# THE INCIDENCE OF THE VALUE ADDED TAX IN A NEOCLASSICAL GROWTH MODEL

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## I. INTRODUCTION

The purpose of this paper is to investigate the long-run incidence effects of the value added tax in a neoclassical growth model. Its main emphasis is on the specific question of how the functional distribution of income would be affected by the introduction of a value added tax into the economy.

The value added tax is the latest fiscal innovation of general scope. It is a new form of taxation which has been devised in recent decades. Despite the recent acceleration of public and professional interest in this tax, little systematic and rigorous analysis has been undertaken to assess its long-run incidence effects. Most public finance economists fail to take advantage of the insights offered by general equilibrium growth theory when evaluating the differential incidence of alternative tax systems. It is hoped that this study will contribute a basic framework of general use in answering the question on the long-run incidence of the value added tax.

The model is presented in Section II. In Section III we use comparative static analysis to examine the differential incidence of a change in tax structure from a personal and corporation income tax system to a value added tax. In Section IV we concluded findings of our analysis.

## II. THE MODEL

We will confine our analysis to a macroeconomic model in which the economy produces only one homogeneous good which may serve either as a consumption good or as an investment good. This good is produced through the use of two factors of production, labor (L) and capital (K). It is assumed that the production function for the economy as a whole possesses all neoclassical properties, among which are strict quasi-concavity and linear homogeneity. The perfect competition prevails in both product and factors markets. Both productive factors are fully employed at all time. It is also assumed that the economy is closed; that is, the economy involves no foreign trade or international capital movements. The government budget is continuously balanced, and is held constant as a proportion of the national income. Furthermore, the government expenditure consists of current consumption only, therefore the capital stock is entirely owned by the private sector. Government expenditures are neutral in the sense that the expenditures benefit capital and labor equally. All enterprises are organized in corporate form. Finally, it is assumed that technical progress is Harrod-neutral.

Under this set of assumptions, the theoretical framework of the economy used in this paper can be described by the following system of equations.

The production function is designated as

(1) $Y=F(K, L);\ K>0,\ L>0.$

where Y is national product, which is a function of the quantities of capital and labor used in production. Both capital and labor are necessary for production.

By definition, national income is the sum of labor's and capitalist's incomes. If no depreciation and no indirect taxes are assumed, then national income equals national product. Thus

(2) $Y=wL+rK$

where w, r are the competitive gross wage and rental rate respectively. This identity will be adopted for simplicity throughout the whole analysis in the present paper.

Under the above assumptions, it can be shown that per capita output is a function of the capital-labor ratio only; i.e.,

(3) $y=f(k)$

where $y=Y/L$, is per capita output and $k=\frac{K}{L}$, is the capital-labor ratio. Equation (3) is the intensive form of the production function.

In order to ensure the existence of a meaningful balanced growth path and a stable equilibrium, it is assumed that the Inada conditions must hold at all time.

Since perfect competition prevails in both factors markets, the gross wage and rental rate are equal to the marginal physical products of labor and capital respectively. Under the assumption of linear homogeneity, both gross wage and rental rate can be expressed as functions of the capital-labor ratio alone.

(4) $w=f(k)-kf'(k)$

(5) $r = f'(k)$

Suppose a personal and corporation income tax system is imposed upon the economy, then the net wage and rental rate become:

(6) $w' = (1-t_w)\ [f(k)-kf'(k)]$

(7) $r' = (1-t_r)\ f'(k)$

where $t_w$ and $t_r$ are the tax rates on wage and rental incomes respectively, and $w'$ and $r'$ are the net wage and rental rates, respectively, under the income tax system. We assume that $t_r > t_w$.

It has been assumee that the government budget is continuously balanced and is held constant as a proportion of the national income. Thus, the total tax revenue must be equal to $\theta Y$, the government budget. This leads to the following equation:

(8) $T' = \theta Y = t_w wL + t_r rK$

$= t_w[f(k)-kf'(k)]L + t_r f'(k)K$

where $T'$ is the total tax revenue under the income tax system, and $\theta$ is the constant proportion. Dividing both sides by L, epuation (8) becomes

(9) $\theta f(k) = t_w[f(k)-kf'(k)] + t_r kf'(k)$

$= t_w f(k) + (t_r - t_w)kf'(k)$

In a neoclassical growth model, labor is assumed to be supplied inelastically and to grow exponentially at a constant rate n. The supply of labor at any given time is

(10) $L = L_o e^{nt}$

where $L_o$ is the initial quantity of labor; and t is the continuous time variable. The labor growth rate is thus given as

(11) $\hat{L} = \dot{L}/L = n$

where L is the labor growth rate; and $\dot{L} = \frac{dL}{dt}$, the derivative of labor with respect to time.

It is assumed that a constant fraction, s, of after-tax national income is saved. Letting $s_w$ and $s_r$ denote, respectively, the savings propensities of labor and capital, we have

(12) $S = s_w w'L + s_r r'K$

$= s_w(1-t_w)[f(k)-kf'(k)]L + s_r(1-t_r)f'(k)K$

where S is the total saving of the economy. $s_w$ and $s_r$ are not necessarily identical.

Assuming that desired saving in the economy is always realized, then the equilibrium condition is

(13) $\dot{S}=\dot{I}$

were I is investment.

It is moreover assumed that capital does not depreciate, capital accumulation is thus given by the equation:

(14) $\dot{K}=I=S$ or $\dot{K}=S$

where $\dot{K}=\frac{dK}{dt}$, time rate of change of the capital stock.

From equations (12) and (14) we get

(15) $\dot{K}=s_w(1-t_w)[f(k)-kf'(k)]L+s_r(1-t_r)f'(k)K$

Dividing both sides by K, equation (15) becomes:

(16) $\hat{K}=s_w(1-t_w)\ [f(k)-kf'(k)]\frac{1}{k}+s_r(1-t_r)f'(k)$

where $\hat{K}=\dot{K}/K$, is the growth rate of the capital stock, or the rate of capital accumulation.

On the full employment steady-state equilibrium growth path, the labor growth rate will equal the growth rate of the capital stock with a fixed capital-labor ratio. That is

(17) $\hat{L}=\hat{K}$

From equation (11), (16) and (17) we get:

(18) $n=s_w(1-t_w)\ [f(k)-kf'(k)]\frac{1}{k}+s_r(1-t_r)f'(k)$

or $nk=s_w(1-t_w)[f(k)-kf'(k)]+s_r(1-t_r)kf'(k)$

Equations (6), (7), (9), and (18) comprise a complete neoclassical balanced growth model with an income tax system. The changes in the tax structure may alter the growth path. Since the returns to the factors of production are functions of the capital-labor ratio, the changes in the tax structure may have some distributional impacts on the economy.

In order to investigate the distributional impacts of a change in the tax strncture on the economy, we will make use of a value added tax as an alternative to the income tax system, and use Musgrave's "differential incidence" concept to evaluate these distributional impacts.

Suppose that the income tax system is repealed, instead we impose an income type of value added tax on the economy, then the net wage and rental rates become:

(19) $w''=(1-t_y)[f(k)-kf'(k)]$

(20) $r''=(1-t_y)f'(k)$

where $t_y$ is the value added tax rate. The total tax revenue is

(21) $T''=t_y wL+t_y rK=t_y Y$

where $T''$ is the total tax revenue under the value added tax.

The assumption of balanced budget leads to

(22) $T'=T''=\theta Y$

From equations (21) and (22) we get

(23) $t_y=\theta$

Equation (23) implies that the value added tax is a tax imposed proportionately on the national income.

From equations (8), (22) and (23), it is obvious that if $t_r=t_w$, then $t_r=t_w=t_y=\theta$. This is to say that if the tax rate on the rental imcome equals that on wage income and the government budget is held constant as a proportion of the national income, then the income tax system is exactly the same as a value added tax. In this model, therefore, the replacement of the income tax system by a value added tax is equivalent to the lowering of the corporate tax rate to the point where $t_1=t_w=\theta$. Thus, investigation of the incidence effects of substituting a value added tax for the income tax system is equivalent to examining the distributional impact of lowering the corporate tax rate to the level at which $t_r=t_w=\theta$.

## III. DETERMINATION OF THE INCIDENCE EFFECTS OF THE VALUE ADDED TAX

In Section II we have constructed a neoclassical growth model with taxation. It has been shown that if a uniform tax rate is imposed on wage income and on corporate income, and the total tax reuenue is held constant as a proportion of the national income, then the income tax system is equivalent to the value added tax. Therfore, determination of the differential incidence effects of the value added tax becomes a problem of determining the effects of lowering $t_r$ and raising $t_w$ while maintaining the government budget constant as a proportion of the national income.

This problem will be analyzed within the context of a neoclassical growth model. Since analysis of neoclassical growth theory has demonstrated that growth paths converge to a balanced growthpath, we are able to use a full employment growth model in which the equilibrium path of the economy is stable before and after changes in the tax structure. Our economy will thus move from one stable equilibrium growth path to another in which the capital-labor ratio will presumably be different while the labor growth rate will be the same. We will neglect the time path of the adjustment process to make mathematical manipulation more manageable. In other words, we will just compare the two economy at the same point in time after all equilibrating adjustments of tax changes have worked themselves out. This will allow us to employ a method of comparative static analysis.

The model is presented as follows:

(6) $w' = (1-t_w)[f(k) - kf'(k)]$

(7) $r' = (1-t_r)f'(k)$

(9) $\theta f(k) = t_w[f(k) - kf'(k)] + t_r kf'(k)$

(18) $nk = s_w(1-t_w)[f(k) - kf'(k)] + s_r(1-t_r)kf'(k)$

Differentiating equations (6), (7), (9) and (18) with respect to $t_r$ and using Cramer's rule to solve these simultaneous equations we obtain:

(24) $$\frac{dk}{dt_r} = \frac{(s_w - s_r)f'(k)}{D}$$

(25) $$\frac{dr'}{dt_r} = \frac{\frac{s_w}{k^2}f'(k)}{D}\{(t_r - \theta)kf'(k) - (1-t_w)[f(k) - kf'(k)]\}$$

(26) $$\frac{dw'}{dt_r} = \frac{\frac{1}{k}f'(k)}{D}\{s_w(1-t_w)[f(k) - kf'(k)] - s_r(t_r - \theta)kf'(k)\}$$

where $D = -s_r(1-t_r)f''(k) + \frac{s_w}{k}\{kf''(k) + \frac{f(k)}{k} - f'(k) + \theta f'(k) - \frac{\theta f(k)}{k} - t_r kf''(k)\}$

Based on equations (24), (25) and (26), we are able to evaluate qualitatively the effects of a change in $t_r$ on k, $r'$ and $w'$; or equivalently, we can determine the differential incidence of the value added tax under the assumptions postulated in Section II.

Equation (24) tells us the effect of a change in $t_r$ on the capital-labor ratio. In the Appendix, we proved that the denominator D is guaranteed positive by the requirement of stability. Thus stability is a sufficient condition for a positive denominator. We assume that the marginal propensity to save from rental income exceeds that from wage income; that is, $s_r > s_w$. Therefore, the numerator is negative. Then we have

(27) $$\frac{dk}{dt_r} < 0 \text{ if } s_w < s_r.$$

Statement (27) means that the equilibrium capital-labor ratio will be raised by lowering the tax rate on rental income whenever the propensity to save out of rental income is greater than the propensity to save out of wages. This is equivalent to say that, given the assumptions of $s_w < s_r$ and balanced government budget as a constant proportion of the national income, when we replace the income tax system by a value added tax, the capital-labor ratio will rise. This implies that, given the constant labor growth rate, this change in the tax structure will increase the time rate of change of the capital stock. Therefore, we conclude that the adoption of the value added tax will expedite capital formation.

Let us now examine equation (25), which tells us the effect of a change in $t_r$ on the net rate of return to capital.

Under the previous assumptions, at equilibrium $t_r = t_w = \theta$, thus the first term in the brackets vanishes. It has been assumed that both inputs are necessary for production, i.e., $K > 0$, $L > 0$. Using the property that $\frac{\partial F}{\partial L} > 0$ on the interior of $R_+^2$, we know that $[f(k) - kf'(k)] = \frac{\partial F}{\partial L} > 0$. Thus we get

$$(28) \quad \frac{dr'}{dt_r} < 0.$$

Therefore, lowering the tax rate on rental income to the level at which $t_r = t_w = \theta$ always raises the net rate of return to capital. As a result, capitalists benefit from a change in the tax structure from income taxes to the value added tax.

Finally, we will devote ourselves to investigating equation (26), which tells us the effect of a change in $t_r$ on the wage rate net of tax. This effect is the central issue of the argument about the differential incidence of the value added tax and the most difficult to identify, deserving our special attention.

Since at equilibrium $t_r = t_w = \theta$, the second term in the brackets vanishes. Furthermore, $[f(k) - kf'(k)] = \frac{\partial F}{\partial L} > 0$. Thus we have

$$(29) \quad \frac{dw'}{dt} > 0.$$

Statement (29) reveals that lowering the tax rate on the rental income will decrease the wage rate net of tax. Therefore, the adoption of the value added tax will hurt labor.

In order to give a satisfactory economic interpretation of statement (29), we will introduce two new concepts.

Remember that $w' = (1 - t_w)\,[f(k) - kf'(k)]$. By assumption and statement

(27), both $t_w$ and k are affected by $t_r$. Thus implicitly, we may write $w' = [t_w(t_r), k(t_r)]$. Therefore

$$(30)\quad \frac{dw'}{dt_r} = \frac{\partial w'}{\partial t_w}\frac{dt_w}{dt_r} + \frac{\partial w'}{\partial k}\frac{dk}{dt_r}$$

The first term on the right side of equation (30) will be called the "*tax-substitution effect*", indicating the effect of a change in $t_r$ on $w'$ through a change in $t_w$. The second term will be designated the "*growth effect*", telling us what effect changing $t_r$ will have on the net wage through a change in the capital-labor ratio.

Since it is assumed that the total tax revenue is held constant as a proportion of the national income, we have to raise $t_w$ while lowering $t_r$. Therefore, based on equation (9) we can show that $\frac{dt_w}{dt_r} < 0$. In addition, it can easily be shown that $\frac{\partial w'}{\partial t_w} < 0$. Thus the "tax-substitution effect", $\frac{\partial w'}{\partial t_w}\frac{dt_w}{dt_r}$, is positive. The "growth effect", $\frac{\partial w'}{\partial k}\frac{dk}{dt_r}$, is negative, because $\frac{\partial w'}{\partial k} = -(1 - t_w)kf''(k) > 0$ but by statement (27) $\frac{dk}{dt_r} < 0$.

The total effect, $\frac{dw'}{dt_r}$, is the sum of these two effects. If the "tax-substitution effect" exceeds the "growth effect", then $\frac{dw'}{dt_r} > 0$; otherwise $\frac{dw'}{dt_r} \leq 0$. The statement (29) implies that the "tax-substitution effect" exceeds the "growth effect."

## IV. CONCLUSIONS AND POLICY IMPLICATIONS

We have investigated the differential incidence effects of substituting a value added tax for a personal and corporation income tax system. The analysis was conducted within the context of a neoclassical growth model. Our conclusions hinge critically on the assumptions concerning the structure of taxation, the nature of the economy, specification of the production function, and the role of government. Insofar as actual behavior diverges from behavior implied by the model, our conclusions must be amended.

Analytical results which we have derived in the previous section indicate that change in the tax structure from an income tax system to a value added tax will raise the capital-labor ratio, which in turn will increase the level of national product and the rate of return to capital. However, this tax change will decrease the net wage rate. We conclude, therefore, that substitution of a value added tax for an income tax system will increase the degree of income inequality in the long run. This adverse distributional impact was an important factor underlying the Richardson Committee's unfavorable report on the value added tax.

Though evaluation of incidence effects is important for any meaningful evaluation of a tax structure, some other effects may be of paramount importance for the decision whether to replace the income tax system by a value added tax. The weight attached to each effect will depend upon the policy goals which the government is trying to achieve. Different policy goals will result in different optimal policies. Therefore, it is advisable to take into account policy goals in seeking a justification for the adoption of a value added tax.

Suppose that the government policy target is to accelerate economic growth with taxation as a policy instrument, while planning to keep its revenue and expenditure at a constant proportion of national product. Though a given volume of the revenue can be raised with various forms of taxation, differences in the tax structure bring about different impacts on the subsequent course of economic growth.

It is shown in this paper that the adoption of a value added tax will expedite capital formation, which in turn accelerate economic growth. In this case, therefore, the value added tax is more efficacious than the income tax system for attaining the policy goal of accelerating economic growth. A positive effect of the value added tax on capital formation may be attractive to the developing countries in which the capital needed for economic development is scarce. They might expedite economic growth at the expense of labor's welfare. However, for the developed countries such as the United States, these effects offer no particular advantage over other forms of taxation, therefore, the proponents of value added taxation will have to seek their justifications elsewhere.

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## APPENDIX

### Determination of the Sign of D

Here we will determine the sign of D, the denominator of equations (24), (25) and (26).

As we have indicated in Section II, on the full employment steady-state equilibrium path the labor growth rate will equal the growth rate of the capital stock with a fixed capital-labor ratio. That is, in equilibrium we must have $\hat{K}=n$ at $k=\bar{k}$.

Define $k=\frac{K}{L}$ and $\dot{k}=\frac{dk}{dt}$, then

$$\dot{k}=\frac{dk}{dt}=\frac{d}{dt}\left(\frac{K}{L}\right)=\frac{L\dot{K}-K\dot{L}}{L^2}=\frac{\dot{K}}{L}-\frac{K}{L}\frac{\dot{L}}{L}$$

$$=\frac{K}{L}\frac{\dot{K}}{K}-\frac{K}{L}\frac{\dot{L}}{L}=\frac{K}{L}\left(\frac{\dot{K}}{K}-\frac{\dot{L}}{L}\right)=k(\hat{K}-n).$$

Obviously, $\dot{k}\gtreqless 0$ as $\hat{K}\gtreqless n$.

Stability requires that

$$\dot{k}>0 \text{ for } k<\bar{k}$$
$$\dot{k}=0 \text{ for } k=\bar{k}$$
$$\dot{k}<0 \text{ for } k>\bar{k}$$

By the above relation, this can be written as:

$$(\hat{K}-n)>0 \text{ for } k<\bar{k}$$
$$(\hat{K}-n)=0 \text{ for } k=\bar{k}$$
$$(\hat{K}-n)<0 \text{ for } k>\bar{k}$$

Since n is constant, stability requires that $\frac{d\hat{K}}{dk}<0$. This can be illustrated by the following diagram.

From equation (16) in Section II we know that

$$\hat{K}=s_w(1-t_w)[f(k)-kf'(k)]\frac{1}{k}+s_r(1-t_r)f'(k)$$

$$=\frac{s_w}{k}[f(k)-kf'(k)]-\frac{s_w}{k}t_w[f(k)-kf'(k)]+s_r(1-t_r)f'(k)$$

$$=\frac{s_w}{k}[(fk)-kf'(k)]-\frac{s_w}{k}[\theta f(k)-t_r kf'(k)]+s_r(1-t_r)f'(k)$$

[By equation (9) in Section II].

Thus $\frac{d\hat{K}}{dk}=\frac{s_w k[f'(k)-f'(k)-kf''(k)]-s_w[f(k)-kf'(k)]}{k^2}$

$$-\frac{s_w k[\theta f'(k)-t_r f'(k)-t_r kf''(k)]-s_w[\theta f(k)-t_r kf'(k)]}{k^2}$$

$$+s_r(1-t_r)f''(k)$$

$$=-s_w f''(k)-\frac{s_w}{k^2}f(k)+\frac{s_w}{k}f'(k)-\frac{s_w}{k}\theta f'(k)+\frac{s_w}{k}t_r f'(k)$$

$$+s_w t_r f''(k)+\frac{s_w}{k^2}\theta f(k)-\frac{s_w}{k}t_r f'(k)+s_r(1-t_r)f''(k)$$

$$=s_r(1-t_r)f''(k)-\frac{s_w}{k}\{kf''(k)+\frac{f(k)}{k}-f'(k)+\theta f'(k)$$

$$-\frac{\theta f(k)}{k}-t_r kf''(k)\}$$

The denominator D of equations (24), (25) and (26) is

$$D=-s_r(1-t_r)f''(k)+\frac{s_w}{k}\{kf''(k)+\frac{f(k)}{k}-f'(k)+\theta f'(k)$$

$$-\frac{\theta f(k)}{k}-t_r kf''(k)\}$$

Thus $D=-\frac{d\hat{K}}{dk}$

Since $\frac{d\hat{K}}{dk}<0$, therefore $D>0$.