Long-Run Risks, Monetary Policy and the Term Structure of Interest Rates

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Abstract

Many previous studies of the term structure of interest rates specify the process for inflation exogenously. Because monetary policy is a crucial driver of inflation, this paper attempts to endogenize the process for inflation through a monetary policy rule and examines the performance of the model. Calibration results suggest that, given the strong stance of monetary policy, the model is able to explain the average upward-sloping nominal yield curve, volatile long rates and the downward-sloping term structure of volatility. The decomposition of the nominal stochastic discount factor in the manner of Alvarez and Jermann (2005) also suggests that the negative correlation of equilibrium inflation and the monetary policy shock help to match the risk premium in the data. However, the model also has difficulty describing inflation and the term structure of yields across different historical regimes.

JEL: E43, E52, G12

1 Introduction

The dynamics of nominal interest rates is an important issue for economists and policy makers since it reflects future expectations for the economy. Empirical studies in past decades have established that the nominal yield curve is usually upward-sloping, the volatility of nominal yields has a downward-sloping term structure, and the term premia on long-term bonds are on average positive and time-varying. Some authors try to rationalize these stylized facts in equilibrium asset pricing models and gain some success, but most of them rely on an exogenous process for inflation in nominal bond pricing. This approach implies that inflation is not structural in these models, so the sources of inflation risk and the associated interest rate risk may not be clear. In addition, it is not possible to explore the role of specific inflation drivers, e.g., monetary policy, in the term structure of nominal yields if the inflation process is exogenous.

This paper attempts to address this issue by endogenizing the inflation process through a monetary policy rule in a consumption-based asset pricing model. If the conduct of monetary policy is a rule describing how the target of a short-term interest rate should be set in response to inflation and macroeconomic activity and the central bank commits to it, the market short-term yield should be adjusted so that it equals the policy target in equilibrium. According to this view, how the process for inflation is determined deeply affects the term structure of interest rates. Economists have arrived at a consensus that inflation is ultimately a monetary phenomenon, and a large portion of the movements in inflation can be attributed to the conduct of monetary policy. Since a surge in inflation substantially depreciates the real payoffs of long-term bonds, variations in monetary policy could be an important source of fluctuations in nominal yields. Hence the model proposed in this paper may help illuminate the role of monetary policy in shaping the term structure of interest rates.

The real side of the model features long-run risks and stochastic volatility in consumption dynamics. The recursive preference in the style of Epstein and Zin (1989) together with a high intertemporal elasticity of substitution implies that agents in this economy desire an early resolution of uncertainties. As indicated by Restoy and Weil (2011), this framework implies that asset risk premia are driven by covariances of asset returns with not only *current* but also *expected future* consumption growth. As long as consumption growth and inflation are negatively correlated, a positive shock to long-run consumption growth volatility leads to the rise of the long-short nominal yield spread and bond risk premia. Thus the average yield curve is upward-sloping. In addition, this framework assumes a small but very persistent component and fluctuating uncertainty in real consumption growth. This component amplifies the consumption volatility channel so that the model can account for the magnitude and variation of bond risk premia quantitatively.

The nominal side of the model characterizes an endogenous inflation process that is jointly determined by the consumption dynamics and an interest rate feedback rule. The resulting model solution indicates that equilibrium inflation is triggered by the long-run consumption risk, stochastic volatility and monetary policy disturbance. Because the real payoff of any nominal asset is affected by expected inflation, these components are also inherited to the nominal bond prices and yields. When long-run consumption growth and monetary policy shocks are quite persistent, the equilibrium nominal yields are highly autocorrelated and volatile. More importantly, the slowly diminishing impact of a monetary policy shock helps to explain the downward-sloping volatility curve of nominal yields and volatile long-term interest rates observed in the data.

Several recent studies have discussed the term structure of interest rates implied by the long-run risk framework. Bansal and Shaliastovich (2008) extend the model by Bansal and Yaron (2004) to resolve the predictability puzzles in bond and currency markets. Hasseltoft (2011) estimates a similar model through the simulated method of moments (SMM) approach to seek joint explanations of key features of equity and bond markets. Doh (2011) incorporates a small persistent variation in the processes of both consumption growth and inflation to explore term structure of bond yields with Bayesian estimation. All of these works can account for some salient facts of yield curves, but their models rely on purely exogenous processes of inflation. Thus the effects of monetary policy on nominal yields are not clear in these models since the dynamics of inflation and real asset prices are independent.

Some authors have attempted to make the inflation process more structural in their recent works on consumption-based asset pricing models. Gallmeyer, Hollifield, Palomino and Zin (2007) argue that a model with recursive preference and endogenous inflation can fit the data more easily than one with exogenous information. However, they focus on comparing model performance and do not further explore the role of monetary policy in nominal yields. Gallmeyer, Hollifield, Palomino and Zin (2009) develop another model with endogenous inflation to explain some salient facts of the term structure of interest rates. The workhorse of their model is a shock that is sensitive to consumption growth and non-systematic taste change, so their model does not feature any importance of long-run risks. Since risks for the long-run successfully explain some key facts of stock prices, it is worth examining if such a model with endogenous inflation is capable of describing the term structure of interest rates.[1]

Calibration with the postwar U.S. data reveals that a long-run risk model with endogenous inflation is able to explain several salient facts of nominal yields. First of all, the model captures the negative correlation of inflation and consumption growth in the data. The implication of this feature is that high expected inflation further depreciates the real payoffs of nominal bonds when consumption growth is low. Thus nominal bonds are risky assets and carry positive risk premia. This explains why the average nominal term structure has a positive slope. Second, the volatility of consumption growth and monetary shock jointly generate volatile long rates and a downward-sloping term structure of volatility. In particular, the monetary policy disturbance with an autoregressive process prevents the volatility of nominal yields from decaying too fast so that the model is able to match the volatility of long-term interest rates. Finally, the decomposition of the nominal stochastic discount factor in the manner of Alvarez and Jermann (2005) shows that the model risk premia are much closer to the consensus than those implied by the model with an exogenous process for inflation. This result comes from the negative correlation of equilibrium inflation and the monetary policy shock, which trims the risk premium on the long end of the term structure. Hence incorporating monetary policy helps to capture the risks implied by nominal yields more precisely.

On the other hand, sub-sample calibration results suggest that the model is not able to

[1] Models with long-run risks have more new developments recently. For example, Yang (2011) and Eraker, Shaliastovich and Wang (2011) focus on long-run risk in durable consumption growth to explain key asset market facts. Dittmar and Palomino (2010) introduce leisure and wage in a long-run risk model and find more plausible results in explaining equity returns. To illuminate the role of monetary policy and keep the tractability of the model, I do not address these issues in this paper.

reconcile the historical regimes of monetary policy and the corresponding term structure of interest rates in some aspects. First of all, most studies of U.S. monetary policy suggest that the conduct of monetary policy was more effective in controlling inflation after the 1980s, but the model implies the opposite because it requires somewhat passive monetary policy to reproduce more volatile nominal yields and a larger term spread in the Volcker-Greenspan era. This result may be attributed to the effect of monetary policy on the process of inflation. The model implies that a more aggressive policy response to inflation reduces the volatility of inflation and nominal yields. Thus monetary policy moves the volatility of the two variables in the same direction. However, the data show volatile inflation and stable nominal yields in the 1970s and the opposite pattern since the 1980s. Hence it is not a surprise that the model fails to track the data.

Second, the term spread of nominal yields is on average larger since the 1980s while the model predicts that a stronger inflation stabilization policy reduces the term spread. In the model economy, inflation is endogenously determined through a credible monetary policy rule. A more aggressive policy response to inflation is interpreted as a clear sign of declining long-run risk, and inflation becomes less volatile. As a result, long-term bonds bear lower inflation risk premia and the term spread should get smaller. However, the data show that the term spread is on average increasing from Burns-Miller to Volcker-Greenspan. This conundrum might be attributed to a less credible monetary policy in the Greenspan era or other factors that are not related to monetary policy, as suggested by Polamino (2012). The model cannot account for these features since it assumes a commitment to the policy rule, and the factors unrelated to monetary policy are not well known.

Finally, the model requires a more than proportional rise in the short rate associated with the increase in consumption growth and inflation, namely, aggressive monetary policy, to replicate the level and slope of the nominal interest rates. Although this policy practice conforms to the Taylor principle and ensures the existence of a unique stationary equilibrium, the view that there has been a sustained strong policy response to inflation in the past decades gets very limited empirical support. Many studies of U.S. monetary policy, such as Clarida, Galí and Gertler (2000), Cogley and Sargent (2001, 2005) and Lubik and Schorfheide (2004), suggest that the Fed was passive in response to inflation in the 1970s. If the conduct of monetary policy did not conform to the Taylor principle in the 1970s, it is not possible for the model to account for the upward-sloping yield curve in that period.

A more fundamental problem of the model regards the inconsistency between Euler equation interest rates and money market rates. The model requires the equivalence of the two rates to clear the bond market and determine the equilibrium process of inflation. However, an analysis by Canzoneri, Cumby and Diba (2007) indicates that many asset pricing models generate systematic negative correlation between the Euler equation rate and the Federal Funds rate. On the other hand, most contemporary central banks conduct monetary tightening by raising the target of a short rate. Thus the interest rates implied by the Euler equation of a model should also rise in response to this policy. However, the model implies that a positive monetary policy shock decreases the Euler equation rates and raises the money market rates based on the Fed's policy reaction rule. This result stems mainly from the negative response of inflation to a monetary policy shock. Forcing this response to be positive cannot solve the problem since it counterfactually implies that disinflation policy exacerbates inflation. This inconsistency problem imposes a fundamental challenge to the long-run risk model with endogenous inflation.

2 Model

2.1 Consumption Dynamics

Consider an endowment economy in which agents have Epstein-Zin preferences:

$$U_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where δ is the subjective discount factor, $\gamma \geq 0$ is the risk aversion parameter, $\theta = (1-\gamma)/(1-1/\psi)$ and $\psi \geq 0$ is the intertemporal elasticity of substitution. Epstein and Zin (1989) derived that the associated logarithm of real stochastic discount factor (SDF) is:

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{2}$$

where Δc_{t+1} denotes the log of real aggregate consumption growth and $r_{c,t+1}$ represents the log return on aggregate wealth. The process for consumption growth follows Bansal and

Yaron (2004):

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \tag{3}$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t \epsilon_{t+1}, \tag{4}$$

$$\sigma_{t+1}^2 = (1 - \nu_1)\sigma^2 + \nu_1 \sigma_t^2 + \sigma_w w_{t+1}. \tag{5}$$

The log of real aggregate consumption growth is determined by an unconditional mean μ, a small persistent component x_t and a short-run consumption risk η_{t+1}. The term x_t follows an autoregressive process with persistence parameter ρ and a long-run consumption risk ϵ_{t+1}. To capture the time-varying bond risk premium, the model features a conditional stochastic volatility of consumption growth σ_t^2, which also follows an autoregressive process and is subject to a macroeconomic volatility risk w_{t+1}. The innovations to consumption growth and stochastic volatility are assumed to have independent standard normal distributions.

2.2 Monetary Policy Rule

Unlike many previous studies, the equilibrium process for inflation in the model is closely related to the conduct of monetary policy. In this economy, the central bank considers movements of consumption growth and inflation to set the target of short-term interest rate i_t as follows:

$$i_t = i_0 + i_c \Delta c_t + i_\pi \pi_t + u_t, \tag{6}$$

where π_t denotes the measure of inflation and u_t characterizes the policy disturbance. The coefficients i_c and i_π measure policy responses to consumption growth and inflation, respectively. The monetary policy shock is assumed to follow an autoregressive process:

$$u_{t+1} = \phi_u u_t + \sigma_\xi \xi_{t+1}, \tag{7}$$

where $\xi_{t+1} \sim N(0, 1)$. Such a *contemporaneous* rule is close to the usual Taylor rule specification. The major deviation is that the central bank responds to consumption growth rather than output gap as there is no production in this economy. The model with a contemporaneous monetary policy rule is denoted as Model C.

Some authors argue that the central bank should formulate interest rate decisions based on expected macroeconomic variables. For example, Clarida, Galí and Gertler (2000) propose a *forward-looking* rule as follows:

$$i_t = i_0 + i_c E_t\left(\Delta c_{t+1}\right) + i_\pi E_t(\pi_{t+1}) + u_t. \tag{8}$$

The disturbance follows the same autoregressive process as (7). The model with a forward-looking monetary policy rule is denoted as Model F. Given that two specifications characterize different reactions to macroeconomic conditions by the central bank, it is interesting to see whether they imply different equilibrium processes for inflation and interest rate dynamics.

2.3 Equilibrium Inflation and Bond Yields

Given that the real SDF contains the unobservable return on aggregate wealth $r_{c,t+1}$, the first step of the solution procedure is to approximate this element using the method proposed by Campbell and Shiller (1988):

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \tag{9}$$

where z_t denotes the log of price to consumption ratio, κ_0 and κ_1 are functions of $\overline{z}$, and $\overline{z}$ is the average of z_t. Bansal and Yaron (2004) conjecture that z_t is a linear function of two state variables x_t and σ_t^2, i.e.,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2, \tag{10}$$

where the coefficients A_0, A_1 and A_2 can be solved using the Euler equation with consumption dynamics.

The second step is to solve for the equilibrium process for inflation. Because inflation is determined through monetary policy, the short-term nominal interest rate derived from the nominal SDF should equal the policy target set by the central bank. Therefore,

$$i_t = -E_t\left(m_{t+1} - \pi_{t+1}\right) - \frac{1}{2} Var_t\left(m_{t+1} - \pi_{t+1}\right), \tag{11}$$

where the right-hand side is obtained by exploiting the properties of the log-normal distribution and π_{t+1} depends on the specification of monetary policy. The specification of the forward-looking policy rule and equation (11) lead to the following conjecture of π_t:

$$\pi_t = \pi_0 + \pi_x x_t + \pi_\sigma \sigma_t^2 + \pi_u u_t. \tag{12}$$

Thus the equilibrium inflation is linear in the state variables x_t, σ_t^2 and u_t. Similarly, the conjecture of the equilibrium inflation in the case of contemporaneous policy rule is given as follows:

$$\pi_{t+1} = \pi_0 + \pi_x x_t + \pi_\sigma \sigma_t^2 + \pi_u u_t + \pi_\eta \sigma_t \eta_{t+1} + \pi_\epsilon \sigma_t \epsilon_{t+1} + \pi_w w_{t+1} + \pi_\xi \xi_{t+1}. \tag{13}$$

The equilibrium process for inflation can be solved by plugging the real SDF and the conjectured process for inflation into (11).

The final step is to solve for nominal bond prices and yields. Let $\widetilde{p}_t^n$ denote the log price of a zero-coupon nominal bond with maturity n at time t. In a frictionless economy, no arbitrage condition implies that the following Euler equation must hold:

$$E_t \left[\exp \left(m_{t+1} - \pi_{t+1} + \widetilde{p}_{t+1}^{n-1} - \widetilde{p}_t^n \right) \right] = 1. \tag{14}$$

Given that all disturbances are Gaussian, the Euler equation (14) can be written as

$$E_t \left(m_{t+1} - \pi_{t+1} + \widetilde{p}_{t+1}^{n-1} - \widetilde{p}_t^n \right) + \frac{1}{2} Var_t \left(m_{t+1} - \pi_{t+1} + \widetilde{p}_{t+1}^{n-1} - \widetilde{p}_t^n \right) = 0. \tag{15}$$

Based on the specification of m_{t+1} and π_{t+1}, I conjecture that $\widetilde{p}_t^n$ is linear in the three state variables x_t, σ_t^2 and u_t, i.e.,

$$\widetilde{p}_t^n = \widetilde{B}_0^n + \widetilde{B}_x^n x_t + \widetilde{B}_\sigma^n \sigma_t^2 + \widetilde{B}_u^n u_t, \tag{16}$$

and the corresponding nominal yields $\widetilde{y}_t^n$ can be defined as

$$\widetilde{y}_t^n = -\frac{1}{n} \widetilde{p}_t^n. \tag{17}$$

The equilibrium bond prices can be solved recursively by plugging all dynamics into (15).

3 Model Calibration and Implications

To study the implications of the model, I calibrate the parameters in consumption dynamics and the monetary policy rule to match the moments of consumption growth, inflation and nominal yields. The data are U.S. quarterly series from 1952:2 to 2011:2. Consumption data are constructed using per capita consumption growth of nondurables and services from the Bureau of Economic Analysis (BEA). Inflation is measured using the price index that corresponds to consumption data, which is also available from BEA. Piazzesi and Schneider (2007) argue that this measure has less high-frequency noise than the commonly used consumer price index (CPI). Bond yields data are from CRSP Fama-Bliss discount bond files.

The parameters are calibrated to match major moments of consumption growth and nominal term structure. Since monetary policy does not affect consumption dynamics in the model, both Model F and Model C share the same real side parameters. Table 1 lists the calibrated parameter values. The value of μ corresponds to 1.97% per annum consumption growth for the entire sample period. Other parameters of consumption growth are chosen to match its volatility and first-order autocorrelation. Following Bansal and Yaron (2004), the intertemporal elasticity of substitution is set at 1.5. The choice of risk aversion parameter $\gamma = 7.5$ is comparable to those in the literature. Parameters of the monetary policy rule are chosen to match the average level and volatility of nominal yields.

3.1 Equilibrium Inflation

Table 2 summarizes the moments of consumption growth and inflation. Model calibration shows that two specifications of the monetary policy rule yield similar inflation dynamics. The average quarterly inflation is 0.76% in Model F and 0.77% in Model C, and the associated volatility is 0.95% in Model F and 0.96% in Model C. Although inflation is slightly lower on average and somewhat more volatile compared to the data, these moments do not seriously deviate from the data since the calibration matches the moments of nominal yields given consumption dynamics. The model also overestimates the first-order autocorrelation of inflation when the monetary policy is described by a forward-looking rule. Because equilibrium inflation is a function of consumption dynamics and monetary policy, its autocorrelation inherits the persistence from the state variables x_t, σ_t^2 and u_t. Highly persistent consumption dynamics and monetary shocks result in a quite persistent process for inflation. This is more obvious in Model F since the central bank responds to future expected macroeconomic variables.

Because the equilibrium process of inflation is linear in the state variables, the factor loadings characterize how inflation is associated with the macroeconomic dynamics and various risks. First of all, equilibrium inflation is inversely related to the small persistent component in consumption growth, i.e.,, $\pi_x < 0$. Holding other things constant, this feature implies a negative contemporaneous correlation of inflation and consumption growth. Given a positive and large i_c, the Fed substantially raises the short rate when consumption growth is high. The aggressive policy reduces current and future inflation, so the model suggests a negative correlation between the two. Table 2 shows that $corr(\Delta c, \pi) = -0.1762$ in Model F and -0.2593 in Model C. In fact, the macroeconomic data reveal that this correlation is -0.1073. Thus the negative correlation of inflation and consumption growth is exactly what we observe in the data.

The coefficients π_σ and π_u are also negative, which indicates that an increase in uncertainty or an unexpected policy tightening decreases the equilibrium inflation. The intuition for $\pi_\sigma < 0$ is the behavior of precautionary saving. When future consumption growth becomes more uncertain, agents in the economy tend to save more. This precautionary motive raises the demand for bonds and lowers the interest rates and inflation. The result $\pi_u < 0$ can be thought of as a consequence of the disinflation policy. Given the strong stance of the Fed on price stability, i.e., $i_\pi > 1$, an unexpected positive policy shock effectively raises the interest rate and reduces inflation in current and future periods. Meanwhile, a negative π_u also implies that an aggressive monetary policy dampens the volatility of inflation.

As equilibrium inflation is determined through a monetary policy rule, the conduct of monetary policy can affect the dynamics of inflation. Table 3 reports how the mean and the standard deviation of inflation change with different values of i_π while all other parameters are kept at their baseline values. This ceteris paribus analysis shows that a larger value of i_π results in lower and more stable inflation. When $i_\pi = 1.2$, the average quarterly inflation is approximately 1.45% (or 5.8% per annum) and the standard deviation is 1.67%. If $i_\pi = 2.2$, the average quarterly inflation substantially shrinks to 0.2% (or 0.8% per annum) and the standard deviation also decreases to roughly 0.3%. Holding other things constant, the model suggests that a stronger policy stance on price stability does achieve a more desirable outcome.

3.2 The Term Structure of Nominal Yields

Table 4 reports the average nominal yield curve and the associated volatility. Given the consumption dynamics, the model reasonably matches the average level and variation for one- to five-year nominal yields. In addition, the model reproduces two important facts of the nominal interest rates. First, the average nominal term structure has a positive slope. The average one-year yield is 5.25% in Model F and Model C, then gradually increases to 5.85% in Model F and 5.81% in Model C for the five-year yield. Second, the average volatility of nominal yields has a downward-sloping term structure. The standard deviation of the one-year rate is 3.08% in Model F and 2.88% in Model C, then slowly decreases to 2.63% in Model F and 2.47% in Model C for the five-year rate. Thus the model captures the high variance of long-term yields in the data even if the volatility curve decreases in maturity. As documented in Shiller (1979), the volatile long-term interest rates imply the failure of the conventional expectation hypothesis.

To understand the upward-sloping average term structure of nominal rates, first recall that the equilibrium inflation is negatively correlated with *current* consumption growth. Since high inflation reduces the real value of any monetary payment, real payoffs on nominal bonds are extremely poor in bad states. In addition, recursive preference implies that nominal bonds are also unattractive in times with news of unexpectedly high inflation. More explicitly, inflation is also bad news for *future* consumption growth. Thus risk-averse investors demand positive risk premia on nominal bonds for compensation. As indicated by Piazzesi and Schneider (2007), this feature results in an upward-sloping nominal yield curve.

It is worth noting that the above result requires an aggressive policy response to consumption growth. Because the relative risk aversion coefficient is greater than the reciprocal of intertemporal elasticity of substitution in the model, the covariance of asset returns with expected future consumption growth have impacts on asset risk premia. A positive and large value of i_c means that the short rate is substantially increased when consumption growth is high. As a result, good news about future consumption growth is associated with low inflation and expected returns on long-term nominal bonds tend to be high. More explicitly, consumption growth is positively correlated with nominal bond returns in this case. Hence nominal bonds are risky assets and long-term bond holders require positive risk premia, which lead to an upward-sloping yield curve. If i_c is too small, the correlation of consumption growth with nominal bond returns becomes negative. Nominal bonds provide insurance against bad states in this case, and the associated negative risk premium implies a downward-sloping yield curve.

However, a positive and large i_c alone is not enough. The upward-sloping nominal term structure also requires an aggressive policy reaction to inflation, i.e., $i_\pi > 1$. A large value of i_π implies that the Fed sharply increases the short rate to fight against high inflation. Long-term bond prices plummet and bond holders suffer substantial capital loss as a result of the rise in yields. Nominal bond investors demand positive risk premia since the returns are poor in bad times. This mechanism explains why an aggressive policy response to inflation is needed to generate a positive slope of nominal term structure. On the contrary, $i_\pi < 1$ makes nominal bond returns negatively correlated with consumption growth. In this case, nominal bonds hedge against consumption risk and the associated risk premium is negative. As a result, the model replicates the positive slope of yield curve only when the short rate target is responsive to both consumption growth and inflation.

The model reproduces the downward-sloping term structure of average volatility of yields and volatile enough long-term interest rates in the data. Because fluctuations of bond prices come from consumption growth and monetary policy shocks, the average volatility of yields decreases in maturity as the shocks decay over time. Note that the persistence of monetary disturbance plays an important role in the volatility of long-term yields. This persistence is transmitted through endogenous inflation to fluctuations of nominal yields so that the long-term rates are still volatile. Table 5 and 6 show how the volatility of nominal yields changes with different values of ϕ_u. It is clear that a very persistent policy shock is necessary for the model to match the volatility at the long end of the term structure. Even a slight decrease in the autoregressive parameter to 0.95 makes the volatility of the five-year yield quickly drops to 1.88% in Model F and 1.70% in Model C, which are far less than 2.78% in the data. Thus the incorporation of monetary policy helps the model to capture the variation of nominal interest rates.

3.3 Bond Risk Premium

Since many asset pricing models can account for various asset return moments, additional criteria may be helpful for further evaluation of their performance. Koijen, Lustig, Van Nieuwerburgh and Verdelhan (2010) propose that the fraction of the variance arising from the martingale component of the SDF can provide useful information of model fit. They apply the method developed by Alvarez and Jermann (2005) to decompose the nominal pricing kernel $\widetilde{N}_t$ as $\widetilde{N}_t = \widetilde{N}_t^P \widetilde{N}_t^T$, where the martingale component $\widetilde{N}_t^P$ satisfies the condition that $E_t(\widetilde{N}_{t+1}^P) = \widetilde{N}_t^P$and the transitory component $\widetilde{N}_t^T$ is defined as

$$\widetilde{N}_t^T = \lim_{\tau \longrightarrow \infty} \frac{\widetilde{\beta}^{t+\tau}}{\widetilde{P}_t^n(\tau)} \tag{18}$$

for some number $\widetilde{\beta}$. Let $\widetilde{m}_{t+1} = m_{t+1} - \pi_{t+1} = \ln\left(\widetilde{N}_{t+1}/\widetilde{N}_t\right)$ be the log of nominal SDF. The ratio of the conditional variance of the martingale component to that of the entire nominal SDF can be defined as follows:

$$\widetilde{\omega}_t = \frac{Var_t\left(\widetilde{m}_{t+1}^P\right)}{Var_t\left(\widetilde{m}_{t+1}\right)}, \tag{19}$$

where $\widetilde{m}_{t+1}^P$ denotes the martingale component of $\widetilde{m}_{t+1}$. Alvarez and Jermann (2005) demonstrate that bond risk premia of all maturities are zero if $\widetilde{m}_{t+1}$ is purely martingale, while the infinite-horizon bond has the largest risk premium if $\widetilde{m}_{t+1}$ does not have a martingale component. Because the size of bond risk premium is relatively small,[2] the variation of $\widetilde{m}_{t+1}$ should mostly come from the martingale component. Thus a data-consistent $\widetilde{\omega}_t$ should be close to one.

I follow Alvarez and Jermann (2005) to compute the conditional variance ratio of the permanent component to the SDF.[3] The ratio is 0.74 for Model F and 0.76 for Model C. These numbers are substantially better than 0.37 reported by Koijen et al. (2010), which is derived from a long-run risk model by Bansal and Shaliastovich (2008). A low value of this variance ratio means that the transitory component contributes too much volatility, making risk premia on long-term bonds unreasonably high. The calculation suggests that the long-

[2] Lustig, Van Nieuwerburgh and Verdelhan (2011) estimate that the five-year nominal bond premium is 0.92% per annum for the period 1953 to 2008, while the equity premium is 6.90%.

[3] The detailed computation is relegated to the appendix.

run risk model with endogenous inflation is more consistent with the estimated price of risk from the data.

Because the Alvarez-Jermann decomposition directly links the volatility of the SDF to the size of the bond risk premium, it is convenient to compute the average five-year risk premium implied by the model and to compare the size with the one implied by the model with an exogenous process for inflation. The expected risk premium $E(rp_{t+1})$ can be expressed as

$$E(rp_{t+1}) = -\varphi_e(m_\epsilon - \pi_x\varphi_e)\sigma^2\widetilde{B}_x^n - \sigma_w(m_w - \pi_\sigma\sigma_w)\widetilde{B}_\sigma^n + \pi_u\sigma_\xi^2\widetilde{B}_u^n \tag{20}$$

for Model F and

$$E(rp_{t+1}) = -\varphi_e(m_\epsilon - \pi_e)\sigma^2\widetilde{B}_x^n - \sigma_w(m_w - \pi_w)\widetilde{B}_\sigma^n + \pi_\xi\sigma_\xi\widetilde{B}_u^n \tag{21}$$

for Model C. Table 7 reports the bond risk premium implied by models with endogenous inflation (Model F and C) and the one with exogenous inflation (BS). At the short end of the term structure, all models reproduce the risk premium estimated from the data and exhibit little difference. As maturity increases, the risk premium implied by BS surges rapidly and far exceeds the data. Koijen et al. (2010) show that the annualized five-year nominal bond premium is 2.97% for BS. In contrast, the risk premia implied by models with endogenous inflation do not increase that drastically. The five-year premium is 1.43% per annum (Model F) or 1.32% (Model C). Since the data suggest that the average five-year premium is around 1%,[4] the risk premium implied by the model with endogenous inflation is closer to the consensus.

To understand the above results, first note that most of the long-run risk models imply a downward-sloping *real* yield curve.[5] To match the observed upward-sloping nominal term structure, the process for inflation has to make the model produce quite a large *nominal* risk premium to overcome the negative real risk premium. Although this strategy reproduces a reasonable nominal yield curve, it also makes the risk premium at the long-end of the term structure counterfactually large. This situation is ameliorated when inflation is determined through a monetary policy rule. Analysis of (20) and (21) shows that the risk premium is

[4] CRSP Fama-Bliss files suggest that the average five-year premium is 0.97% per annum. Lustig, Van Nieuwerburgh and Verdelhan (2011) get an estimate of 0.92% using the data from 1952 to 2008.

[5] See Bansal and Shaliastovich (2008) and Hasseltoft (2011) for more details.

primarily driven by long-run consumption risk, which is characterized by the negative and large value of m_ϵ. Hence the contribution of long-run consumption risk is to keep the average risk premium positive and increasing with maturity. On the other hand, the risk premium is also determined by the reaction of equilibrium inflation to monetary policy disturbances. The calibration suggests that $\pi_u = -2.89$ (Model F) or $\pi_u = -2.75$ (Model C). The result implies that the reaction is negative since a positive monetary policy shock raises the short rate and suppresses inflation. Thus the effect of a policy disturbance partially reduces the inflation variability delivered to bond yields. Overall, the average risk premium remains positive, but its magnitude does not grow very fast as maturity increases.

4 Different Monetary Policy Regimes

A long strand of research on the conduct of U.S. monetary policy suggests that the Fed's reaction to inflation was very different in the pre-Volcker and Volcker-Greenspan periods. Clarida, Galí and Gertler (2000), Cogley and Sargent (2001, 2005) and Lubik and Schorfheide (2004) conclude that U.S. monetary policy became much more aggressive in controlling inflation since the early 1980s.[6] If this view is correct, monetary policy regime switching may lead to different profiles of the term structure of interest rates in various historical periods. Because inflation can be endogenously determined through a monetary policy rule in a long-run risk model, this framework provides a venue to explore how changes to monetary policy affect the nominal term structure and whether these effects are consistent with the data for each policy regime. Consequently, a few calibration exercises are conducted using the data from three historical periods which roughly correspond to the terms of several Federal Reserve Board Chairmen. These periods are Burns-Miller (1970:1 to 1979:2), Volcker-Greenspan (1979:3 to 2005:4) and Greenspan-only (1987:3 to 2005:4).

Table 8 and 9 report calibration results for different periods using Model F and C, respec-

[6] Some authors do not agree with this view. For example, Orphanides (2004) suggests that the great inflation in the 1970s was the consequence of excessive activist policy response to real economic activity rather than weak policy stance on price stability. Canova and Gambetti (2009) argue that the same policy rule dominated for several decades and their analysis does not support a more aggressive policy in fighting inflation in the Volcker-Greenspan era.

tively. The first calibration strategy is to match the level and the slope of the nominal yield curve given consumption dynamics, which is exactly the same method used for the baseline calibration. This is denoted Case 1 in Table 8 and 9. The results show that the Fed's reaction to macroeconomic activity increased from $i_c = 1.10$ in Burns-Miller to $i_c = 2.20$ in Greenspan-only. Since the data indicate that consumption growth became lower and less volatile in recent decades, the model suggests that the Fed reduced macroeconomic fluctuations by adjusting the short rate in a countercyclical manner. In contrast, the Fed's response to inflation was not very different among these periods, though the coefficient is somewhat larger in Burns-Miller ($i_\pi = 1.25$ in Model F and $i_\pi = 1.21$ in Model C) while relatively smaller during Greenspan-only ($i_\pi = 1.10$ in both Model F and C). Thus the model counterfactually implies that a more proactive stance of price stability is associated with a regime of high and volatile inflation in the 1970s. In addition, the volatilities of inflation and nominal yields predicted by the model are obviously too high, especially in the Greenspan-only period.

Because Case 1 calibration has difficulty matching variations of inflation and nominal yields, another strategy is to match the volatility of the nominal yields given the consumption dynamics. This is denoted Case 2. The results show that a more aggressive attitude of the Fed in controlling inflation is necessary to produce the volatility of nominal yields in the data. As a result, the fluctuations of inflation are also much lower compared to those in Case 1. However, it is also clear that Case 2 calibration fails to match the slope of the yield curve. More explicitly, nominal rates at the long end are substantially underestimated in all sub-sample periods. For example, the average five-year rate was 5.84% in the era of Chairman Greenspan while Model F simply gives a 5.08% five-year yield and Model C produces 5.10%. In addition, the model still suggests excessive volatile inflation during the terms of Chairman Volcker and Greenspan.

Compared with the literature, these calibration results suggest a quite different pattern of monetary policy in the postwar United States. Meanwhile, the changes of the term structure in response to the monetary policy shifts are also inconsistent with the data in several aspects. The first discrepancy involves the change of policy response to inflation from Burns-Miller to Volcker-Greenspan. A strand of research suggests that, if there was a structural change in the U.S. monetary policy rule in the early 1980s, the conduct of monetary policy was more effective in controlling inflation *after* that time. However, the sub-sample calibration exercises imply a slightly less aggressive attitude toward price stability by the Fed after the 1980s. This discrepancy may be explained by the structure of the model. Since inflation is endogenously determined through a monetary policy rule, a more aggressive response to the change of price level lowers the volatility of inflation, and the channel of nominal bond pricing also leads to more stable nominal yields. Table 8 and 9 clearly show that more stable inflation and nominal yields are associated with a stronger policy stance on controlling inflation. However, the macroeconomic data reveal that stable interest rates were associated with volatile inflation in Burns-Miller while volatile rates were associated with stable inflation in Greenspan-only. To match the volatile interest rates after the 1980s, the policy attitude toward price stability has to be passive so that the model can generate a volatile process of inflation. As a result, the model implies that i_π is on average larger in the 1970s and smaller after the 1980s, which is different from the pattern suggested in the literature.

The second conundrum concerns the change in the policy response to controlling inflation on the term spread. Table 8 and 9 show that $i_\pi = 1.25$ in Burns-Miller corresponds to a 0.38% spread between the five-year and one-year yields, and $i_\pi = 1.20$ in Volcker-Greenspan is associated with a 0.83% spread. Thus the model suggests that a more aggressive response to inflation shrinks the term spread. Because equilibrium inflation is driven by monetary policy in the model, investors believe that a stronger stance on price stability ultimately leads to lower and less volatile inflation. As a result, long-term bonds are exposed to less inflation risk, and a lower risk premium implies a smaller term spread. However, the spread between the five-year and one-year rates in the CRSP Fama-Bliss file is 0.41% in Burns-Miller, 0.83% in Volcker-Greenspan and 0.90% in Greenspan-only. If monetary policy was more aggressive in controlling inflation after the 1980s, a larger i_π should be associated with a smaller slope of the yield curve. Thus the relationship between the disinflation policy and the term spread implied by the model seems not consistent with the data.

Ang, Boivin, Dong and Loo-Kung (2009) argue that a surprise increase in the Fed's response to inflation has a larger effect on the long end of the yield curve. Their impulse response analysis shows that such a policy shift raises the five-year yield almost twice as much as the short rate, and even a small increase in inflation loading leads to a sharp increase in the risk premium on long-term bonds. Thus the authors conclude that a stronger stance on price stability during the Volcker-Greenspan period carries a positive price of risk. More explicitly, an unexpected increase in the Fed's response to inflation exposes the entire yield curve to higher inflation risk and increases the term spread. This argument seems appealing at first glance given that the five-year term spread is clearly larger on average in Volcker-Greenspan, and many studies suggest a strong intention to suppress inflation during that time. However, there is ample evidence that volatilities of many macroeconomic variables and compensations for various sources of uncertainties, including inflation risk, are significantly lower after the mid-1980s. Therefore, it is not clear if a sharp increase in the short rate target should be interpreted as introducing more inflation risk on long-term bonds.

Palomino (2012) also finds it difficult to reconcile the increasing average term spread with the improved credibility of monetary policy from the 1950s to the early 2000s. His analysis also shows that the higher credibility of monetary policy during the Bretton Woods regime accounts for a smaller term spread on average. However, the larger average term spread in the Greenspan era cannot be rationalized by the same story. Palomino suggests that one possible interpretation of this phenomenon could be a suspicion in the policy credibility improvements in the Greenspan era. Although monetary policy is believed to have been more aggressive in controlling inflation during the Volcker-Greenspan period, the Fed did not explicitly state a nominal anchor. Hence an essentially lower credibility of monetary policy is not out of the question given that the public may not understand the policy well. On the other hand, other factors that are not related to monetary policy could also be a reason. The model cannot account for these factors if we do not know what they are and how they affect the term structure of interest rates. In sum, providing a coherent explanation for the relatively large term spread in the quiescent Volcker-Greenspan period continues to be challenging.

The third problem focuses on the magnitude of the policy response to inflation and consumption growth. To reproduce the positive slope of the nominal yield curve, the model imposes the restriction $i_\pi > 1$ and disciplines i_c to be positive and sufficiently large. The condition $i_\pi > 1$ is in line with the Taylor principle, which ensures the existence of a unique stationary equilibrium. The empirical work by Orphanides (2004) suggests a similarly strong policy reaction to inflation, i.e., $i_\pi > 1$, in the pre- and post-Volcker periods. However, most other studies of postwar U.S. monetary policy do not suggest that the Fed always adhered to the Taylor principle. Clarida, Galí and Gertler (2000) apply the generalized method of moments (GMM) to estimate a simple forward-looking monetary policy rule similar to (8). Their baseline estimate of i_π is 2.15 for the Volcker-Greenspan era while the estimate for the pre-Volcker period is merely 0.68. Their estimates of i_c are much smaller than i_π and are statistically insignificant for the Volcker-Greenspan era. In addition, the estimates of policy responses to inflation and output gap are quite divergent in the literature. Ang, Dong and Piazzesi (2007) use a no-arbitrage pricing technique to estimate various specifications of the Taylor rule for the period June 1952 to December 2004. Their benchmark estimates are $i_c = 0.509$ and $i_\pi = 0.238$ for a contemporaneous rule and $i_c = 0.590$ and $i_\pi = 0.292$ for a forward-looking rule. In sum, these studies are far from unanimous on the Fed's stance of price and output stability, and few of them support the prevailing aggressive policy in the past decades.

In addition to the above problems, the inconsistency between Euler equation interest rates and money market rates seems to be a more fundamental challenge. To clear the bond market, the short rate implied by the Euler equation has to equal the target rate set by the Fed. This condition not only determines the equilibrium inflation but also illuminates the effects of monetary policy on the term structure of interest rates. However, Canzoneri, Cumby and Diba (2007) find that the behavior of the Federal Funds rate is quite different from the short rate implied by many asset pricing models. Their analysis shows that the gap between the two rates links to the stance of monetary policy. More explicitly, a tightening policy increases the money market rate and reduces the Euler equation rate. Thus the correlation of the Euler equation rate and the money market rate is systematically negative, which casts doubt on the equivalence of the two rates in many asset pricing as well as macroeconomic models.

Unfortunately, the long-run risk model with endogenous inflation is not immune to this critique. The Euler equation rate from the model, which is the right-hand side of (11), can be expressed as

$$\begin{aligned}&(\pi_0 - m_0) - \frac{1}{2}\left[(m_w - \pi_w)^2 + \pi_\xi^2\right] + (\pi_x - m_x)\, x_t + \pi_u u_t \\ &+ \left\{(\pi_\sigma - m_\sigma) - \frac{1}{2}\left[(m_\eta - \pi_\eta)^2 + (m_\epsilon - \pi_\epsilon)^2\right]\right\}\sigma_t^2. \end{aligned} \tag{22}$$

Given that $\pi_u < 0$, it is clear that a positive monetary policy shock decreases the Euler equation rate. On the other hand, the positive values of i_c and i_π imply an increase in the money market rate in response to the same shock. Thus the model is not free from this inconsistency problem. If we address this issue by calibrating a positive π_u, the model will not be able to account for some stylized facts of the term structure and inflation. For example, $\pi_u > 0$ leads to a counterfactually downward-sloping nominal yield curve and also erroneously implies that an aggressive disinflation policy amplifies the volatility of inflation.

5 Conclusion

Because inflation is not structural in most consumption-based asset pricing models, it is difficult to explore how inflation drivers affect the dynamics of nominal yields. To address this issue, I augment a baseline long-run risk model with a monetary policy rule so that the equilibrium process for inflation is endogenously determined. Calibration with postwar U.S. data suggests that, given the aggressive monetary policy responses to inflation and economic growth, the model captures the negative correlation of inflation and consumption growth in the data. This implies positive risk premia carried by nominal bonds and the average upward-sloping nominal term structure of interest rates. The decomposition of nominal SDF also shows that the risk premia implied by the model with endogenous inflation are much closer to the empirical estimates than those implied by the model with exogenous inflation. However, calibration exercises for three historical periods indicate some discrepancies between the model and the data. The model implies that a passive monetary policy is associated with volatile nominal yields and a large term spread in the era of Chairmen Volcker and Greenspan, while many studies suggest that the conduct of monetary policy essentially shows a stronger stance on price stability during that period. Meanwhile, the model requires the policy rule coefficients on inflation and economic growth to exceed unity to account for the positive slope of the term structure. Unfortunately, it seems that this condition lacks empirical support. Last but not least, the model is not free from the fundamental problem that the Euler equation and money market rates are not consistent.

The model developed in this paper does not allow any feedback from inflation to consumption growth. In some recent studies, a production economy with investment wedges or sticky prices can endogenously generate long-run risk. This framework is able to incorporate complicated interplays among inflation and other real variables. For example, Gavazzoni (2012) shows that long-run risk arises in a New Keynesian model with recursive preference and monetary policy inertia. This approach addresses the non-neutrality of inflation, but the calibration in Gavazzoni (2012) implies a counterfactually downward-sloping nominal term structure. On the other hand, the inconsistency between the Euler equation and money market rates also attracts more attention. Collard and Dellas (2012) show that a preference with non-separability between leisure and consumption partially improves the model fit. However, this strategy does not eradicate the inconsistency problem. The solutions to these problems are left for future research.

6 Appendix

6.1 Model Solution

To solve for the model, first plug equation (9) and (10) into (2). The real SDF can be expressed as follows:

$$m_{t+1} = m_0 + m_x x_t + m_\sigma \sigma_t^2 + m_\eta \sigma_t \eta_{t+1} + m_\epsilon \sigma_t \epsilon_{t+1} + m_w w_{t+1}, \tag{23}$$

$$m_x = -\frac{1}{\psi}, \tag{24}$$

$$m_\sigma = (1-\theta) A_2 (1 - \kappa_1 \upsilon_1), \tag{25}$$

$$m_\eta = -\gamma, \tag{26}$$

$$m_\epsilon = -(1-\theta)\kappa_1 A_1 \varphi_e, \tag{27}$$

$$m_w = -(1-\theta)\kappa_1 A_2 \sigma_w, \tag{28}$$

$$m_0 = \theta \ln \delta - \gamma\mu - (1-\theta)\kappa_0 + (1-\theta)(1-\kappa_1)A_0 - (1-\theta)\kappa_1 A_2 (1-\upsilon_1)\sigma^2, \tag{29}$$

where κ_0, κ_1, A_0, A_1 and A_2 are the same as those in Bansal and Yaron (2004):

$$\kappa_1 = \frac{\exp(\overline{z})}{1 + \exp(\overline{z})}, \tag{30}$$

$$\kappa_0 = \ln\left[1 + \exp(\overline{z})\right] - \kappa_1 \overline{z}, \tag{31}$$

$$A_1 = \frac{1 - 1/\psi}{1 - \kappa_1 \rho}, \tag{32}$$

$$A_2 = \frac{\left[(1-\gamma)^2 + (\theta A_1 \kappa_1 \varphi_e)^2\right]}{2\theta(1-\kappa_1 \upsilon_1)}, \tag{33}$$

$$A_0 = \frac{1}{1-\kappa_1}\left[\ln \delta + \left(1 - \frac{1}{\psi}\right)\mu + \kappa_0 + \kappa_1 A_2 (1-\upsilon_1)\sigma^2 + \frac{1}{2}\theta(\kappa_1 A_2 \sigma_w)^2\right]. \tag{34}$$

Plug (23) and the conjectured process for inflation (12) or (13) gives the expression of $m_{t+1} - \pi_{t+1}$, which is the nominal SDF. Thus the equilibrium process for inflation is solved by plugging nominal SDF into (11). For Model F (forward-looking policy rule), the associated coefficients are

$$\pi_x = \frac{m_x + i_c}{\rho(1 - i_\pi)}, \tag{35}$$

$$\pi_\sigma = \frac{1}{\upsilon_1 (1 - i_\pi)}\left\{m_\sigma + \frac{1}{2}\left[m_\eta^2 + (m_\epsilon - \pi_x \varphi_e)^2\right]\right\}, \tag{36}$$

$$\pi_u = \frac{1}{\phi_u(1-i_\pi)}, \tag{37}$$

$$\pi_0 = \frac{1}{1-i_\pi}\left[m_0 - (1-i_\pi)(1-\upsilon_1)\pi_\sigma\sigma^2 + \frac{1}{2}(m_w - \pi_\sigma\sigma_w)^2 - \pi_u^2\sigma_\xi^2 + i_0 + i_c\mu\right]. \tag{38}$$

For Model C (contemporaneous rule), the associated coefficients are

$$\pi_x = \frac{i_c + \rho m_x}{\rho - i_\pi}, \tag{39}$$

$$\pi_\sigma = \frac{1}{\upsilon_1 - i_\pi}\left\{m_\sigma + \frac{1}{2}\upsilon_1\left[(m_\eta - \pi_\eta)^2 + (m_\epsilon - \pi_\epsilon)^2\right]\right\}, \tag{40}$$

$$\pi_u = \frac{\phi_u}{\phi_u - i_\pi}, \tag{41}$$

$$\pi_\eta = -\frac{i_c}{i_\pi}, \tag{42}$$

$$\pi_\epsilon = \frac{\varphi_e\left(\pi_x - m_x\right)}{i_\pi}, \tag{43}$$

$$\pi_w = \frac{1}{i_\pi}\left\{\pi_\sigma - m_\sigma - \frac{1}{2}\sigma_w\left[(m_\eta - \pi_\eta)^2 + (m_\epsilon - \pi_\epsilon)^2\right]\right\}, \tag{44}$$

$$\pi_\xi = \frac{\sigma_\xi\left(\pi_u - 1\right)}{i_\pi}, \tag{45}$$

$$\begin{aligned}\pi_0 \;&=\; \frac{1}{1-i_\pi}\left\{m_0 + i_0 + i_c\mu + \frac{1}{2}\left[(m_w - \pi_w)^2 + \pi_\xi^2\right]\right\} \\ &\quad + \frac{\sigma^2(1-\upsilon_1)}{1-i_\pi}\left[m_\sigma - \pi_\sigma + \frac{1}{2}\left(m_\eta - \pi_\eta\right)^2 + \frac{1}{2}\left(m_\epsilon - \pi_\epsilon\right)^2\right],\end{aligned} \tag{46}$$

To solve for equilibrium nominal bond prices, plug the expression of nominal SDF and (16) into (15). For Model F, the coefficients are

$$\widetilde{B}_x^n = m_x - \rho\left(\pi_x - \widetilde{B}_x^{n-1}\right), \tag{47}$$

$$\widetilde{B}_\sigma^n = m_\sigma - \upsilon_1\left(\pi_\sigma - \widetilde{B}_\sigma^{n-1}\right) + \frac{1}{2}\left\{m_\eta^2 + \left[m_\epsilon - \varphi_e\left(\pi_x - \widetilde{B}_x^{n-1}\right)\right]^2\right\}, \tag{48}$$

$$\widetilde{B}_u^n = -\phi_u\left(\pi_u - \widetilde{B}_u^{n-1}\right), \tag{49}$$

$$\begin{aligned}\widetilde{B}_0^n \;&=\; m_0 - \left(\pi_0 - \widetilde{B}_0^{n-1}\right) - \left(\pi_\sigma - \widetilde{B}_\sigma^{n-1}\right)(1-\upsilon_1)\sigma^2 \\ &\quad + \frac{1}{2}\left[m_w - \sigma_w\left(\pi_\sigma - \widetilde{B}_\sigma^{n-1}\right)\right]^2 + \frac{1}{2}\sigma_\xi^2\left(\pi_u - \widetilde{B}_u^{n-1}\right)^2.\end{aligned} \tag{50}$$

For Model C, the coefficients are

$$\widetilde{B}_x^n = m_x - \pi_x + \rho \widetilde{B}_x^{n-1}, \tag{51}$$

$$\widetilde{B}_\sigma^n = m_\sigma - \pi_\sigma + \upsilon_1 \widetilde{B}_\sigma^{n-1} + \frac{1}{2}\left[(m_\eta - \pi_\eta)^2 + \left(m_\epsilon - \pi_\epsilon + \varphi_e \widetilde{B}_x^{n-1}\right)^2\right], \tag{52}$$

$$\widetilde{B}_u^n = -\pi_u + \phi_u \widetilde{B}_u^{n-1}, \tag{53}$$

$$\begin{aligned} \widetilde{B}_0^n &= m_0 - \pi_0 + \widetilde{B}_0^{n-1} + \widetilde{B}_\sigma^{n-1}(1-\upsilon_1)\sigma^2 \\ &+ \frac{1}{2}\left[\left(m_w - \pi_w + \sigma_w \widetilde{B}_\sigma^{n-1}\right)^2 + \left(\sigma_\xi \widetilde{B}_u^{n-1} - \pi_\xi\right)^2\right]. \end{aligned} \tag{54}$$

The associated nominal yields can be derived using (17).

6.2 Decompose the SDF

According to Alvarez and Jermann (2005), the transitory component of the nominal SDF can be obtained by

$$\frac{\widetilde{N}_{t+1}^T}{\widetilde{N}_t^T} = \lim_{n \longrightarrow \infty} \widetilde{\beta} \exp\left(\widetilde{p}_t^n - \widetilde{p}_{t+1}^{n-1}\right), \tag{55}$$

where $\widetilde{\beta}$ is a number. For Model F, we can infer that

$$\widetilde{B}_x^\infty = \frac{m_x - \rho \pi_x}{1-\rho}, \tag{56}$$

$$\widetilde{B}_u^\infty = \frac{-\phi_u \pi_u}{1-\phi_u}, \tag{57}$$

$$\widetilde{B}_\sigma^\infty = \frac{1}{1-\upsilon_1}\left\{m_\sigma - \upsilon_1 \pi_\sigma + \frac{1}{2} m_\eta^2 + \frac{1}{2}\left[m_\epsilon - \varphi_e\left(\pi_x - \widetilde{B}_x^\infty\right)\right]^2\right\}. \tag{58}$$

Thus the transitory component of the nominal SDF becomes

$$\begin{aligned} \frac{\widetilde{N}_{t+1}^T}{\widetilde{N}_t^T} &= \widetilde{\beta} \exp\{(1-\rho)\, \widetilde{B}_x^\infty x_t + (1-\upsilon_1)\left(\sigma_t^2 - \sigma^2\right) \widetilde{B}_\sigma^\infty \\ &\quad + (1-\phi_u)\, \widetilde{B}_u^\infty u_t - \varphi_e \widetilde{B}_x^\infty \sigma_t \epsilon_{t+1} - \widetilde{B}_\sigma^\infty \sigma_w w_{t+1} - \widetilde{B}_u^\infty \sigma_\xi \xi_{t+1}\}. \end{aligned} \tag{59}$$

Since the limit of $\widetilde{B}_0^{n-1} - \widetilde{B}_0^n$ is finite, the constant $\widetilde{\beta}$ is chosen to offset this term as n grows:

$$\widetilde{\beta} = \exp\left[\lim_{n \longrightarrow \infty}\left(\widetilde{B}_0^n - \widetilde{B}_0^{n-1}\right)\right]. \tag{60}$$

The martingale component is then defined as

$$\frac{\widetilde{N}_{t+1}^{P}}{\widetilde{N}_{t}^{P}} = \frac{\widetilde{N}_{t+1}}{\widetilde{N}_{t}} \left(\frac{\widetilde{N}_{t+1}^{T}}{\widetilde{N}_{t}^{T}} \right)^{-1}. \tag{61}$$

Some algebras lead to the following result:

$$\begin{aligned} \widetilde{m}_{t+1}^{P} &= -\frac{1}{2} \left\{ m_{\eta}^{2} + \left[m_{\epsilon} - \varphi_{e} \left(\pi_{x} - \widetilde{B}_{x}^{\infty} \right) \right]^{2} \right\} \sigma_{t}^{2} + m_{\eta} \sigma_{t} \eta_{t+1} \\ & + \left[m_{\epsilon} - \varphi_{e} \left(\pi_{x} - \widetilde{B}_{x}^{\infty} \right) \right] \sigma_{t} \epsilon_{t+1} + \left[m_{w} - \sigma_{w} \left(\pi_{\sigma} - \widetilde{B}_{\sigma}^{\infty} \right) \right] w_{t+1} \\ & + \left(\widetilde{B}_{u}^{\infty} - \pi_{u} \right) \sigma_{\xi} \xi_{t+1} - \frac{1}{2} \left[m_{w} - \sigma_{w} \left(\pi_{\sigma} - \widetilde{B}_{\sigma}^{\infty} \right) \right]^{2} \\ & - \frac{1}{2} \left[\sigma_{\xi} \left(\pi_{u} - \widetilde{B}_{u}^{\infty} \right) \right]^{2}. \end{aligned} \tag{62}$$

Thus the ratio of the conditional variance of the martingale component to the conditional variance of the whole SDF is

$$\widetilde{\omega}_{t} = \frac{\left\{ m_{\eta}^{2} + \left[m_{\epsilon} - \varphi_{e} \left(\pi_{x} - \widetilde{B}_{x}^{\infty} \right) \right]^{2} \right\} \sigma_{t}^{2} + \left[m_{w} - \sigma_{w} \left(\pi_{\sigma} - \widetilde{B}_{\sigma}^{\infty} \right) \right]^{2} + \left[\sigma_{\xi} \left(\pi_{u} - \widetilde{B}_{u}^{\infty} \right) \right]^{2}}{\left[m_{\eta}^{2} + \left(m_{\epsilon} - \varphi_{e} \pi_{x} \right)^{2} \right] \sigma_{t}^{2} + \left(m_{w} - \sigma_{w} \pi_{\sigma} \right)^{2} + \sigma_{\xi}^{2} \pi_{u}^{2}}. \tag{63}$$

For Model C, the same procedure is implemented and the associated conditional variance ratio is:

$$\widetilde{\omega}_{t} = \frac{\left[\left(m_{\eta} - \pi_{\eta} \right)^{2} + \left(m_{\epsilon} - \pi_{\epsilon} + \varphi_{e} \widetilde{B}_{x}^{\infty} \right)^{2} \right] \sigma_{t}^{2} + \left(m_{w} - \pi_{w} + \sigma_{w} \widetilde{B}_{\sigma}^{\infty} \right)^{2} + \left(\sigma_{\xi} \widetilde{B}_{u}^{\infty} - \pi_{\xi} \right)^{2}}{\left[\left(m_{\eta} - \pi_{\eta} \right)^{2} + \left(m_{\epsilon} - \pi_{\epsilon} \right)^{2} \right] \sigma_{t}^{2} + \left(m_{w} - \pi_{w} \right)^{2} + \pi_{\xi}^{2}}. \tag{64}$$

This completes the decomposition of the volatility of the SDF for both model specifications.

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Common Parameters		Forward-Looking Rule		Contemporaneous Rule	
μ	0.0049	i_0	-0.00518	i_0	-0.00532
ρ	0.9671	i_c	1.5	i_c	1.5
σ	0.0034	i_π	1.35	i_π	1.35
φ_e	0.2447	ϕ_u	0.99	ϕ_u	0.99
υ_1	0.9771	σ_ξ	2.5×10^{-4}	σ_ξ	2.5×10^{-4}
σ_w	0.02×10^{-5}				
δ	0.998				
γ	7.5				
ψ	1.5				

Table 1: Calibrated Parameter Values

	Data	Model F	Model C
$E(\Delta c)$	0.0049	0.0049	0.0049
$sd(\Delta c)$	0.0048	0.0047	0.0047
$AC1(\Delta c)$	0.4648	0.4648	0.4648
$E(\pi)$	0.0089	0.0076	0.0077
$sd(\pi)$	0.0064	0.0095	0.0096
$AC1(\pi)$	0.7957	0.9737	0.8340
$corr(\Delta c, \pi)$	-0.1073	-0.1762	-0.2593

Table 2: Moments of Consumption Growth and Inflation

This table reports some moments of consumption dynamics and inflation. "Model F" means model with forward-looking policy rule, "Model C" means model with contemporaneous policy rule, "$AC1$" means the first-order correlation coefficient and "$corr$" means linear correlation coefficient.

Model F

i_π	1.2	1.5	1.8	2.2
$E(\pi)$	0.0146	0.0051	0.0031	0.0020
$sd(\pi)$	0.0167	0.0067	0.0042	0.0028

Model C

i_π	1.2	1.5	1.8	2.2
$E(\pi)$	0.0145	0.0052	0.0032	0.0021
$sd(\pi)$	0.0155	0.0072	0.0049	0.0036

Table 3: The Stance of Monetary Policy and Equilibrium Inflation

This table shows the mean and the standard deviation of inflation with different values of i_π.

	1-year	2-year	3-year	4-year	5-year
Data	5.2392	5.4386	5.6156	5.7560	5.8545
	(3.0168)	(2.9755)	(2.8944)	(2.8473)	(2.7823)
Model F	5.2485	5.4253	5.5835	5.7252	5.8525
	(3.0773)	(2.9515)	(2.8356)	(2.7285)	(2.6294)
Model C	5.2523	5.4153	5.5614	5.6925	5.8103
	(2.8834)	(2.7684)	(2.6623)	(2.5642)	(2.4734)

Table 4: The Level and Volatility of Nominal Yields

The average levels of nominal yields are reported in the first line of each panel, and the associated volatilities are reported in the parentheses. All values are reported in per annum percentage points. "Data" comes from the CRSP Fama-Bliss file and only maturities of one through five years are available.

	1-year	2-year	3-year	4-year	5-year
Data	3.0168	2.9755	2.8944	2.8473	2.7823
$\phi_u = 0.99$	3.0773	2.9515	2.8356	2.7285	2.6294
$\phi_u = 0.95$	2.4376	2.2788	2.1342	2.0024	1.8820
$\phi_u = 0.50$	2.2737	2.1309	2.0026	1.8850	1.7769
$\phi_u = 0.10$	2.2685	2.1287	2.0011	1.8839	1.7759

Table 5: Volatilities of Nominal Yields and the Persistence of Monetary Policy Shock: Model F

The table exhibits the volatilities of nominal yields with different persistence of monetary policy shock implied by the model. The monetary policy is described by a forward-looking rule. All values are reported in per annum percentage points.

	1-year	2-year	3-year	4-year	5-year
Data	3.0168	2.9755	2.8944	2.8473	2.7823
$\phi_u = 0.99$	2.8834	2.7684	2.6623	2.5642	2.4734
$\phi_u = 0.95$	2.2022	2.0598	1.9300	1.8115	1.7032
$\phi_u = 0.50$	2.0734	1.9462	1.8294	1.7220	1.6232
$\phi_u = 0.10$	2.0732	1.9461	1.8293	1.7220	1.6231

Table 6: Volatilities of Nominal Yields and the Persistence of Monetary Policy Shock: Model C

The table exhibits the volatilities of nominal yields with different persistence of monetary policy shock implied by the model. The monetary policy is described by a contemporaneous rule. All values are reported in per annum percentage points.

	1-year	2-year	3-year	4-year	5-year
Data	0.35	0.55	0.73	0.87	0.97
BS	0.33	0.93	1.59	2.27	2.97
Model F	0.37	0.69	0.97	1.22	1.43
Model C	0.34	0.64	0.91	1.14	1.34

Table 7: Nominal Bond Risk Premium

The table reports the average nominal bond risk premium per annum. BS means Bansal and Shaliastovich (2008), calculated by Koijen et al. (2010).

	1970Q1 − 1979Q2			1979Q3 − 2005Q4			1987Q3 − 2005Q4		
	Data	Model		Data	Model		Data	Model	
		Case 1	Case 2		Case 1	Case 2		Case 1	Case 2
i_c		1.10	1.42		1.80	1.80		2.20	1.50
i_π		1.25	1.60		1.20	1.35		1.10	1.40
$E(\Delta c)$	0.0059			0.0047			0.0045		
$sd(\Delta c)$	0.0047			0.0040			0.0033		
$AC1(\Delta c)$	0.4767			0.3633			0.2826		
$E(\pi)$	0.0161	0.0105	0.0107	0.0094	0.0100	0.0105	0.0076	0.0059	0.0071
$sd(\pi)$	0.0060	0.0093	0.0052	0.0057	0.0168	0.0096	0.0032	0.0337	0.0059
$AC1(\pi)$	0.7660	0.9806	0.9745	0.8055	0.9736	0.9736	0.5281	0.9736	0.9802
$E(y_t^4)$	6.6545	6.6492	6.6566	6.4850	6.4899	6.4821	4.9386	4.9770	4.9377
$E(y_t^{20})$	7.0672	7.0302	6.8841	7.3196	7.3231	6.9677	5.8432	5.6235	5.0830
$sd(y_t^4)$	1.4644	3.3117	1.4749	3.3066	6.1838	3.3070	2.1160	13.0029	2.0976
$sd(y_t^{20})$	0.9317	2.9855	1.3067	2.9447	5.1991	2.7969	1.7523	10.8126	1.8734
$\widetilde{\omega}_t$		1.0454	0.8944		1.1025	0.7190		9.8078	1.0769
$E(rp_{t+1})$		0.0092	0.0051		0.0226	0.0118		0.0346	0.0036

Table 8: Experiments on Inflation Response: Model F

The table presents calibration exercises for Model F with some sub-sample periods. The moments of nominal yields are reported in per annum percentage points.

	1970Q1 − 1979Q2			1979Q3 − 2005Q4			1987Q3 − 2005Q4		
	Data	Model		Data	Model		Data	Model	
		Case 1	Case 2		Case 1	Case 2		Case 1	Case 2
i_c		1.10	1.40		1.85	1.90		2.20	1.60
i_π		1.21	1.58		1.20	1.35		1.10	1.40
$E(\Delta c)$	0.0059			0.0047			0.0045		
$sd(\Delta c)$	0.0047			0.0040			0.0033		
$AC1(\Delta c)$	0.4767			0.3633			0.2826		
$E(\pi)$	0.0161	0.0106	0.0108	0.0094	0.0101	0.0105	0.0076	0.0062	0.0071
$sd(\pi)$	0.0060	0.0107	0.0059	0.0057	0.0161	0.0104	0.0032	0.0275	0.0067
$AC1(\pi)$	0.7660	0.9061	0.7318	0.8055	0.8946	0.8087	0.5281	0.9403	0.7618
$E(y_t^4)$	6.6545	6.6566	6.6578	6.4850	6.4747	6.4719	4.9386	4.9438	4.9419
$E(y_t^{20})$	7.0672	7.0924	6.8722	7.3196	7.2580	6.9695	5.8432	5.6171	5.1017
$sd(y_t^4)$	1.4644	3.7312	1.4656	3.3066	5.6668	3.2869	2.1160	10.4034	2.0890
$sd(y_t^{20})$	0.9317	3.3640	1.3022	2.9447	4.7800	2.7676	1.7523	8.7909	1.8528
$\widetilde{\omega}_t$		1.0974	0.9011		1.0155	0.6833		6.8134	1.0178
$E(rp_{t+1})$		0.0105	0.0049		0.0206	0.0120		0.0267	0.0039

Table 9: Experiments on Inflation Response: Model C

The table presents calibration exercises for Model C with some sub-sample periods. The moments of nominal yields are reported in per annum percentage points.