Consumption Euler Equation: The Theoretical and Practical Roles of Higher-Order Moments

Abstract

The theme of this thesis seeks to explore the importance of higher-order moments in the consumption Euler equation, both theoretically and empirically. Applying log-linearized versions of Euler equations has been a dominant approach to obtaining sensible analytical solutions, and a popular choice of model specifications for estimation. The literature however by now has been no lack of conflicting empirical results that are attributed to the use of the specific version of Euler equations. Important yet natural questions whether the higher-order moments can be safely ignored, or whether higher-order approximations offer explanations to the stylized facts remain unanswered. Such inquires as in the thesis thus can improve our understanding toward consumer behaviors over prior studies based on the linear approximation.

1. What Do We Gain from Estimating Euler Equations with Higher-Order Approximations?

Despite the importance of estimating structural parameters governing consumption dynamics, such as the elasticity of intertemporal substitution, empirical attempts to unveil these parameters using a log-linearized version of the Euler equation have produced many puzzling results. Some studies show that the approximation bias may well constitute a compelling explanation. Even so, the approximation technique continues to be useful and convenient in estimation of the parameters, because noisy consumption data renders a full-fledged GMM estimation unreliable. Motivated by its potential success in reducing the bias, we investigate the economic significance and empirical relevance of higher-order approximations to the Euler equation with simulation methodology. The higher-order approximations suggest a linear relationship between expected consumption growth and its higher-order moments. Our simulation results clearly reveal that the approximation

bias can be significantly reduced when the higher-order moments are introduced into estimation, but at the cost of efficiency loss. It therefore documents a clear tradeoff between approximation bias reduction and efficiency loss in the consumption growth regression when higher-order approximations to the Euler equation is considered. A question of immediate practical interest arises "How many higher-order terms are needed?" The second part of our Monte-Carlo studies then deals with this issue. We judge whether a particular consumption moment should be included in the regression by the criterion of mean squared errors (MSE) that accounts for a trade-off between estimation bias and efficiency loss. The included moments leading to smaller MSE are regarded as ones to be needed. We also investigate the usefulness of the model and/or moment selection criteria in providing guidance in selecting the approximation order. We find that improvements over the second-order approximated Euler equation can always be achieved simply by allowing for the higher-order moments in the consumption regression, with the approximation order selected by these criteria.

2. Uncovering Preference Parameters with the Utilization of Relations between Higher-Order Consumption Moments

Our previous attempt to deliver more desirable estimation performance with higher-order approximations to the consumption Euler equation reveals that the approximation bias can be significantly reduced when the higher-order moments are introduced into estimation, but at the cost of efficiency loss. The latter results from the difficulty in identifying independent variation in the higher-order moments by sets of linear instruments used to identify that in variability in consumption growth, mainly consisting of individual-specific characteristics. Thus, one major challenge in the study is how to obtain quality instruments that are capable of doing so. With the numerical analysis technique, we first establish the nonlinear equilibrium relation between consumption risk and higher-order consumption moments. This nonlinear relation is then utilized to form quality instruments that can better capture variations in higher-order moments. A novelty of this chapter lies in adopting a set of nonlinear instruments that is to cope with this issue.

They are very simple moment transformations of the characteristic-related instruments, thereby easy to obtain in practice. As expected, our simulations demonstrate that for a comparable amount of the bias corrected, applying the nonlinear instruments does entail an inclusion of fewer higher-order moments in estimation. A smaller simulated MSE that reveals the improvement over our previous estimation results can thus be achieved.

3. Precautionary Saving and Consumption with Borrowing Constraint

This last chapter offers a theoretical underpinning for the importance of the higher-order moments in a simple environment where economic agents have a quadratic-utility preference. The resulting Euler equation gives rise to a linear policy function in essence, or a random-walk consumption rule. The twist in our theory comes from a presence of borrowing constraint facing consumers. The analysis shows that the presence of the constraint induces precautionary motives for saving as responses from consumers to income uncertainties, even there has been no such motives inherent in consumers' preference. The corresponding value function now displays a convexity property that is virtually only associated with more general preferences than a quadratic utility. The analytical framework allows us to be able to characterize saving behaviors that are of precautionary motives, and their responses to changes in different moments of income process. As empirical implications, our analysis shed new light on the causes of excess sensitivity, the consequences of sample splitting between the rich and the poor, as well as the relevance of the higher-order moments to consumption dynamics, specifically skewness and kurtosis.

消費尤拉方程式：高階動差的理論與實證重要性

摘要

本論文共分三章, 全數圍繞在消費尤拉方程式中, 消費成長的高階動差在理論與實證上的重要性。分別說明如下:

本論文第一章討論消費高階動差在實證估計消費結構性參數之重要性。消費尤拉方程式是消費者極大化問題的一階條件, 而自 Hall (1978) 起, 估計消費結構參數如跨期替代彈性時, 也多是利用這個尤拉方程式所隱涵的消費動態關係, 進行估計。但是由於消費資料存在嚴重的衡量誤差問題, 實證上多將尤拉方程式進行對數線性化, 或是二階線性化後進行估計。

然而前述一、二階線性化, 固然處理了資料的衡量誤差問題, 卻也造成了參數估計上的近似誤差 (approximation bias)。其原因來自於線性化過程中所忽略的高階動差實爲內生, 而與迴歸式中的二階動差相關。這使得即便用工具變數進行估計, 仍然無法產生具有一致性的估計結果。這當中的原因在於足以解釋二階動差, 卻又與殘差項中的高階動差直交的良好 (valid) 的工具變數無法取得。

我們認爲在資料普遍存在衡量誤差的情況下, 線性化估計尤拉方程式不失爲一可行又易於操作的方法。於是我們嘗試在線性化的尤拉方程式中, 將高階動差引入, 並檢視這種高階近似是否能有效降低近似誤差。我們的模擬結果首先證實, 過去二階近似尤拉方程式的估計, 確實存在嚴重近似誤差。利用工具變數雖然可以少部份降低該誤差, 但由於高階動差的內生性質, 誤差仍然顯著。我們也發現, 將高階動差引入模型, 確實可以大幅降低近似誤差, 但是在偏誤降低的同時, 參數估計效率卻也隨之降低。

高階動差的引入, 除了降低近似偏誤外, 卻也必須付出估計效率降低的代價。我們因此並不建議無限制地放入高階動差。則近似階次選取, 乃爲攸關估計績效的重要因素。本章的第二部份, 即著眼於該最適近似階次選取。我們首先定義使參數估計均方誤 (mean squared error, MSE) 爲最小的近似階次, 爲最適近似階次。我們發現, 該最適階次與樣本大小、效用函數的彎曲程度都有直接的關係。

然而在實際進行估計時, 由於參數眞值無法得知, MSE準則自然無法作爲階次選取之依據。我們於是利用目前在模型與階次選取上經常被使用的一些準則進行階次選取, 並比較這些不同準則下參數估計的 MSE。我們發現利用這些準則, 確實可以使高階近似尤拉方程式得到 MSE 遠低於目前被普遍採用的二階近似的估計結果, 而爲估計消費結構參數時更佳的選擇。

本論文第二章延續前一章的模擬結果, 嘗試利用消費高階動差間的非線性關係, 進一步改善高階

近似消費尤拉方程式的估計表現。由第一章的研究結果, 我們發現高階近似估計確有助大幅降低近似誤差, 但這其中可能產生的估計效率喪失, 卻是輕乎不得的。這個效率喪失, 很大一部份來自於我們所使用的工具變數, 雖然可以有效掌握消費成長二階動差的變動, 但是當這同一組工具變數被用來解釋如偏態與峰態等這些更高階動差時, 預測力卻大幅滑落。這使待得當我們將這些配適度偏低的配適後高階動差, 放到迴歸式中進行估計時, 所能提供的額外情報也就相當有限。而所造成的共線性問題, 也自然使得估計效率大幅惡化。

於是在其他合格的工具變數相對有限的情況下, 我們利用高階動差間所存在的均衡關係, 將原來的工具變數進行非線性轉換, 以求得對高階動差的較佳配適。由於消費動差間之關係, 尚未見諸相關文獻。於是我們首先透過數值分析, 進一步釐清消費高階動差間之關係。這其中尤爲重要的是由消費二階動差所衡量的消費風險, 與更高階動差間之關係。因爲這些關係將爲我們轉換工具變數之依據。

我們發現與二階動差相一致地, 消費者對這些高階動差之預期, 都隨其財富水準的提高而減少。這隱涵消費風險與更高階動差間之正向關係。更進一步檢視消費風險與高階動差間之關係也發現, 二者間確實存在非線性之正向關係。而這也解釋了何以前一章線性的工具變數, 雖可適切捕捉消費風險, 但對高階動差的解釋力卻異常薄弱。

利用這些非線性關係, 我們將原始的工具變數進行非線性轉換後, 用以配適更高階動差。透過模擬分析, 我們證實了這些非線性工具變數, 確實大幅改善高階近似尤拉方程式的估計表現。除了仍保有與線性工具變數般的一些特性, 諸如隨樣本的增加, 最適近似階次也隨之增加之外, 相較於線性工具變數, 非線性工具變數可以在較低的近似階次下, 就使得估計偏誤大幅下降。在近似階次愈高估計效率愈低的情況下, 這自然大幅度地提高了估計效率。比較兩種工具變數估計結構數參數所產生的 MSE 也證實, 非線性工具變數確實有遠低於原始線性工具變數的 MSE 表現。

然而我們同時也發現, 利用非線性工具變數估計, 若未適當選擇近似階次, 效率喪失的速度, 可能更甚於線性工具變數時。這凸顯了選擇近似階次的重要性。於是我們同樣檢視了前述階次選擇準則在目前非線性工具變數環境下的適用性。而總結第一、二章的研究結果, 我們凸顯了高階動差的重要性, 確實助益重要消費結構參數估計。而利用過去尚未被討論過的高階動差間非線性關係, 更可大幅度改善估計績效。

本論文的最後一章, 則旨在理論上建立高階動差的重要性。我們在二次式的效用函數 (quadratic utility function) 設定下, 推導借貸限制下的最適消費決策。二次式的效用函數, 由於其邊際價值函數 (marginal value function) 爲一線性函數, 因此所隱涵的消費決策, 具有確定相等 (certainty equivalence) 的特性。這表示消費者只關心未來的期望消費水準, 二階以上的更高階動差, 都不影響

其消費決策。然而這種確定相等的特性, 將因爲借貸限制的存在而不復存在, 而高階動差的重要性也就因此凸顯。

我們證明, 確定相等特性的喪失, 其背後的理論原因在於, 借貸限制的存在, 使得二次式效用函數的邊際價值函數, 產生凸性。消費者因而因應未來的不確定性, 進行預防性儲蓄。透過分析解的求得, 我們也得以進一步分析更高階動差的對消費決策的理論性質。同時我們也引申理論推導的實證意涵, 其中較重要者諸如未受限消費者因預防性儲蓄行爲所引發的消費過度敏感性現象, 實證上樣本分割法的選取, 以及高階動差的引入模型。