6 Appendix A

Following the work in Battocchio and Menoncin (2002), the inflation rates can be considered as a background risk affecting only the wealth growth rate, without altering the amount of wealth that can be invested. Actually, fund managers must invest the nominal fund, even though they are interested in maximizing the growth rate of the real fund. Then, we have to consider two different measures for the same fund. In particular, we call F_N the nominal fund and F the real fund.

To model how the real fund behaves, a commonly used approximation is the following: the growth rate of the real fund is given by the difference between the nominal fund growth rate and the consumption price growth rate. If we call P the level of consumption prices, then we can write:

$$\frac{dF}{F} \simeq \frac{dF_N}{F_N} - \frac{dP}{P}.$$

This is the so called Fisher equation but it gives a log-approximation of the exact relation which must hold between F_N and F. Actually, the true relation comes from an arbitrage hypothesis. Considering the inflation rate in this framework means considering a possible arbitrage between the financial and the real market. In fact, the nominal interest rate must compensate the opportunity cost of investing in financial assets. The investor who puts his money in the financial market misses the return he could have obtained from a real investment. If the investor buys today a real good and sells it after one period, he gains the inflation rate. If he buys today a financial asset and sells it after one period, he gains a nominal return. Now, we suppose that a particular market, called the real-financial market, exists. If this is the case, then the corresponding "real-financial" return must be such that the investor is indifferent between the two following opportunities:

1. investing one nominal monetary unit in the financial market and missing the return he could have obtained on the real market;

2. investing one nominal monetary unit in the real-financial market.

Accordingly, if we call ϕ the real-financial return, ϕ_N the nominal financial return, and π the inflation rate, then the true equation that must hold between the nominal and the real fund is as follows:

$$F_\phi = F_N \cdot \phi_N - F_N \cdot \pi,$$

which means that the return on the real wealth must equate the return on the nominal wealth reduced by the loss due to the increase in the price level. By definition it must be true that:

$$\phi \equiv \frac{dF}{F}, \quad \phi_N \equiv \frac{dF_N}{F_N}, \quad \pi \equiv \frac{dP}{P},$$

and so, after substituting in the arbitrage condition, we can write:

$$dF = dF_N - F_N \frac{dP}{P}.$$