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狀態相依跳躍風險與美式選擇權評價：

黃金期貨市場之實證研究

State-Dependent Jump Risks and American Option Pricing:

An Empirical Study of the Gold Futures Market

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State-Dependent Jump Risks and American Option Pricing:

An Empirical Study of the Gold Futures Market

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中文摘要

本文實證探討黃金期貨報酬率的特性並在標的黃金期貨價格遵循狀態轉換跳躍擴散過程時實現美式選擇權之評價。在這樣的動態過程下，跳躍事件被一個複合普瓦松過程與對數常態跳躍振幅所描述，以及狀態轉換到達強度是由一個其狀態代表經濟狀態的隱藏馬可夫鏈所捕捉。考量不同的跳躍風險假設，我們使用Merton 測度與 Esscher 轉換推導出在一個不完全市場設定下的風險中立黃金期貨價格動態過程。為了達到所需的精確度，最小平方蒙地卡羅法被用來近似美式黃金期貨選擇權的價值。基於實際市場資料，我們提供實證與數值結果來說明這個動態模型的優點。

關鍵詞：美式黃金期貨選擇權、狀態轉換跳躍擴散過程、Merton 測度、Esscher 轉換、最小平方蒙地卡羅法

Abstract

This dissertation empirically investigates the characteristics of gold futures returns and achieves the valuation of American-style options when the underlying gold futures price follows a regime-switching jump-diffusion process. Under such dynamics, the jump events are described as a compound Poisson process with a log-normal jump amplitude, and the regime-switching arrival intensity is captured by a hidden Markov chain whose states represent the economic states. Considering the different jump risk assumptions, we use the Merton measure and Esscher transform to derive risk-neutral gold futures price dynamics under an incomplete market setting. To achieve a desired accuracy level, the least-squares Monte Carlo method is used to approximate the values of American gold futures options. Our empirical and numerical results based on actual market data are provided to illustrate the advantages of this dynamic model.

Keywords: American gold futures option; Regime-switching jump-diffusion process; Merton measure; Esscher transform; Least-squares Monte Carlo method

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Chapter 1

Introduction

Gold is a precious metal, which in recent years is considered to be an investment tool alternative to equity and bond markets. It provides similar functions as money in that it acts as a preserver of wealth, a medium of exchange and a unit of value. Unlike other commodities, gold is a special asset with renewable, relatively transportable, universally acceptable and easily authenticated. The unique and diverse drivers of gold price behavior not highly correlate with changes in other financial assets. As a consequence, this precious metal can contribute in a saving role by acting as a type of insurance against extreme movements and jumps in the value of traditional assets during times of economic and market stress (Capie et al., 2005; Baur and Lucey, 2010; Baur and McDermott, 2010; Reboredo, 2013; Zagaglia and Marzo, 2013). Gold is a liquid asset, continuously quoted on spot and futures markets and easy to trade. In addition to Beckers (1984) and Ball et al. (1985) empirically investigate the gold options market under the Black–Scholes framework, others, for instance, Ogden et al. (1990) study gold spot and futures options. It is well known that the presence of jumps in the underlying asset price can have significant implications on pricing derivatives, but these aforementioned papers do not address such jump phenomena.

The increasing number of jump events, especially after the subprime financial crisis of 2008, has created large fluctuations in the gold futures prices and related derivatives (e.g., American gold futures options). For the market development, it is crucial in capturing the dynamic jump process appropriately and evaluate American gold futures options corresponding to the changing prices of gold futures.

Most exchange-traded option contracts are American style and therefore, many numerical methods have been presented attempting to evaluate American options, including the lattice (Cox et al., 1979), finite difference (Brennan and Schwartz, 1977; Hull and White, 1990), and Monte Carlo simulation methods (Boyle et al., 1997). Increasing the number of steps comes at the cost of exponential growth in the size of the lattice pricing methods. Longstaff and Schwartz (2001) propose an algorithm for pricing American options called least-squares Monte Carlo (LSM) approach. This technique proceeds by simulating forward paths using the Monte Carlo simulation, and then performs backward iterations by applying least-squares approximation of the continuation function over a collection of basic functions. This algorithm is simple to implement within existing Monte Carlo frameworks, and has the additional advantages that the continuation functions are constructed explicitly and it is easy to calibrate to existing market prices. Therefore, we adopt this approach to evaluate

American gold futures options.

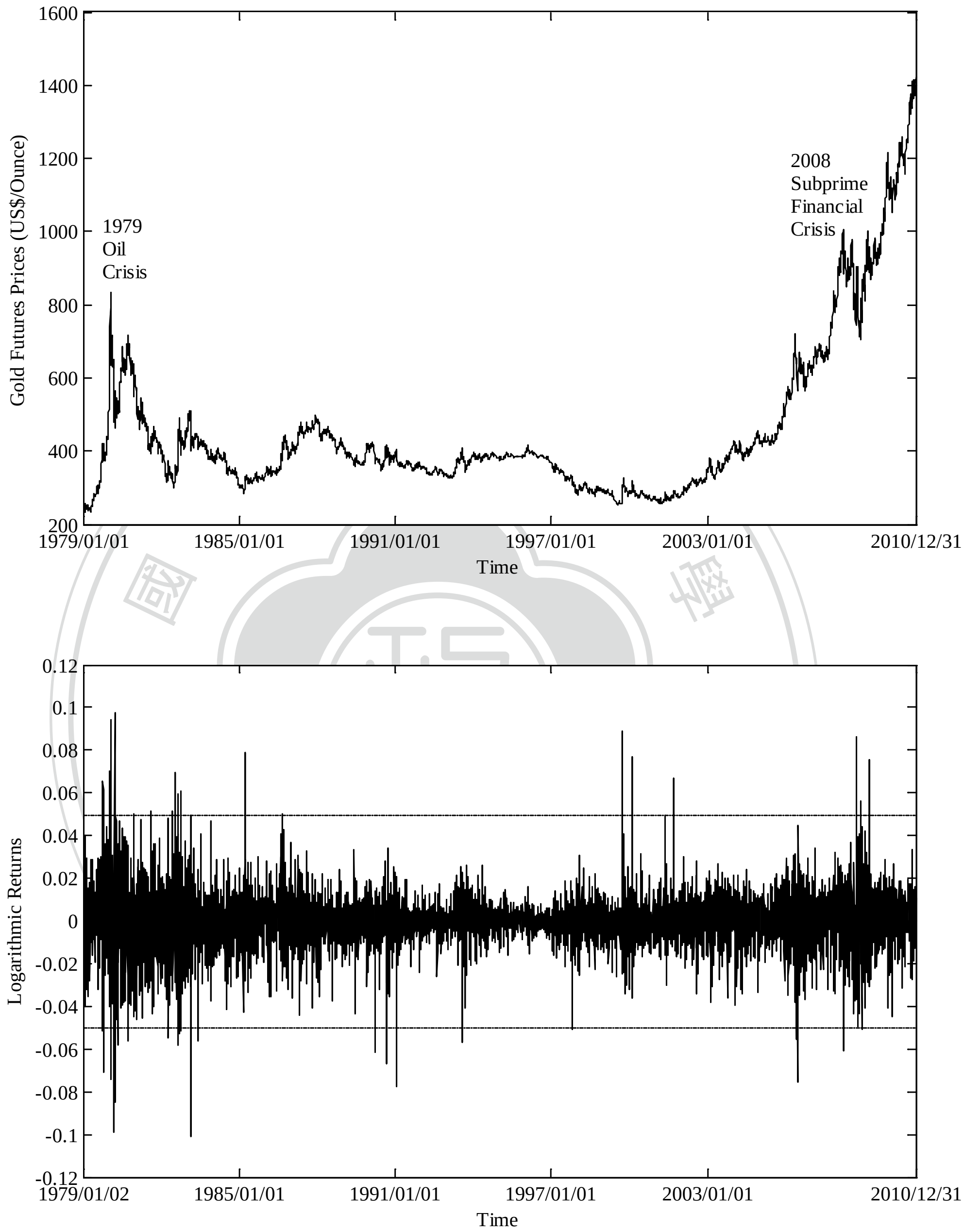

Figure 1.1 Time series (top panel) and logarithmic returns (bottom panel) of daily gold futures prices Note that the dotted lines in the bottom panel denote the gold futures returns over $\pm 5\%$ in magnitude are jumps. The gold futures data is the one month forward contract attained from the continuous futures series from Datastream with switchovers at the first day of a new month. The futures contracts are the daily CMX-Gold-100-Oz.

The top panel of Figure 1.1 draws some significant price jumps of gold futures in the daily data.[1] In particular, it shows that there are larger jumps (returns) in several time periods. In the oil crisis of 1979 and subprime financial crisis of 2008, for example, gold futures prices had larger jumps. The empirical data have revealed that the geometric Brownian motion (GBM) is not completely consistent with the reality, that is, the jumps do exist in the gold futures price realizations. Hence, incorporating sudden random shocks into a dynamic model is necessary and significant (Carr et al., 2002; Eraker et al., 2003; Eraker, 2004; Maheu and McCurdy, 2004). In line with the changing gold futures returns in the bottom panel of Figure 1.1, we could identify two regimes of the gold futures market. The first state is defined as the relatively low-volatility regime and can be viewed as the ordinary state. The second state is defined as the relatively high-volatility regime and can be regarded as the volatile state. In addition, we also find that, in the gold futures market, the bottom panel of Figure 1.1 exhibits different arrival rates of jump events in different time periods. It is an empirical fact that there exists the so-called jump and volatility clustering in the logarithmic return series of gold futures prices caused by a period of time of high (low) arrival rates tend to be followed by a period of time of continued high (low) arrival rates. Nevertheless, the existing jump-diffusion processes, such as in Merton (1976),

[1] The empirical data are from Datastream and cover the period from 01/01/1979 to 12/31/2010.

Amin (1993), and Kou (2002), are unable to address the phenomenon of volatility clustering. Duan et al. (2006) evaluate options when there are jumps in the pricing kernel and correlated jumps in asset prices and volatilities. The models capture leptokurtosis and volatility clustering but do not show the regime-switching phenomenon. Chan and Maheu (2002) propose a time-varying Poisson jump model to describe the jump dynamics of the stock prices in the discrete time circumstance. In the light of our empirical observations in Figure 1.1 with economic intuition, the change of gold futures prices represents that the economy stays in each state for a period of time and then transitions to the other, as well as the jump intensity changes over time according to the different regimes of the economy.

In order to incorporate both unanticipated jump events and regime-switching arrival rates in the oscillating gold futures market simultaneously, we model the gold futures price dynamics using a regime-switching jump-diffusion model (RSJM) proposed by Chang et al. (2013). Under such a model, the jump events are described as a compound Poisson process with the log-normal jump amplitude setting applied by Merton (1976), and the regime-switching arrival intensity is governed by a continuous-time finite-state Markov chain. The states of the Markov chain can be interpreted as the hidden states of an economy. The regime-switching models are used to describe the asset price dynamics and are applied for option pricing. Elliott et al. (2007) investigate the option prices under a generalized Markov regime-switching jump-diffusion economy for the use of the Esscher transform employed by Elliott et al. (2005) and Elliott and Osakwe (2006) for valuing options under a regime-switching environment. Elliott and Siu (2011) incorporate structural changes in economic conditions in asset price dynamics and American option valuation. None of these discussions provide the empirical tests for the observed features and model fit of their dynamic processes. In this dissertation, we demonstrate that the dynamic model describes the existence of jump events, leptokurtosis, asymmetry, and volatility clustering for the gold futures returns, and illustrate the presence of regime-switching arrival rates and jump clustering, and verify the superior empirical fit over competing models.

The market is incomplete in such a Markov regime-switching jump-diffusion economy, and we therefore need to select a pricing kernel for option valuation. Previous researches empirically illustrate that gold is either a zero or negative beta asset (McCown and Zimmerman, 2006; McCown and Zimmerman, 2007; Baur and Lucey, 2010; Baur and McDermott, 2010). This means that, under the CAPM assumptions, the jump component of the gold futures returns shows non-systematic or systematic risk. Thus, we make use of two specific approaches for choosing the martingale pricing measure to derive the risk-neutral gold futures price dynamics. First, we assume that the jump component of the gold futures returns presents non-systematic and diversifiable risk, that is, under the assumption that the jump risk is not priced, as in Merton (1976). Second, we relax this assumption to allow for the jump risk, regarding which as systematic and non-diversifiable. Then, the Esscher transform technique developed by Gerber and Shiu (1994, 1996) is considered to determine a risk-neutral pricing measure. After determining such martingale dynamics of the gold futures prices, we use the LSM algorithm to approximate the values of American gold futures options. Considering the jump events and economic states are unobserved when estimating the parameters of the RSJM, the model parameters are estimated using the expectation maximization (EM) algorithm (Lange, 1995a, b) and the standard errors of parameter estimators are obtained using the supplemented expectation maximization (SEM) algorithm (Meng and Rubin, 1991). From the empirically estimated parameters in the dynamic model and the LSM-simulated option prices, we show that the model is more accurate than competing models in pricing American gold futures put options.

The main results from this dissertation can be summarized as follows. First, we empirically analyze the gold futures returns for understanding the market operation and the risks involved. Our findings are valuable for gold futures price modeling and for other gold derivative asset pricing. Second, we derive the risk-neutral gold futures price dynamics via the Merton measure and Esscher transform under the different jump risk settings. Finally, we use actual market data to investigate the pricing performance of the LSM method for American gold futures put options and illustrate the importance of incorporating state-dependent jump risks into the dynamic price model on gold futures.

The remainder of this dissertation is organized as follows. Chapter 2 presents the model setting and numerical method. Chapter 3 illustrates the measure change of two specific approaches and the valuation of American gold futures options. Chapter 4 discusses the results of empirical analyses and numerical illustrations. Chapter 5 presents the conclusions of this dissertation.

Chapter 2

Model Framework and Pricing Method

The empirical data of Figure 1.1 demonstrate that the frequency of sudden random shocks can be significantly different under different regimes of the gold futures price. Based on these observations, it seems inadequate to assume that the jump arrival intensity follows a pure Poisson process, and we therefore use a Markov-modulated Poisson process to model the jump risk components. For valuing American gold futures options, the LSM method is adopted to approximate the option values by simulation when the underlying gold futures price follows a continuous-time, Markov, regime-switching jump-diffusion process.

2.1 Markov-modulated Poisson process

The Markov-modulated Poisson process $\Phi(t)$ is a Poisson process in which the underlying state is governed by a hidden Markov chain $X(t)$ (Last and Brandt, 1995). More precisely, instead of the constant (average) arrival intensity under a Poisson process used in the Merton-type jump-diffusion model (JDM) (Merton, 1976), the jump component of the gold futures price is set to be a Markov-modulated Poisson process whereby the intensity process of jump events is different in different states.

The regime-switching arrival intensity $\lambda_{X(t)}$ of $\Phi(t)$ is modulated by $X(t)$, with the transition function $P_{ij}(t)$ on the finite state space $\mathrm{X}=\{1,2,...,I\}$. For $i, j \in \mathrm{X}$, we denote the transition rate $\Psi(i, j)$ from state $X(0)=i$ to state $X(t)=j$ of $\Phi(t)$ as

$$\Psi(i, j) = \begin{cases} \psi(i, j), & i \neq j, \\ -\sum_{j, j\neq i} \psi(i, j), & otherwise, \end{cases} \tag{2.1}$$

The notation $\Psi = (\Psi(i, j))_{I\times I}$ represents the $I\times I$ matrix of the transition rate with diagonal elements $\psi_{ii} = -\sum_{j=1, j\neq i}^{I} \psi_{ij} = -v_i$. v_i is the departure rate at which the process leaves state i. Since the Markov chain has a finite number of states, the Poisson arrival intensity takes discrete values corresponding to each state. From the Markov structure of $X(t)$, Last and Brandt (1995) give the moment-generating function for the joint distribution function of $X(t)$ and $\Phi(t)$ via the Laplace inverse transform as follows:

$$P^{*}(\zeta, t) = \sum_{n=0}^{\infty} P(n,t)\zeta^{n}, \quad 0 \leq \zeta \leq 1, \tag{2.2}$$

where ζ is a complex number. $P(n,t) = (P_{ij}(n,t))_{I\times I}$ represents the $I\times I$ transition probability matrix and $P_{ij}(n,t) = P(X(0)=i, X(t)=j, \Phi(t)=n)$ denotes the transition probability with n jump times from state $X(0)=i$ to state $X(t)=j$.

Here, $P(n,0)=(1_{\{n=0\}}D_{ij})$, where $D_{ij}=1$ if $i=j$ and 0, otherwise. Using the Kolmogorov forward equation, the derivative of $P(n,t)$ becomes $\frac{d}{dt}P(n,t)=(\Psi-\Lambda)P(n,t)+1_{\{n\geq 1\}}\Lambda P(\mathrm{n}-1,\mathrm{t})$. Thus, the unique solution of $P^{*}(\zeta,t)$ can be obtained as

$$P^{*}(\zeta,t)=\exp\left(\left(\Psi-(1-\zeta)\Lambda\right)t\right), \tag{2.3}$$

where $\Lambda=(\lambda_{X(t)})_{I\times I}$ denotes the $I\times I$ diagonal matrix of the arrival intensity with diagonal elements λ_i. The exponential power series, are given as $e^{A}=\sum_{n=0}^{\infty}\frac{A^{n}}{n!}$ for any $I\times I$ matrix A and $A^{0}=(D_{ij})$. Applying the Laplace inverse transform of Equation (2.2) and the unique solution of Equation (2.3), we have the joint distribution of $X(t)$ and $\Phi(t)$ at time t with the following equation:

$$P(n,t)=\frac{\partial^{n}}{\partial\zeta^{n}n!}P^{*}(\zeta,t)\Big|_{\zeta=0}, \tag{2.4}$$

2.2 Gold futures price modeling

Let (Ω,F,P) be a complete probability space, where P is the physical probability measure. For each $t\in[0,T]$, we consider that the sample path for the gold futures price is continuous except on finite points in time, and the arrival intensity of jump events depends on the hidden states of an economy. The RSJM for the underlying gold futures price at time t, $F(t)$, can be written as

$$\frac{dF(t)}{F(t-)} = (\mu - \Lambda\kappa)\,dt + \sigma dW(t) + d\left[\sum_{k=1}^{\Phi(t)} \left(\exp(Y_k) - 1\right)\right], \tag{2.5}$$

where the appreciation rate μ and the volatility σ are constants and $W(t)$ is a Wiener process under P. Specifically, the jump risk components are indicated by the Markov-modulated Poisson process $\Phi(t)$ with the arrival intensity matrix $\Lambda = (\lambda_{X(t)})_{I\times I}$, and the jump amplitude is supposed to follow a log-normal distribution, as in Merton (1976). $\{Y_k : k = 1, 2, ...\}$ are the jump amplitudes, which are assumed to be independently identically distributed non-negative random variables with the density function $f_Y(y)$. If a jump event occurs at time k, the jump amplitude Y_k is normally distributed with mean μ_y and variance σ_y^2. As a consequence, the mean percentage jump amplitude of the gold futures price is $\kappa = E\left[\exp(Y_k) - 1\right] = \exp\left(\mu_y + \frac{1}{2}\sigma_y^2\right) - 1 = \phi_Y(1) - 1$. In Equation (2.5), we assume that all random shock processes $W(t)$, $\Phi(t)$, $X(t)$, and Y_k are mutually independent.

2.3 Least-squares Monte Carlo approach

Longstaff and Schwartz (2001) show that an American option can be evaluated using the LSM technique to achieve a desired accuracy level. This advantage supports us to use the LSM approach for pricing American gold futures options under the RSJM, in contrast with the shortcomings of the lattice pricing methods. Let ω denote a sample path of the underlying asset price generated by the Monte Carlo simulation over a discrete set of τ exercise times $0 < t_1 \leq t_2 \leq \ldots \leq t_\tau = T$. The continuous exercise property of an American option is approximated by taking sufficiently large τ. Let $C(\omega, s; t, T)$ denote the path of cash flows generated assuming the option is not exercised at or before time t and the option holder follows the optimal exercise policy for all subsequent $s \in (t, T]$. At maturity, the investor exercises the option if it is in the money. At time t_k prior to expiration, the option holder must decide whether to exercise at that point or to continue and revisit the decision at the next time point. Here, although the option holder knows the immediate exercise payoff, he has no exact idea of the expected cash flows from continuation. According to the no-arbitrage pricing theory, $F(\omega; t_k)$, the continuation value at time t_k for path ω, is formally given by

$$F(\omega; t_k) = E^Q\left[\sum_{j=k+1}^{\tau} \exp\left(-r\left(\omega, t_j; t_k\right)\right) C\left(\omega, t_j; t_k, T\right) \middle| F_{t_k}\right], \tag{2.6}$$

where $r\left(\omega, t_j; t_k\right)$ is the risk-free discount rate, Q is a martingale pricing measure, and the expectation of the cash flows is taken conditional on the information set F_{t_k} at time t_k. Supposing the continuation value is estimated, we can decide whether it is optimal to exercise at time t_k or continue by comparing the immediate exercise value with the estimate of the continuation value. The procedure is repeated until exercise decisions have been determined for each exercise point on every path. Thus by estimating the conditional expectation function for each exercise date, a complete specification of the optimal exercise strategy can be obtained along each path. Once the exercise strategy has been estimated, the valuation of an American option is approximately achieved.

Chapter 3

Valuation of American Gold Futures Options in a Markov Regime-Switching Jump-Diffusion Economy

The security economy described by Equation (2.5) is incomplete, meaning that under the assumption of no arbitrage opportunities in this market, there are infinitely equivalent martingale measures with which to price options. Therefore, we need to determine a risk-neutral pricing measure Q, under which the gold futures prices discounted at the risk-free rate are Q-martingales. In order to reflect the argument of previous studies that gold is either a zero or negative beta asset (McCown and Zimmerman, 2006; McCown and Zimmerman, 2007; Baur and Lucey, 2010; Baur and McDermott, 2010), we consider the different assumptions for a jump risk, and then use the Merton (1976) measure and the Esscher transform adopted from Gerber and Shiu (1994, 1996) for the RSJM to select the martingale pricing measure such that risk-neutral gold futures price dynamics are obtained.

3.1 Merton measure for the Markov regime-switching jump-diffusion process

In this section, we follow the assumption of a diversifiable jump risk made by Merton (1976), which implies that no premium is paid for such a risk. We then use the Merton's approach for the RSJM to identify a risk-neutral pricing measure by changing the drift of the Wiener process but leaving the other ingredients unchanged. To determine the martingale pricing measure, we decompose the gold futures logarithmic return $Z(t)=\log\left(F(t)/F(0)\right)=C(t)+J(t)$ into a continuous diffusion part $C(t)=\left(\mu-\frac{1}{2}\sigma^2-\Lambda\kappa\right)t+\sigma W(t)$ and a jump part $J(t)=\sum_{k=1}^{\Phi(t)}Y_k$, for all $t\in[0,T]$. Here, let F_t^Z and F_t^X for the P-augmentation of the natural filtrations generated by $Z(t)$ and $X(t)$, respectively. For each $t\in[0,T]$, we define $F_t=F_t^Z\vee F_t^X$ as the σ- algebra. In this case, the Radon–Nikodym derivative is given by

$$\xi^M(t)=\left.\frac{dQ^M}{dP}\right|_{F_t}=\frac{\exp\left(\sigma W(t)\right)}{E\left[\exp\left(\sigma W(t)\right)\Big|_{F_0^Z}\right]}=\exp\left(\sigma W(t)-\frac{1}{2}\sigma^2 t\right), \tag{3.1}$$

where Q^M is the risk-neutral pricing measure (Merton measure) resulting from this approach. Under these assumptions, we can obtain that $W^M(t)=W(t)+\left(\frac{\mu-r}{\sigma}\right)t$ is a Wiener process under Q^M, which means that the investors receive a premium

$\left(\frac{\mu - r}{\sigma}\right)$ for the continuous diffusion risk at time t, and thus the Wiener process is affected by the measure change. In addition, under Q^M, we also have the distribution of the Markov-modulated Poisson process as

$$P^M\left(\Phi(t)=n \middle| X(t), t>0\right) = E\left[\xi^M(t) 1_{\left\{\Phi(t)=n \middle| X(t), t>0\right\}}\right] = P(\Phi(t)=n \mid X(t), t>0)$$

$$= \frac{\left(\int_0^t \lambda_{X(s)}\, ds\right)^n}{n!} \exp\left(-\int_0^t \lambda_{X(s)} ds\right), \quad P\text{-a.s.} \tag{3.2}$$

which means that the investors receive a zero premium for the jump risk, and hence the transition probability matrix $P^M(n,t) = P(n,t)$ and the arrival intensity matrix $\Lambda^M = \Lambda$ are unchanged by the measure change, that is, the risk-neutral properties of the jump component of the gold futures price are supposed to be the same as its statistical properties. In particular, the independently identically distributed jump amplitudes $Y_k^M \overset{i.i.d.}{\sim} N\left(\mu_y, \sigma_y^2\right)$ are also unchanged. Appendix A shows the detailed proof. Accordingly, the risk-neutral process for the gold futures price dynamics under Q^M is

$$\frac{dF(t)}{F(t-)} = \left(r - \Lambda^M \kappa\right) dt + \sigma dW^M(t) + d\left(\sum_{k=1}^{\Phi^M(t)} \left(\exp\left(Y_k^M\right) - 1\right)\right), \tag{3.3}$$

3.2 Esscher transform for the Markov regime-switching jump-diffusion process

In this section, we relax the model assumptions of Merton (1976), and then apply the Esscher transform (Gerber and Shiu, 1994, 1996) used by Elliott et al. (2005) and Elliott and Osakwe (2006) for the RSJM to determine a risk-neutral pricing measure. Under the given filtered probability space $\left(\Omega, F, P, \{F_t\}_{t\in[0,T]}\right)$, the Radon–Nikodym derivative of the Esscher transform can be expressed

$$\xi^{\theta^m}(t) = \left.\frac{dQ^{\theta^m}}{dP}\right|_{F_t} = \frac{\exp\left(\theta^C \sigma W(t)\right)}{E\left[\exp\left(\theta^C \sigma W(t)\right)\Big|_{F_0^Z}\right]} \cdot \frac{\exp\left(\theta^J \sum_{k=1}^{\Phi(t)} Y_k\right)}{E\left[\exp\left(\theta^J \sum_{k=1}^{\Phi(t)} Y_k\right)\Bigg|_{F_0^X}\right]}$$

$$= \exp\left(\theta^C \sigma W(t) - \frac{1}{2}(\theta^C \sigma)^2 t\right) \cdot \exp\left(\theta^J \sum_{k=1}^{\Phi(t)} Y_k - \Lambda \kappa^{\theta^J} t\right), \qquad (3.4)$$

where Q^{θ^m} is the Esscher measure and $\theta^m \in R$ for $m = \{C, J\}$, in which θ^C and θ^J are the Esscher parameters of the continuous diffusion part $C(t)$ and the jump part $J(t)$ for the gold futures logarithmic return $Z(t)$, respectively. As a consequence, the mean percentage jump amplitude of the gold futures price becomes $\kappa^{\theta^J} = E\left[\exp\left(\theta^J Y_k\right) - 1\right] = \exp\left(\theta^J \mu_y + \frac{1}{2}(\theta^J \sigma_y)^2\right) - 1 = \phi_Y(\theta^J) - 1$. Furthermore, the concrete form of the Esscher transform density process $\xi^{\theta^m}(t)$ is an exponential

F_t -martingale.

According to the general theory of derivative pricing, the absence of arbitrage opportunities is equivalent to the existence of an equivalent martingale measure under which the discounted asset price processs is a martingale. For the no-arbitrage valuation of American gold futures options, there exists a risk-neutral pricing measure such that the Markov, regime-switching jump-diffusion process for the gold futures price is an F_t - martingale under this measure. Let the Esscher transform be defined by Equation (3.4). Then, the martingale condition is satisfied if and only if

$$\theta^C = \frac{r-\mu+\Lambda\kappa}{\sigma^2} \tag{3.5}$$

and

$$\theta^J = \frac{-\mu_y - \frac{1}{2}\sigma_y^2}{\sigma_y^2}. \tag{3.6}$$

Appendix B shows the detailed proof.

An equivalent martingale measure can be treated as the Esscher measure Q^{θ^m} with respect to the measure P. We begin with identifying the dynamic process for gold futures prices under the risk-neutral pricing measure Q^{θ^m}. Let θ^C and θ^J be the Esscher parameters of the risk-neutral Esscher measure. Then, under Q^{θ^m} and conditional on F_t^Z

$$W^{\theta^m}(t) = W(t) - \theta^C \sigma t\,, \tag{3.7}$$

is a Wiener process. Furthermore, under Q^{θ^m}, the transition probability matrix $P^{\theta^m}(n,t)$ of the continuous-time finite-state Markov chain $X^{\theta^m}(t)$, the arrival intensity matrix Λ^{θ^m} of the Markov-modulated Poisson process $\Phi^{\theta^m}(t)$, and the jump amplitude $Y_k^{\theta^m}$ are respectively given by

$$P^{\theta^m}(n,t) = P(n,t)\left(\phi_Y(\theta^J)\right)^n \exp\left(-\Lambda\kappa^{\theta^J} t\right), \tag{3.8}$$

$$\Lambda^{\theta^m} = \Lambda\phi_Y(\theta^J) = \Lambda\exp\left(\theta^J\mu_y + \frac{1}{2}\left(\theta^J\sigma_y\right)^2\right), \tag{3.9}$$

and

$$Y_k^{\theta^m} \overset{i.i.d.}{\sim} N\left(\mu_y + \theta^J\sigma_y^2, \sigma_y^2\right), \tag{3.10}$$

where $W^{\theta^m}(t) = W(t) - \theta^C\sigma t$ is changed by the Esscher transform, which means that the investors receive a premium $-\theta^C\sigma$ for the continuous diffusion risk at time t, and then the Wiener process is affected by the measure change. Through the change of measures, the risk-neutral transition probability matrix becomes $P^{\theta^m}(n,t)$ with transition rate matrix Ψ and arrival intensity matrix Λ^{θ^m}. Under Q^{θ^m}, the jump risk can be formulated by the Esscher transform intensity of the Markov-modulated Poisson process. The arrival intensity matrix $\Lambda^{\theta^m} = \Lambda\phi_Y(\theta^J)$ is altered by the Esscher transform, which means that the investors receive a premium $\phi_Y(\theta^J)$ for the jump risk at time t, and thus the arrival intensity is affected by the measure change. If $\phi_Y(\theta^J) = 1$, the jump risk is not priced as in Merton (1976), and the arrival intensity and distribution are unaffected by the measure change. Under Q^{θ^m}, if a jump event occurs at time k, the jump amplitude $Y_k^{\theta^m}$ is normally distributed with mean $\mu_y + \theta^J\sigma_y^2$ and variance σ_y^2. Appendix C presents the detailed proof.

Using a pair of solutions of Esscher parameters given by Equations (3.5) and (3.6), we further get

$$W^{\theta^m}(t) = W(t) + \left(\frac{\mu - r - \Lambda\kappa}{\sigma}\right)t, \tag{3.11}$$

$$\Lambda^{\theta^m} = \Lambda\exp\left(-\frac{\mu_y^2}{2\sigma_y^2} + \frac{\sigma_y^2}{8}\right), \tag{3.12}$$

and

$$Y_k^{\theta^m} \overset{i.i.d.}{\sim} N\left(-\frac{1}{2}\sigma_y^2, \sigma_y^2\right), \tag{3.13}$$

where $\left(\frac{\mu - r - \Lambda\kappa}{\sigma}\right)$ and $\exp\left(-\frac{\mu_y^2}{2\sigma_y^2} + \frac{\sigma_y^2}{8}\right)$ are the market prices of the continuous diffusion risk and jump risk at time t, respectively. In addition, under Q^{θ^m}, the jump amplitude $Y_k^{\theta^m}$ is normally distributed with mean $-\frac{1}{2}\sigma_y^2$ and variance σ_y^2. Consequently, the gold futures price dynamics under Q^{θ^m} is

$$\frac{dF(t)}{F(t-)} = rdt + \sigma dW^{\theta^m}(t) + d\left(\sum_{k=1}^{\Phi^{\theta^m}(t)}\left(\exp\left(Y_k^{\theta^m}\right) - 1\right)\right), \tag{3.14}$$

where $r = \mu - \Lambda\kappa + \theta^C\sigma^2 + \Lambda^{\theta^m}\kappa^{\theta^J}$, for all $t \in [0, T]$.

3.3 Valuing American gold futures options

The valuation and optimal exercise of derivatives with discrete American-style exercise features is one of the most important practical problems in option pricing. These types of derivatives could be found within the commodity market. Assume that we are interested in pricing American put options on gold futures, where the risk-neutral gold futures price dynamics follow the stochastic differential Equations (3.3) and (3.14). The sample paths of such dynamic processes are generated by LSM simulations. For each path the optimal stopping point is determined using the estimator function with $F(\omega; t_\tau, t_{\tau-1}, \dots, t_2, t_1) = E_{t_1}[Z] = \hat{a}_\tau + \hat{b}_\tau F(t_1) + c_\tau\ F^2(t_1)$, where Z represents the discounted cash flow of an option. In line with the option pricing theory, an optimal exercise policy will generate exactly one cash flow for each path. The American gold put options are then priced by discounting the resulting cash flows back to time zero, and averaging the discounted cash flows over all paths.

Chapter 4

Empirical Analyses and Numerical Illustrations

To investigate that the gold futures price follows the RSJM, we perform a strict empirical analysis for the gold futures returns. Applying the estimated parameters from the RSJM, this dissertation further provides some numerical illustrations using actual option market data to assess the impacts of both Merton measure and Esscher transform on state-based jump-event premia. We also present a comparative analysis between alternative stochastic processes with respect to their pricing accuracy.

4.1 Empirical results

The dataset used in the descriptive analysis of Table 4.1 consists of the daily gold futures prices.[2] Analyzing returns from different periods makes us to examine the potential effect of different jump behaviors over time. The skewness and kurtosis coefficients suggest a leptokurtic distribution with positively skewed returns in the gold futures market. In 2008 and 2009, we can observe extreme movements and jumps in the daily returns, respectively. These can be regarded as the volatile state (state 2), whereas other periods can be viewed as the ordinary state (state 1). Taking

[2] The daily data are from Bloomberg and cover the period from 01/02/2007 to 12/31/2010.

the gold futures returns that are over ±5% in magnitude as an example, Table 4.1 shows that the mean frequency of the jumps in the entire period is 2.25, where the mean frequencies of the jumps in state 1 and state 2 are 0 and 4.5, respectively. State 1, therefore, is deduced as being below the long-term average jump activity. State 2 is inferred as being above the long-term average jump activity. As shown in Table 4.1, the period 2007 is in state 1, but the regime transitions to state 2 in the periods 2008 and 2009, and then transitions back to state 1 in the period 2010. A further implication of the figures in Table 4.1 is that during the test period, the economy switches between two states of jump rates, showing the characteristic that the state-dependent nature of the jump dynamics.

Table 4.1 Descriptive statistics of the gold futures returns

Classification	2007	2008	2009	2010	Total
Trading days	251	253	252	252	1008
Mean	0.0007	−0.0002	0.0008	0.0010	0.0006
Std. Dev.	0.0105	0.0191	0.0138	0.0101	0.0138
Skewness	−0.5418	0.2982	0.2708	−0.6290	0.0925
Kurtosis	4.6612	5.0181	5.6249	5.0150	6.6019
Days exceeding ±3%	5	22	7	3	37
Days exceeding ±5%	0	8	1	0	9
Days exceeding ±7%	0	1	1	0	2

Note: The descriptive statistics are reported for the gold futures returns from 2007 to 2010. This table shows the number of days each year in which the gold futures yield a logarithmic return series over ±3%, ±5%, and ±7% in magnitude.

The previous analysis of Table 4.1 sheds doubts on the validity of the GBM assumption. Motivated by these findings, we examine the ability of alternative stochastic processes for modeling the gold futures prices. In the empirical analyses, we employ the Black–Scholes model (BSM) as a benchmark for the actual data analyzed. For simplicity, we assume that the hidden Markov chain $X(t)$ has two states: state 1 and state 2. This means that, an economy switches between the ordinary and volatile states. The transition probability matrix of the two-state Markov chain $X(t)$ is given by $\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & 1-P_{11} \\ 1-P_{22} & P_{22} \end{bmatrix}$. Due to the economic state is hidden at time zero, the stationary distribution can be evaluated by the transition probability. Therefore, the stationary distributions of state 1 and state 2 are $\pi_1 = v_2 / (v_1 + v_2)$ and $\pi_2 = v_1 / (v_1 + v_2)$, respectively. Given the gold futures price dynamics defined in Equation (2.5), we express the logarithmic return in discrete time as follows:

$$R(t) = \tilde{\mu} + \tilde{\sigma} Z + \begin{cases} \sum_{k=1}^{N_1(\Delta t)} Y_k & if \quad X(t) = 1 \\ \sum_{k=1}^{N_2(\Delta t)} Y_k & if \quad X(t) = 2 \end{cases}, \tag{4.1}$$

where $\tilde{\mu} = \left(\mu - \frac{1}{2}\sigma^2 - \Lambda\kappa \right)\Delta t$ with $\Lambda\kappa = \left(p_{11}\lambda_1 + p_{22}\lambda_2 \right)\left(\phi_Y(1) - 1 \right)$, $\tilde{\sigma} = \sigma\sqrt{\Delta t}$, $Z \sim N(0,1)$, $Y_k \sim N(\mu_J, \sigma_J^2)$, and $N_i(\Delta t)$ is a Poisson process with the intensity rate λ_i in the interval time Δt when the Markov chain $X(t)$ remains in state i. The states and the jump arrivals are unobserved. We apply the EM algorithm (Lange, 1995a, b) to calculate the maximum likelihood estimations. In the first step, given the observed return data $\boldsymbol{R}$ and the former one-period parameters $\Theta^{(k-1)}$, we compute the conditional expectation of the log complete-data likelihood function as $\Gamma\left(\Theta\ ,\Theta^{(k-1)}\right)=E\left[\log\Pr(\boldsymbol{R},\boldsymbol{X},\boldsymbol{N}\mid\Theta)\mid\boldsymbol{R},\Theta^{(k-1)}\right]$. Then, for the second step, we maximize the Γ-function to use the parameter set as $\Theta^{(k)}=\arg\max\Gamma\left(\Theta\ ,\Theta^{(k-1)}\right)$. By the iteration and recursive computation of these two steps, the parameters converge the Γ-function to the local maximum in the incomplete-data likelihood function. Applying the code of the EM algorithm and the complete-data information matrix, we get the standard errors of parameter estimators by the SEM algorithm (Meng and Rubin, 1991). Khalaf et al. (2003) combine bounds and Monte Carlo simulation techniques to test the generalized autoregressive conditional heteroskedasticity (GARCH) class of models with nuisance parameters. To determine whether the JDM outperforms the BSM, and whether the data fit the RSJM better than the JDM, we apply the likelihood ratio test as follows:

$$LR\mathrm{T}=2\left(\ln L_1(\Theta)-\ln L_0(\Theta)\right)\xrightarrow{asy}\chi^2_{d,1-\alpha}, \tag{4.2}$$

where $L_m(\Theta)$ represents the likelihood function under the hypothesis H_m for $m=\{0,1\}$, and d denotes the difference of the parameters between the H_0 and

H_1 constraints. If $LR\mathrm{T} > \chi^2_{d,1-\alpha}$, H_0 is rejected. The respective null hypotheses are that the BSM and JDM hold.

The estimated parameters and corresponding standard errors (in parentheses) for each one of the alternative stochastic processes, and the likelihood ratios under study are reported in Table 4.2. The results provide several interesting findings. In the case of the JDM vs. RSJM, we can determine the value of d by the difference in the number of parameters between these two models. At the confidence level of $1-\alpha = 0.95$, the critical value for the aforementioned test is $\chi^2_{4,0.95} = 9.49$. As shown in Table 4.2, we can observe that the RSJM outperforms the JDM by the likelihood ratio $LR\mathrm{T}_2$ of 108.03 > 9.49 at the 5% statistical significance level, implying that the addition of the Markov-modulated Poisson process clearly dominates the pure Poisson process, that is, there exists the regime-switching jump feature due to the state-dependent nature of the RSJM. By a similar procedure, we find that the JDM clearly dominates the BSM by the likelihood ratio $LR\mathrm{T}_1$ of 126.68> $\chi^2_{3,0.95} = 7.82$ at the 5% statistical significance level, as shown in Table 4.2. In addition, these two transition probabilities of the gold futures market for the RSJM are almost 1, emphasizing that the economy stays in each state for a period of time and then transitions to other. Chang et al. (2013) indicate when the sum of these two transition probabilities is nearly 2, the autocorrelation of volatility (volatility clustering) is most significant. Table 4.2 shows that the sum of these two values is nearly 2, and we therefore find the volatility clustering is substantial, as in Chang et al. (2013). The mean and standard deviation of the gold futures returns for the RSJM are 0.0020 and 0.0069, respectively. The additional jump component prescribes a drift of −0.0011 and a volatility of 0.0105. The arrival intensity is found to be 0.6277 in state 1 and 3.7508 in state 2, which clearly demonstrates different arrival rates in different states. These findings indicate that the gold futures price has a GBM structure with Markov-modulated Poisson processes, that is, the jumps are subject to regime-switching movements that cannot be explained by existing jump-diffusion processes, such as in Merton (1976), Amin (1993), and Kou (2002). They are consistent with the findings in the descriptive analysis of Table 4.1, namely the non-normality of returns and the existence of changing jump rates according to the economic states.

Table 4.2 Estimated results of continuous-time models

Parameter	BSM	JDM	RSJM
p_{11}			0.9969 (0.0028)
p_{22}			0.9879 (0.0117)
μ	0.0006 (0.0004)	0.0021 (0.0005)	0.0020 (0.0004)
μ_y		−0.0019 (0.0008)	−0.0011 (0.0005)
σ	0.0138 (0.0003)	0.0067 (0.0003)	0.0069 (0.0006)
σ_y		0.0130 (0.0003)	0.0105 (0.0011)
λ_1		0.8244 (0.2611)	0.6277 (0.1848)
λ_2			3.7508 (1.1795)
LRT_1		126.68	
LRT_2			108.03

Note: This table presents the empirical results of dynamic models, reporting the estimated parameters and corresponding standard errors. The estimation settings for the BSM and JDM/RSJM are determined via the maximum likelihood (ML) approach and EM algorithm, respectively. The standard errors of parameter estimators for the BSM and JDM/RSJM obtained by the ML approach and SEM algorithm, respectively, are reported in parentheses. LRT_1 represents the likelihood ratio test for maximum likelihood functions with the null hypothesis that there is no jump event, that is, the dynamic model is BSM. LRT_2 shows the likelihood ratio test for maximum likelihood functions with the null hypothesis that there is no switching regime, that is, the dynamic model is JDM. Performance is evaluated in terms of both the statistical accuracy and likelihood ratio test.

Table 4.3 presents the mean, variance, skewness, and kurtosis for the original data and alternative stochastic processes. We further compare these model-determined values with empirical results. To investigate the leptokurtic and asymmetric features in gold futures returns, we use the formulas for the skewness and kurtosis of the JDM (Becker, 1981; Ball and Torous, 1983) and of the RSJM (Chang et al., 2013). As shown in Table 4.3, the mean and variance for the original data and alternative stochastic processes are the same. Comparing with the original data, the RSJM is better than BSM and JDM in terms of kurtosis.

Table 4.3 Distributional statistics for data and continuous-time models

Classification	Data	BSM	JDM	RSJM
Mean	0.0006	0.0006	0.0006	0.0006
Variance	0.0002	0.0002	0.0002	0.0002
Skewness	0.0925	0	−0.3076	−0.3966
Kurtosis	6.6019	3	5.1067	6.0758

Note: This table reports the distributional statistics for the original data and continuous-time models.

Figure 4.1 (a) plots the dynamic process of daily gold futures price data. Figure 4.1 (b) apparently exhibits different arrival rates with changing volatility over the test period. The time-varying volatility is captured by the RSJM in the form of regime-switching behavior, that is, the fluctuating periods of high and low arrival rates. Using the Baum-Welch algorithm adopted by Chang et al. (2013), as shown in Figure 4.1 (c), we find that the first 150 and the last 450 days of the period seem to predominantly characterized by a low arrival rate (the probability of state 1 is close to 1), while the other days are characterized by a high jump intensity (as in state 2). Our test period ends with a period of high arrival rates, corresponding to the U.S. subprime financial crisis of 2008. Figure 4.1 (d) shows the probability of jumps in a period, when there exists jumps, the probability is nearly 1. Compared with the Figure 4.1 (b), the aforementioned turmoil is an example of such jump-sensitive periods.

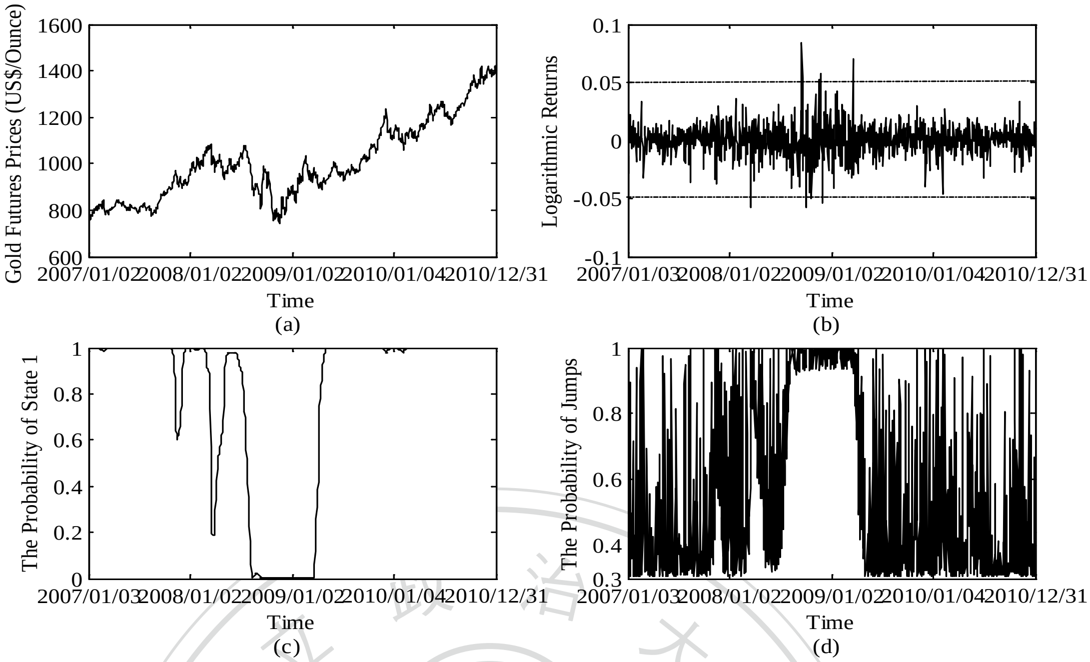

Figure 4.1 Graph of (a) gold futures prices, (b) logarithmic returns, (c) State probability, and (d) Jump probability

Note that the dotted lines in (b) denote the gold futures returns over $\pm 5\%$ in magnitude are jumps. (c) shows the probability of being in state 1, whereas the economy is in state 2 when the probability is zero.

Regarding the so-called volatility clustering, we make use of the autocorrelation function of the squared asset returns calculated by Chang et al. (2013) to investigate this phenomenon. Figure 4.2 shows a substantially positive autocorrelation in the squared logarithmic returns of gold futures in which the trend steadily declines as the lag length increases. The RSJM therefore captures not only the existence of volatility clustering but also the magnitude and decay of this phenomenon. Under the RSJM framework, the volatility clustering occurs from the jump clustering caused by the jump arrival intensity changes over time according to the states of an economy. These empirical results suggest that the RSJM provides the adequate description for the gold futures returns. It is an evidence that the model addresses the previously mentioned empirical characteristics, including the presence of jumps, leptokurtosis, asymmetry, and volatility clustering phenomena. The RSJM overcomes the shortcomings of GBM and jump-diffusion processes (Merton, 1976; Amin, 1993; Kou, 2002) for modeling the underlying asset price. This dynamic model will be general enough to cater for price jumps more appropriate for gold futures.

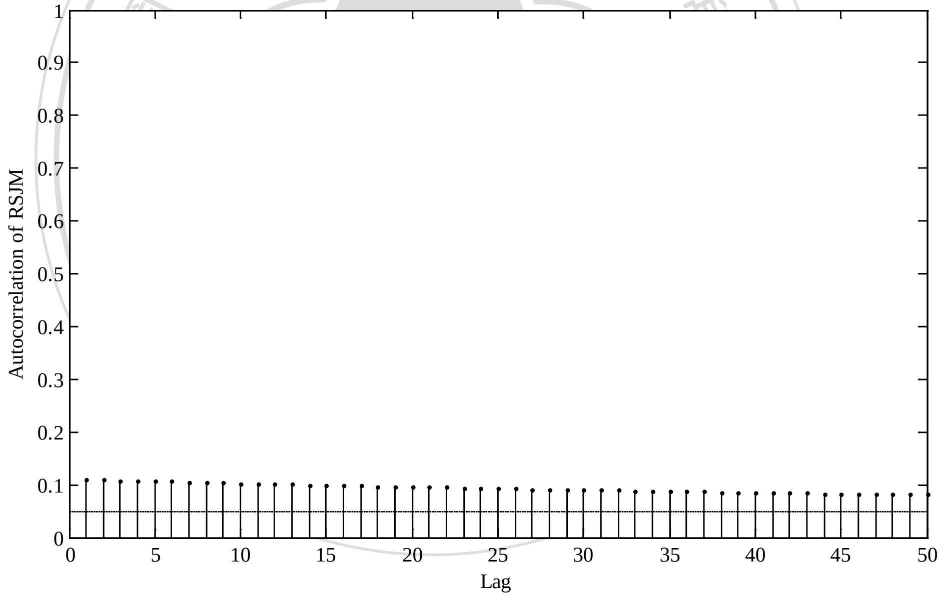

Figure 4.2 Model autocorrelation of the squared logarithmic returns

4.2 Pricing performance

In this section, we evaluate the pricing performance of in-sample and out-of-sample periods using actual option market data[3] from the Commodity Exchange (COMEX). As a benchmark, we apply the BSM to price American put options on gold futures. The one-year U.S. treasury bill rate is used as a proxy for the risk-free rate. The relative mean square errors (RMSEs) are adopted for model evaluation. Due to actual data limitations, an analysis across moneyness levels is not possible. The parameters of each model are calibrated over periods of four years and then the estimated parameters are used to evaluate the pricing performance of in-sample and out-of-sample periods for each option.

The pricing errors presented in Table 4.4 correspond to RMSEs across both in-sample and out-of-sample data analyzed. On the in-sample analysis of the option pricing models in terms of RMSEs. The results show that the American gold put options are more accurately priced using the RSJM under the Esscher measure. For $K = 780$ and $K = 800$, the pricing errors of the RSJM under the Esscher measure are just slightly higher than those under the Merton measure in terms of RMSEs.

[3] The option data are from Bloomberg. These data correspond to the gold futures prices and cover the period between 10/01/2010 and 03/31/2011 with the expiration date 05/25/2011.

Taking the strike price $K = 860$ as an example, the largest improvement offered by the RSJM under the Esscher measure over the Merton measure is 0.0411 in the reduction of RMSEs. Turning now to the out-of-sample analysis, we find a similar picture. The results show that the RSJM under the Esscher measure is again more accurate than under the Merton measure in pricing American gold put options. For $K = 780$, the pricing error of the RSJM under the Esscher measure is just slightly higher than that under the Merton measure in terms of RMSEs. When $K = 840$, the improvement is largest for the RSJM between the Esscher and Merton measures with a reduction of 0.0672 in terms of RMSEs.

Through the analysis of in-sample and out-of-sample pricing errors with different strike prices K, these numerical illustrations indicate that pricing errors under the RSJM are all smaller than those competing models in terms of RMSEs. Specifically, the reduction of the RMSEs between the JDM and RSJM is more substantial than that of the RMSEs between the BSM and JDM. One can infer that for this reason, the Markov component contributes more to the superior pricing performance rather than the pure jump process. The numerical results show that the RSJM is more accurate than the BSM and JDM in pricing American gold put options. In other words, whether the Merton measure or Esscher transform is employed to derive the risk-neutral gold futures price dynamics, the RSJM strongly outperforms the BSM and JDM. Overall, the evidence presented seems to suggest that it is worth accounting for state-dependent jump risks when pricing American put options on gold futures. Once more data become available, it is necessary to engage a more extensive comparative analysis between alternative stochastic processes, not only with respect to their pricing accuracy but also in terms of their hedging performance.

Table 4.4 In-sample and out-of-sample pricing errors of American gold put option pricing models

K	BSM	JDM	RSJM	
		Merton measure		Esscher measure
	In-sample pricing errors (%) (Observations=64)			
780	0.9700	0.9659	0.3549	0.3583
800	0.9587	0.9542	0.2910	0.2935
820	0.9429	0.9376	0.3176	0.3008
840	0.9231	0.9174	0.3512	0.3202
860	0.8985	0.8930	0.3836	0.3425
	Out-of-sample pricing errors (%) (Observations=62)			
780	0.9971	0.9955	0.7050	0.7089
800	0.9945	0.9925	0.7622	0.7423
820	0.9909	0.9882	0.8288	0.8100
840	0.9859	0.9822	0.9100	0.8428
860	0.9796	0.9737	0.8454	0.7900

Note: This table presents the RMSEs of various option pricing models. We calibrate each model in the period from 01/03/2007 to 12/31/2010 and then use the estimated parameters to evaluate the in-sample performance in the period from 10/01/2010 to 12/31/2010 as well as the out-of-sample performance in the period from 01/03/2011 to 03/31/2011. The RMSEs are estimated by minimizing the sum of the squared pricing errors between the LSM-simulated prices and the market prices (divided by the market prices) for each option. The RMSE (pricing error) expressed in percentage. Pricing performance is evaluated for the aggregate sample on the basis of the RMSEs.

Chapter 5

Conclusions and Future Extensions

According to the empirical analysis of the gold futures returns, we verify the existence of unanticipated jump events with different jump rates of gold futures prices in different time periods. The empirical results show that the gold futures price is better approximated by a continuous-time, Markov regime-switching jump-diffusion process. Compared with the standard jump-diffusion processes, the main advantage is that we incorporate state-dependent jump risks into the dynamic model. This stochastic process also appropriately characterizes the leptokurtosis, asymmetry, and volatility clustering across the empirical data analyzed. In the light of the empirically favored RSJM for the gold futures price under the different jump risk considerations, we adopt the Merton measure and Esscher transform to derive the risk-neutral gold futures price dynamics. After determining such dynamic processes, the values of American gold futures options are approximated using the LSM method. In this dissertation, we empirically investigate the in-sample and out-of-sample pricing performance of the LSM algorithm for American gold futures options with alternative stochastic processes, including GBM, JDM, and RSJM. The numerical results indicate that whether the Merton measure or Esscher transform is employed to derive the risk-neutral gold futures price dynamics, the RSJM strongly outperforms the BSM and JDM. As a consequence, we find that the Markov-modulated Poisson process is more accurate than the pure Poisson process when valuing American gold futures put options, and jump risks implied by the RSJM have a more significant impact on the option prices. To some extent, the comparison has indicated the importance of incorporating state-dependent jump risks into the gold futures price model.

This dissertation has several possible extensions and potential improvements. First, due to the discrete nature of jump risks, the riskless hedging with jump risks under an incomplete gold market remains an important challenge. Second, we might employ an alternative distribution instead of a log-normal distribution used in the jump amplitude. Third, it is interesting to examine how the changing transition rate impacts the transition probability under the measure change. Fourth, more simple, yet powerful numerical algorithms are required to evaluate the American-style options. As a potential future work, we might consider incorporating other Markov regime-switching market parameters, such as interest rates, the appreciation rate and the volatility of the underlying asset price, the jump amplitude of a compound Poisson process, into the dynamic model studied in this dissertation. Comparing the effect of risk premia on the option prices may be a new interesting topic.

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Appendix A: Distributional properties of RSJM under Q^M

Proof. Based on Equation (3.1), apply the Girsanov theorem and by the Itô's rule, we immediately get that $W^M(t) = W(t) + \left(\frac{\mu - r}{\sigma}\right)t$ is a Wiener process under Q^M. We then denote the moment-generating function of the random variable Y_k by $\phi_Y(1) = E\left[\exp(Y_k)\right] = \kappa + 1$. This does not depend on the index k because $\{Y_k : k = 1, 2, \ldots\}$ all have the same distribution. Then, we have

$$E\left[\exp\left(\sum_{k=1}^{\Phi(t)} Y_k\right)\right] = P\left(\Phi(t) = 0\right) + \sum_{n=1}^{\infty} E\left[\exp\left(\sum_{k=1}^{n} Y_k\right) \middle| \Phi(t) = n\right] P\left(\Phi(t) = n\right)$$

$$= \sum_{n=0}^{\infty} \left(E\left[\exp(Y_k)\right]\right)^n P(n,t) = \sum_{n=0}^{\infty} \sum_{i=1}^{I} \sum_{j=1}^{I} \pi_i \left(\phi_Y(1)\right)^n P_{ij}(n,t)$$

$$= \exp(\Lambda \kappa t), \tag{A.1}$$

where π_i denotes the stationary distribution in state i. This limiting distribution can be computed by $\psi_{jj}\pi_i = \sum_{k=1,k\neq j}^{I} \psi_{kj}\pi_j$ along with the constraint $\sum_{j=1}^{I} \pi_j = 1$. Furthermore, we note that $\sum_{k=1}^{\Phi(t)} Y_k - \Lambda \kappa t$ is a martingale at time t. Given $\Phi(t) = n$, the Radon–Nikodym derivative of the transition probability can be written as

$$\left.\frac{dQ^M_{prob.}}{dP_{prob.}}\right|_{\Phi(t)=n}=1, \tag{A.2}$$

then we get $dP^M(n,t)=dP(n,t)$, where $P^M(n,t)$ denotes the transition probability matrix under Q^M. Moreover, we use Equation (2.2) and its unique solution given by Equation (2.3). Letting $P^M(n,t)=P(n,t)$, we can get

$$\begin{aligned} P^{M*}(\zeta,t) &= \sum_{n=0}^{\infty} P^M(n,t)\zeta^n = \sum_{n=0}^{\infty} P(n,t)\zeta^n = P^*(\zeta,t) \\ &= \exp\left(\left(\Psi-(1-\zeta)\Lambda\right)t\right), \end{aligned} \tag{A.3}$$

Therefore, the transition rate matrix Ψ of the transition probability matrix $P^M(n,t)$ and the arrival intensity matrix $\Lambda^M=\Lambda$ are unchanged by the measure change. Finally, we investigate the jump amplitude, where $\{Y_1,Y_2,\ldots,Y_n\}$ are independently identically distributed random variables. Thus, the Radon–Nikodym derivative of each specific jump amplitude can be set as

$$\left.\frac{dQ^M_Y}{dP_Y}\right|_{F_t^x}=1, \tag{A.4}$$

then we obtain $dQ^M_Y=dP_Y=\frac{1}{\sqrt{2\pi\sigma_y^2}}\exp\left(-\frac{\left(Y_k-\mu_y\right)^2}{2\sigma_y^2}\right)$. Under Q^M, if a jump event occurs at time k, the jump amplitude Y_k^M is normally distributed with mean μ_y

and variance σ_y^2. Furthermore, under Q^M, the density function of each specific jump amplitude Y_k^M is $f_Y^M(y) = f_Y(y)$, and therefore the density function is unchanged by the change of measures.

Appendix B: Solving the Esscher parameters

Proof. Let E^{θ^m} denote the mathematical expectation operator with respect to the Esscher measure Q^{θ^m} equivalent to P. It is possible to select the risk-neutral Esscher measure as the measure Q^{θ^m} such that the discounted gold futures price process is a Q^{θ^m}-martingale. This is obtained by determining the Esscher parameters θ^C and θ^J as solutions of $E^{\theta^m}\left[\exp(-rt)F(t)\middle|F_0\right]=F(0)$. Applying Equation (3.4), we have

$$
\begin{aligned}
F(0) &= \exp(-rt)E^{\theta^m}\left[F(t)\middle|F_0\right] = \exp(-rt)E\left[\frac{dQ^{\theta^m}}{dP}F(t)\middle|F_0\right] \\
&= F(0)E\left[\exp\left(\left(\mu - r - \frac{1}{2}\sigma^2 - \Lambda\kappa\right)t + \sigma W(t) + \sum_{k=1}^{\Phi(t)} Y_k\right)\frac{dQ^{\theta^m}}{dP}\right] \\
&= F(0)\exp\left(\left(\mu - r - \frac{1}{2}\sigma^2 - \Lambda\kappa\right)t + \frac{1}{2}\left(\left(1+\theta^C\right)\sigma\right)^2 t - \frac{1}{2}\left(\theta^C\sigma\right)^2 t\right) \\
&\quad \cdot \exp\left(\Lambda\left(\kappa^{\left(1+\theta^J\right)} - \kappa^{\theta^J}\right)t\right),
\end{aligned}
\tag{B.1}
$$

From the mutual independence of random shocks $W(t)$, $\Phi(t)$, and Y_k, and then the martingale condition holds if and only if the Esscher parameters θ^C and θ^J satisfy

$$
\mu - r - \Lambda\kappa + \theta^C\sigma^2 = 0 \tag{B.2}
$$

and

$$\mu_y + \frac{1}{2}\sigma_y^2 + \theta^J \sigma_y^2 = 0, \tag{B.3}$$

for all $t \in [0,T]$. Therefore, we can define a pair of solutions of Esscher parameters for the martingale condition by Equations (3.5) and (3.6).

Appendix C: Distributional properties of RSJM under Q^{θ^m}

Proof. By Equation (3.4), apply the Girsanov theorem and from the mutual independence of random shocks $W(t)$, $\Phi(t)$, and Y_k, we can obtain that $W^{\theta^m}(t) = W(t) - \theta^C \sigma t$ is a Wiener process under Q^{θ^m}. Next, we denote the moment-generating function of the random variable Y_k by $\phi_Y(\theta^J) = E\left[\exp\left(\theta^J Y_k\right)\right] = \kappa^{\theta^J} + 1$. This does not depend on the index k because $\{Y_k : k = 1, 2, ...\}$ all have the same distribution. Then, we have

$$E\left[\exp\left(\theta^J \sum_{k=1}^{\Phi(t)} Y_k\right)\right] = P\left(\Phi(t) = 0\right) + \sum_{n=1}^{\infty} E\left[\exp\left(\theta^J \sum_{k=1}^{n} Y_k\right) \middle| \Phi(t) = n\right] P\left(\Phi(t) = n\right)$$

$$= \sum_{n=0}^{\infty} \left(E\left[\exp\left(\theta^J Y_k\right)\right]\right)^n P(n,t) = \sum_{n=0}^{\infty} \sum_{i=1}^{I} \sum_{j=1}^{I} \pi_i \left(\phi_Y(\theta^J)\right)^n P_{ij}(n,t)$$

$$= \exp\left(\Lambda \kappa^{\theta^J} t\right), \tag{C.1}$$

where π_i denotes the stationary distribution in state i. This limiting distribution can be computed by $\psi_{jj}\pi_i = \sum_{k=1,k\neq j}^{I} \psi_{kj}\pi_j$ along with the constraint $\sum_{j=1}^{I} \pi_j = 1$. Specifically, we note that $\theta^J \sum_{k=1}^{\Phi(t)} Y_k - \Lambda \kappa^{\theta^J} t$ is a martingale at time t. Given $\Phi(t) = n$, the Radon–Nikodym derivative of the transition probability can be set as

$$\left.\frac{dQ_{prob.}^{\theta^m}}{dP_{prob.}}\right|_{\Phi(t)=n} = \left(\phi_Y(\theta^J)\right)^n \exp\left(-\Lambda\kappa^{\theta^J} t\right), \tag{C.2}$$

then we get $dP^{\theta^m}(n,t) = dP(n,t)\left(\phi_Y(\theta^J)\right)^n \exp\left(-\Lambda\kappa^{\theta^J} t\right)$, where $P^{\theta^m}(n,t)$ denotes the transition probability matrix under Q^{θ^m}. We use Equation (2.2) and its unique solution given by Equation (2.3). Letting $P^{\theta^m}(n,t) = P(n,t)\left(\phi_Y(\theta^J)\right)^n \exp\left(-\Lambda\kappa^{\theta^J} t\right)$, we can get

$$\begin{aligned} P^{\theta^m *}(\zeta,t) &= \sum_{n=0}^{\infty} P(n,t)\left(\phi_Y(\theta^J)\right)^n \exp\left(-\Lambda\kappa^{\theta^J} t\right)\zeta^n = \sum_{n=0}^{\infty} P(n,t)\left(\zeta\phi_Y(\theta^J)\right)^n \exp\left(-\Lambda\kappa^{\theta^J} t\right) \\ &= \exp\left(\left(\Psi - (1-\zeta)\Lambda\phi_Y(\theta^J)\right)t\right), \end{aligned} \tag{C.3}$$

where the transition rate matrix Ψ of the transition probability matrix $P^{\theta^m}(n,t)$ is unaffected by the Esscher transform. Under Q^{θ^m}, the jump risk can be formulated by the Esscher transform intensity $\Lambda^{\theta^m} = \Lambda\phi_Y(\theta^J) = \Lambda\exp\left(\theta^J\mu_y + \frac{1}{2}\left(\theta^J\sigma_y\right)^2\right)$. Finally, we investigate the jump amplitude, where $\{Y_1, Y_2, \ldots, Y_n\}$ are independently identically distributed random variables. Hence, the Radon–Nikodym derivative of each specific jump amplitude can be written as

$$\left.\frac{dQ_Y^{\theta^m}}{dP_Y}\right|_{F_t^X} = \frac{\exp\left(\theta^J Y_k\right)}{E\left[\left.\exp\left(\theta^J Y_k\right)\right|_{F_0^X}\right]}, \tag{C.4}$$

then we obtain $dQ_Y^{\theta^m} = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{\left(Y_k - \left(\mu_y + \theta^J \sigma_y^2\right)\right)^2}{2\sigma_y^2}\right)$. Furthermore, under the physical probability measure P, the density function of each specific jump amplitude Y_k is $f_Y(y)$. Through the change of measures, under Q^{θ^m}, the density function of each specific jump amplitude $Y_k^{\theta^m}$ is $f_Y^{\theta^m}(y) = f_Y(y) \cdot \left.\frac{dQ_Y^{\theta^m}}{dP_Y}\right|_{F_t^X}$.