行政院國家科學委員會專題研究計畫成果報告

多階製程下多等級產品之製程管制
Multiple Processes Control for Multiple Grades Products

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主持人: 楊素芬　國立政治大學統計系所
E-Mail: YANG@ NCCU.EDU.TW

1. ABSTRACT

An approach to controlling multiple grades products produced on the multiple dependent process steps is developed. It not only distinguish which process step is out of control but also monitors the common assignable cause which affects overall process mean and the grade-specific assignable cause which affect grade-specific process mean effectively. A standardized EWMA control chart and grade-specific EWMA control charts are proposed to control grade-specific and standardized combined incoming data stream, respectively. Besides, a standardized cause-selecting control chart and grade-specific cause-selecting control charts are proposed to control grade-specific and standardized out-going combined data streams, respectively. A simulation results shows the effect of the proposed six control charts from statistical viewpoint. Finally, the economic scheme of the six proposed control charts is also proposed.

Keywords: Multiple Grades Products, Multiple Process Steps, Assignable Causes, Markov Chain, Renewal Reward Processes

中文摘要

本研究提出多等級產品在相依製程下的監控方法。它能有效區分那個製程階段是失控的及診斷出引起所有製程平均值平移的非機遇因素和引起特定製程平均值平移的非機遇因素，以使失控的製程迅速回復穩定中。一個標準化的 EWMA 管制圖和各特定等級的 EWMA 管制圖被發展來管制標準化和不同等級的 incoming 資料。另外，一個標準化的選控圖和各特定等級的選控圖被發展來管制標準化和不同等級的 out-going 資料。這些管制圖的偵測力以模擬結果說明。最後，這些管制圖的經濟設計也以 Markov chain 和更新報酬方法推導出。

關鍵字: 多等級產品，相依製程，非機遇因素，馬可夫鏈，更新報酬過程。

2. RESEARCH MOTIVATION AND OBJECTIVE

The object of statistical process control (spc) is the rapid detection of changes in a process mean and/or variability. Early detection of a change improves the likelihood of problem identification and corrective action. In the continuous process industries, multiple grades product are often made from multiple dependent process steps. The production process may be out of control because of the occurrence of the common assignable cause which affects overall process mean, and the occurrence of the grade-specific assignable causes which affect specific process means on each process step. The SPC approach is using control charts to monitor a process for changes in mean. For example, we may use specific EWMA control chart to detect if specific process mean shifts for incoming quality. The problem for the specific control chart is that it may only detect the grade-specific assignable cause not the common assignable cause. We may also use a standardized EWMA control chart to detect the common assignable cause for the incoming quality, but it is not easy to detect the effect of the specific assignable cause. Doganaksoy, Schmee and Vandeven (1996) proposed a simple method of combining and monitoring data from multiple grades allowing speeding detection of changes affecting all grades and yet still permitting differentiation of overall process shifts from grade-specific shifts using EWMA charts. In practice, there may exist that (1) products may be produced from multiple dependent process steps. How to distinguish which subprocess is out of control? (2) only the specific assignable cause which influences the grade-specific product on each process step; (3) the process may be influenced by the common and grade-specific assignable causes both on each step; (4) quality engineers may try to detect the common and grade-specific assignable causes both speedily and with minimal cost. So far, the methods to solve these problems have not been addressed. The purposes in the study are as follows. (1) To propose an effective approach to distinguish which process step is out-of –control. (2) To propose the appropriate control charts to detect the multiple grade products when only grade-specific assignable causes may occur in the process, and design their charts from statistical and economic viewpoint. (3) To propose the appropriate control charts to detect common and grade-specific assignable causes both speedily on each process step. (4) Using some numerical results and examples to show the effects of our proposed control charts.

3. RESULTS AND DISCUSSION

3.1 The Design of standardized control chart and grade-specific EWMA control charts on each process step

為追蹤第一階段 G1 的品質特性值X_{1j}是否受到 AC1 的影響;G2 的品質特性值X_{2j}是否受到 AC2 的影響，我們分別根據X_{1j}、X_{2j}在製程穩定下的分配，建立個別的 EWMA 管制圖。為追蹤第一階段 G1 和 G2 的品質特性值X_{1j}和X_{2j}是否同時受到 AC3 的影響，我們根據X_{1j}和X_{2j}在製程穩定下標準化後的分配建立合併的 EWMA 管制圖。為追蹤第二階段 G1 的品質特性值ε_{1j}是否受到 AC4 的影響；G2 的品質特性ε_{2j}值是否受到 AC5 的影響，我們分別根據ε_{1j}、ε_{2j}在製程穩定下的分配，建立個別的選控圖。為追蹤第二階段 G1 和 G2 的品質特性值ε_{1j}和ε_{2j}是否同時受到 AC6 的影響，我們根據ε_{1j}和ε_{2j}在製程穩定下標準化後的分配建立合併的選控圖。於是在製程穩定時，(1)X_{1j}~N(μ_1, σ_1^2)，所以監控 AC1 是否發生的 EWMA1 管制圖的中心線為μ_1，管制上下界為

$\mu_1 \pm k_1\sigma_1\sqrt{\frac{\lambda_1}{2-\lambda_1}\left[1-(1-\lambda_1)^{2j}\right]}$，其中 λ_1 表 EWMA 管制圖的權數、k_1 表 EWMA 管制圖管制界線的係數；(2) X_{2j} ~N(μ_2, σ_2^2)，所以監控 AC2 是否發生的 EWMA2 管制圖的中心線為 μ_2，管制上下界為 $\mu_2 \pm k_2\sigma_2\sqrt{\frac{\lambda_2}{2-\lambda_2}\left[1-(1-\lambda_2)^{2j}\right]}$；(3) 將 X_{1j} 標準化為 $S_{1j}=\frac{X_{1j}-\mu_1}{\sigma_1}$ ~N(0, 1)，X_{2j} 標準化為 $S_{2j}=\frac{X_{2j}-\mu_2}{\sigma_2}$ ~N(0, 1)，所以監控 AC3 是否發生的合併的 EWMA3 管制圖的中心線為 0，管制上下界為 $\pm k_3\sqrt{\frac{\lambda_3}{2-\lambda_3}\left[1-(1-\lambda_3)^{2j}\right]}$。(4)假設 X_{ij} 和 Y_{ij} 之間具有簡單線性迴歸模式，即 E(Y_{ij} | X_{ij})= $a_{i0}+a_{i1}X_{ij}=\mu_{ij}$。選控圖是對製程在穩定狀態下的選控值 ε_{ij} 建立 EWMA 形式的管制圖，因為選控值 ε_{ij} ~N(0, $\sigma_{Y_i}^2$)，所以監控 AC4 是否發生的 EWMA4 管制圖的中心線為 0，管制上下界為 $\pm k_4\sigma_{Y_1}\sqrt{\frac{\lambda_4}{2-\lambda_4}\left[1-(1-\lambda_4)^{2j}\right]}$。同理監控 AC5 是否發生的 EWMA5 管制圖的中心線為 0，管制上下界為 $\pm k_5\sigma_{Y_2}\sqrt{\frac{\lambda_5}{2-\lambda_5}\left[1-(1-\lambda_5)^{2j}\right]}$；(5)將 ε_{1j} 標準化為 $Z_{1j}=\frac{\varepsilon_{1j}-0}{\sigma_{Y_1}}$ ~N(0, 1)，ε_{2j} 標準化為 $Z_{2j}=\frac{\varepsilon_{2j}-0}{\sigma_{Y_2}}$ ~N(0, 1)，所以監控 AC6 是否發生的 EWMA6 管制圖的中心線為 0，管制上下界為 $\pm k_6\sqrt{\frac{\lambda_6}{2-\lambda_6}\left[1-(1-\lambda_6)^{2j}\right]}$。

於是這六個管制圖可以有效的診斷哪個子製程失控並知道是共同非機遇因素或(及)是特定等級之非機遇因素發生。另外，模擬結果顯示這六個管制圖之檢定力比其他管制圖好。

3.2 The economic design of standardized control chart and grade-specific EWMA control charts on each process step

在製程穩定時，(1) X_{1j} ~N(μ_1, σ_1^2)，所以監控 AC 是否發生在第一階段影響 G1 品質特性值的 EWMA1 管制圖的中心線為 μ_1，管制上下界為 $\mu_1 \pm k_1\sigma_1\sqrt{\frac{\lambda_1}{2-\lambda_1}\left[1-(1-\lambda_1)^{2j}\right]}$，其中 λ_1 表 EWMA 管制圖的權數、k_1 表 EWMA 管制圖管制界線的係數；(2) X_{2j} ~N(μ_2, σ_2^2)，所以監控 AC 是否發生在第一階段影響 G2 品質特性值的 EWMA2 管制圖的中心線為 μ_2，管制上下界為 $\pm k_2\sigma_2\sqrt{\frac{\lambda_2}{2-\lambda_2}\left[1-(1-\lambda_2)^{2j}\right]}$；(3) 將 X_{1j} 標準化為 $S_{1j}=\frac{X_{1j}-\mu_1}{\sigma_1}$ ~N(0, 1)，X_{2j} 標準化為 $S_{2j}=\frac{X_{2j}-\mu_2}{\sigma_2}$ ~N(0, 1)，所以監控 AC 是否發生在第一階段同時影響 G1 和 G2 品質特性值的合併的 EWMA3 管制圖的中心線為 0，管制上下界為 $\pm k_3\sqrt{\frac{\lambda_3}{2-\lambda_3}\left[1-(1-\lambda_3)^{2j}\right]}$。(4)假設 X_{ij} 和 Y_{ij} 之間具有簡單線性迴歸模式，即 E(Y_{ij} | X_{ij})= $a_{i0}+a_{i1}X_{ij}=\mu_{ij}$。選控圖是對製程在穩定狀態下的選控值 ε_{ij} 建立 EWMA 形式的管制圖，因為選控值 ε_{ij} ~N(0, $\sigma_{Y_i}^2$)，所以監控 AC 是否發生在第二階段影響 G1 的品質特性值的 EWMA4 管制圖的中心線為 0，管制上下界為 $\pm k_4\sigma_{Y_1}\sqrt{\frac{\lambda_4}{2-\lambda_4}\left[1-(1-\lambda_4)^{2j}\right]}$。同理監控 AC 是否發生在第二階段影響 G2 的品質

特性值的 EWMA5 管制圖的中心線為 0，管制上下界為 $\pm k_5\sigma_{Y_2}\sqrt{\frac{\lambda_5}{2-\lambda_5}\left[1-(1-\lambda_5)^{2j}\right]}$；(5)將 ε_{1j} 標準化為 $Z_{1j}=\frac{\varepsilon_{1j}-0}{\sigma_{Y_1}}$ ~N(0, 1)，ε_{2j} 標準化為 $Z_{2j}=\frac{\varepsilon_{2j}-0}{\sigma_{Y_2}}$ ~N(0, 1)，所以監控 AC 是否發生在第二階段同時影響 G1 和 G2 品質特性值的 EWMA6 管制圖的中心線為 0，管制上下界為 $\pm k_6\sqrt{\frac{\lambda_6}{2-\lambda_6}\left[1-(1-\lambda_6)^{2j}\right]}$。

上述六個 EWMA 管制圖的管制係數 k_1 ~ k_6 和參數 λ_1 ~ λ_6 是未知的。為建立以最小成本追蹤製程的經濟管制圖，我們必須決定使成本最低的最適的 λ_i 和 k_i 值；即 λ_i^* 和 k_i^*，i=1, 2, ……, 6。在決定 λ_i^* 和 k_i^* 後，將其代入六個 EWMA 管制圖，則六個經濟 EWMA 管制圖得以建立。

成本模式乃利用更新理論方法和更新報酬過程而得。成本模式之推導已十分不易，幸能突破。另外，六個經濟 EWMA 管制圖之檢定力乃利用模擬獲得，頗費功夫。

最後，我們以例子說明這些經濟 EWMA 管制圖的應用及診斷決策。

4. EVALUATION

All the problems described in the project are figured out as expected. Two papers will be submitted to journals, and we expect that they will be accepted. We only consider the single quality characteristics in the study. All the proposed approaches are expected to solve the problem of multivariate quality characteristics. The economic process control for multiple grade products with Weibull distributions will be discussed in next year's project.

5. REFERENCES

1. 楊素芬（1998），"多等級產品的製程管制"，國科會專題計劃，NSC 87-2118-M-004-008。
2. 楊素芬和劉泰亨（1999），"不同等級產品的經濟製程管制"，技術報告，國立政治大學。
3. Banerjee, P. and Rahim, M.M19877 , "The Economic Design of Control Chartl A Renewal Theory Approach ", Engineering Optimization,Vol.12, pp.63-73.
4. Doganaskoy, N., Schmee, J. and Vandeven, M. M 1996 7 ,"Process Monitoring with Multiple Product Grades", Journal of Quality Technology, Vol. 28, No. 3, pp.346-355.
5. Duncan, A.M19567 ,"The Economic Design of $\overline{X}$ Chart Used to Maintain Current Control of a Process", Journal of the American Statistical Association, Vol. 51, pp.228-242.
6. Lorenzen, T. and Vance, L. M 19867 ,"The Economic Design of Control Chartsl A Unified Approach", Technometrics, Vol. 28, No. 1, pp.3-10.
7. Montgomery, D. M 1980 7 ," The Economic Design of Control Chartsl A Review and Literature Survey", Journal of Quality Technology, Vol. 12, No. 2, April, pp.75-86.
8. Montgomery, D.M19967 , Introduction to Statistical Quality Control, John WileyWSons, Inc., New York.
9. Panagos, M., Heikes, R. and Montgomery, D. (19857 ,"Economic Design of $\overline{X}$ Control Charts for Two Manufacturing Process Models", Naval Research Logistics Quarterly, Vol. 32,

pp.631-646.

10. Roberts, S.M 19597 ,"Control Chart Tests Based on Geometric Moving Average", Technometrics, Vol. 1, pp.239-250.
11. Ross, S. M 1993 7 , Introduction to Probability Models, Academic Press.
12. Shewhart, W. M 1931 7 , Economic Control of Quality of Manufactured Product ,D. Van Nostrand Company, Inc.
13. Yang, S. M 19977 , "The Economic Design of Control Charts When There are Dependent Process Steps", International Journal of Quality W Reliability Management, Vol. 14, No. 6, pp.606-615.
14. Zhang, G.M19847 ,"A New Type of Control Charts and a Theory of Diagnosis with Control Charts", World Quality Congress Transactions, American Society for Quality Control, Milwaukee, WI, pp.75-85.