7 Appendix B

Here we carry out the original computation for the modified process of L. Similarly, we need to compute the matrix product

$$[-M'(\Gamma\Gamma')^{-1}\Gamma\Omega + \beta_2(F_N\Phi' + \gamma L\Lambda')(I - \Gamma'(\Gamma\Gamma')^{-1}\Gamma)\Omega]'.$$

According to what has already been presented above, we can write:

$$[-M'(\Gamma\Gamma')^{-1}\Gamma\Omega + \beta_2(F_N\Phi' + \gamma L\Lambda')(I - \Gamma'(\Gamma\Gamma')^{-1}\Gamma)\Omega]' = \begin{bmatrix} w_1 \\ Lw_2 \end{bmatrix},$$

where w_1 and w_2 are given by

$$\begin{aligned} w_1 &\equiv \sigma_r \cdot \lambda_r \\ w_2 &\equiv \gamma\beta_2 L\sigma_L^2 - \beta_2 F_N \sigma_L \sigma_\pi - \frac{2\sigma_{L,m}\sigma_{S,r}\lambda_r}{\sigma_{S,m}} - \sigma_{L,m}\lambda_m + \sigma_{L,r}\lambda_r. \end{aligned}$$

Thus, the modified differential of the state variables $\widetilde{z}_s$ can be written as

$$\begin{bmatrix} d\widetilde{r} \\ \frac{d\widetilde{L}}{\widetilde{L}} \end{bmatrix} = \begin{bmatrix} a(b-\widetilde{r}) - w_1 \\ \mu_L^i - w_2 \end{bmatrix} dt + \begin{bmatrix} \sigma_r & 0 & 0 \\ \sigma_{L,r} & \sigma_{L,m} & \sigma_L \end{bmatrix} \begin{bmatrix} dW^r \\ dW^m \\ dW^L \end{bmatrix}.$$

In particular, for $s \geq t$, the solution of the interest rate process is

$$\widetilde{r}(s) = \widetilde{r}(t)e^{a(t-s)} + \frac{ab - w_1}{a}(1 - e^{a(t-s)}) + \sigma_r e^{-as}\int_t^s e^{a\tau} dW^r(\tau).$$

But since there is no closed-form solution of the labor incomes process, we have to use the numerical method to simulate $\widetilde{L}(s)$.