550	-0.6192	2.5336	-0.4747	3.1680

Table XIX. The performance of estimated function $\ln G$ setting $M(\cdot) = \sqrt{y_1} + \ln(1 + y_1)$
 $(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$

(N, T)	Model A				Translog	
	G1		G2A		G2T	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0061	0.0296	-0.0030	0.0383	-0.0028	0.0356
(50, 10)	0.0005	0.0150	-0.0032	0.0184	-0.0022	0.0162
(50, 20)	-0.0026	0.0085	-0.0039	0.0103	0.0017	-0.0051
(100, 6)	-0.0010	0.0125	-0.0038	0.0160	-0.0030	0.0144
(100, 10)	-0.0026	0.0085	-0.0040	0.0105	-0.0041	0.0086
(100, 20)	0.0002	0.0038	0.0001	0.0050	0.0001	0.0036
(200, 6)	-0.0017	0.0070	-0.0028	0.0089	-0.0029	0.0074
(200, 10)	0.0002	0.0038	-6.95E-06	0.0050	-0.0008	0.0039
(200, 20)	-0.0012	0.0019	-0.0008	0.0025	-0.0019	0.0018

Table XX. The performance of estimated AE setting $M(\cdot) = \sqrt{y_1} + \ln(1 + y_1)$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A				Translog	
	AE1		AE2A		AE2T	
	Bias	MSE	Bias	MSE	Bias	MSE
(50, 6)	0.0031	0.0027	0.0149	0.3992	0.0082	0.4110
(50, 10)	0.0010	0.0015	0.0038	0.0008	0.0020	0.0012
(50, 20)	0.0002	0.0007	0.0014	0.0004	-0.0006	0.0006
(100, 6)	0.0004	0.0012	0.0030	0.0007	0.0011	0.0011
(100, 10)	0.0002	0.0007	0.0013	0.0004	0.0002	0.0006
(100, 20)	2.89E-06	0.0003	0.0009	0.0002	-0.0001	0.0003
(200, 6)	0.0001	0.0006	0.0013	0.0003	0.0004	0.0005
(200, 10)	2.89E-06	0.0003	0.0007	0.0002	0.0001	0.0003
(200, 20)	0.0004	0.0002	0.0006	0.0001	0.0001	0.0001

Table XXI. The performance of estimated TE scores setting $M(\cdot) = \sqrt{y_1} + \ln(1 + y_1)$

$$(\gamma, \sigma^2, \lambda) = (0.025, 1.88, 1.66)$$

(N, T)	Model A		Translog	
	Bias	MSE	Bias	MSE
(50, 6)	0.0030	0.0093	0.0649	0.0833
(50, 10)	0.0048	0.0033	0.0508	0.0672
(50, 20)	0.0214	0.0018	-0.2723	0.2523
(100, 6)	0.0015	0.0065	0.0263	0.0452
(100, 10)	0.0024	0.0029	0.0288	0.0342
(100, 20)	0.0204	0.0017	-0.1862	0.2019
(200, 6)	-0.0003	0.0060	0.0022	0.0111
(200, 10)	0.0009	0.0026	0.0044	0.0073
(200, 20)	0.0200	0.0016	0.0145	0.0722

Table XXII. Parameter estimates of the Translog model

	Parametric Estimate	Standard Error
Intercept	0.0086	0.2602
$\ln(y_1)$	0.8704***	0.0305
$\ln(y_2)$	0.3639***	0.0457
$0.5 \times [\ln(y_1)]^2$	0.0762***	1.88E-03
$0.5 \times [\ln(y_2)]^2$	0.1146***	0.0052
$\ln(y_1) \ln(y_2)$	-0.1102***	0.0024
$\ln(w_3 / w_1)$	-0.0049	0.0305
$0.5 \times [\ln(w_3 / w_1)]^2$	0.1321***	0.0079
$\ln(w_2 / w_1) \ln(w_3 / w_1)$	-0.0564***	0.0023
$\ln y_1 \ln(w_3 / w_1)$	0.0529***	0.0036
$\ln y_2 \ln(w_3 / w_1)$	0.0055	0.0041
$\ln(w_2 / w_1)$	-0.0004	0.0241
$0.5 \times [\ln(w_2 / w_1)]^2$	0.0245***	0.0017
$\ln y_1 \ln(w_2 / w_1)$	-6.45E-03***	2.21E-03
$\ln y_2 \ln(w_2 / w_1)$	1.26E-02***	3.02E-03
γ	0.0286***	0.0020
σ^2	2.0454***	0.1751
λ	2.5310***	0.1272
log-likelihood	-2971.95	

Table XXIII. Average AE and TE scores of the Translog model

	Mean	St. Dev.
AE (%)	89.21	0.1013
TE (%)	61.15	0.2033

國科會補助計畫衍生研發成果推廣資料表

日期:2011/07/18

國科會補助計畫	計畫名稱: A study of the economic efficiencies in East European countries using semiparametric approaches
	計畫主持人: 陳冠臻
	計畫編號: 98-2420-H-004-174-DR 學門領域: 財務與金融
無研發成果推廣資料	

98 年度專題研究計畫研究成果彙整表

計畫主持人：陳冠臻		計畫編號：98-2420-H-004-174-DR				計畫名稱：A study of the economic efficiencies in East European countries using semiparametric approaches	
成果項目		量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）	
		實際已達成數（被接受或已發表）	預期總達成數（含實際已達成數）	本計畫實際貢獻百分比			
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	1	1	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
博士後研究員		0	0	100%			
專任助理		0	0	100%			
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		章/本
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
博士後研究員		0	0	100%			
專任助理		0	0	100%			

<p>其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)</p>	<p>無</p>
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	成果項目	量化	名稱或內容性質簡述
科 教 處 計 畫 加 填 項 目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與(閱聽)人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

達成目標

未達成目標（請說明，以 100 字為限）

實驗失敗

因故實驗中斷

其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文： 已發表 未發表之文稿 撰寫中 無

專利： 已獲得 申請中 無

技轉： 已技轉 洽談中 無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）