Chapter 2

A real options model of FDI under exchange rate uncertainty

1. Introduction

Ever since the breakdown of the Bretton Woods system in the 1970s, the exchange rates of many countries have been fluctuating considerably over time. A large body of recent research deals with the implications of exchange rate uncertainty for the real economy. Regarding the effects of exchange rate uncertainty on foreign direct investment (FDI, hereafter), while many theoretical and empirical studies indicate that exchange rate volatility has had a significant effect on FDI movements, the impact of exchange rates is found to be heterogeneous across countries and types of investment, and varies over time.[15]

Previous theoretical studies demonstrate that exchange rate volatility affects FDI activity through two main channels: firms' attitude towards risk and the option value of investment flexibility. It has been suggested that, for a risk-averse firm, higher volatility lowers the certainty equivalent value of the investing firm.[16] Hence, FDI decreases as exchange rate volatility increases. By contrast, Itagaki (1981), Cushman (1985) and Goldberg and Kolstad (1995) illustrate the importance of considering the post-FDI changes in the exposure of a firm's profits to exchange rate risk. If the investing firm can choose to serve foreign markets via exports or FDI, then an

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[15] For instance, Amuedo-Dorantes and Pozo (2001), Bell and Campa (1997), Campa (1993, 1994), Darby et al. (1999), Crowley and Lee (2002), and Kiyota and Urata (2004) find that exchange rate uncertainty deters FDI activity. However, Cushman (1985), Goldberg and Kolstad (1995), and Pain and Van Welsum (2003) illustrate that exchange rate stimulates FDI flows.

[16] See, for instance, Wihlborg (1978).

increase in exchange rate volatility might lead the firm to substitute FDI for exports, since FDI activity reduces the exposure of its profits to exchange rate risk.

The studies mentioned above are based on the traditional investment theory, which assumes that an investment decision is to be taken now or never. They ignore the option of delaying an investment. Beginning in the 1980s a real options theory has been developed to analyze investment behavior. Under the assumptions of uncertainty and irreversible investment, the real options theory emphasizes the option value of the flexibility that a firm has in possibly delaying an investment decision in order to obtain more information about the future. Dixit (1989a,b) indicates that the waiting value increases as the uncertainty rises even for a risk-neutral firm. Hence, an increase in exchange rate uncertainty will defer the FDI activity of the firm. Using Dixit-Pindyck's (1994) model, however, Darby et al. (1999) illustrate that, for a risk-averse firm, the impact of exchange rate uncertainty on the timing of FDI is ambiguous.

A limitation of Dixit-Pindyck (1994) and Darby et al. (1999) is their treatment of firms' risk aversion. The risk aversion is incorporated into their model through a risk premium added to the private discount rate. This approach ignores an important feature in the traditional theory that allows the exposure of the investing firm's profits to exchange rate risk to vary with different types of FDI.[17] To fill the gap in the literature, the purpose of this chapter is theoretically to reexamine the relationship between exchange rate uncertainty and FDI.

This chapter develops an integrated framework of FDI under uncertainty in which a firm's attitude towards risk and the option value of investment flexibility are

[17] Although Erdal (2001) considers different motives of firms, the risk neutrality assumption in their model makes their results same as Dixit's analysis. The "exposure problem" also has not been discussed in their framework.

incorporated simultaneously. In this regard, Dixit-Pindyck's (1994) real options model is extended to consider possible changes in the post-FDI exposure to exchange rate risk. It is shown that the relationship between exchange rate uncertainty and FDI varies with the extent of the exposure to exchange rate risk, which is determined by investing motives.

The remainder of this chapter proceeds as follows. In section 2, a real options model of FDI under exchange rate uncertainty is developed, followed in the subsequent sections 3, 4 and 5 by a presentation of the effects of exchange rate volatility respectively on the FDI activity of export-substituting firms, market-seeking firms, and reverse-importing firms. Section 6 discusses the effects of other determinants of FDI. Brief concluding remarks are given in the final section.

2. The model

An orthodox investment theory, the net present value (NPV) theory, assumes that an investment decision is to be taken now or never. This theory ignores the option of delaying an investment. Given the inadequacy of such an orthodox investment theory, since the 1980s a real options theory has been developed to analyze investment behavior. The real options theory emphasizes three important characteristics of investment. First, investment is at least partially irreversible, implying that some investment costs cannot be completely recovered by selling capital. Second, investment decisions have to be made in an uncertain world. Third, it is possible to delay the investment decision in order to obtain more information about the future.

Investment spending is like a financial call option and its exercise price is the sunk costs involved in the investment. The return of executing the investment is the expected present discounted value of future profits. The call option's value is the value of the option for waiting and entering the market in the future. FDI decisions are made in a more uncertain environment than in a domestic investment especially if the firm faces a larger exchange rate risk. Furthermore, FDI generally incurs substantial sunk costs.[18] Hence, a real options approach is more relevant for analyzing the determinants of FDI timing.

In this section, Dixit-Pindyck's (1994) real options model is extended to reexamine the relationship between exchange rate uncertainty and FDI. To illustrate the importance of the diversity of motives in investigating the determinants of the timing of FDI, the model developed here focuses on three extreme cases according to motives of investing firms, namely, export-substituting FDI, market-seeking FDI, and reverse-importing FDI. Market-seeking FDI refers to the situation in which a domestic firm, originally not serving a foreign market via exports, chooses to set up a foreign subsidiary to produce and sell in a given foreign market. Thus, the motive of market-seeking FDI is to create a new market for its product. By contrast, export-substituting FDI refers to the situation in which an exporting firm, originally producing at its home country and serving a foreign market via exports, relocates its *whole* production abroad to serve the foreign market.[19] Furthermore, reverse-importing FDI refers to the situation in which a firm sets up a foreign subsidiary to produce and exports its output back to the home country. Thus, the motives for export-substituting FDI and reverse-importing FDI are to reduce the firm's production costs.

Suppose that the firm is a price taker and it produces a unit flow of output at fixed marginal cost per period. For simplicity, we assume that the variable costs

[18] Laar (2000) illustrates several types of sunk costs for executing a foreign investment project: irreversible orientation costs, such as the cost of the country specific literature and seminars during the decision making process; irreversible set-up costs, such as infrastructure investments; and recurrent fixed costs, such as the rent or depreciation of the building and machinery.

[19] The conclusion remains the same even if we allow the firm to substitutes FDI for exports partially.

comprise labor cost only and the input-output coefficient is fixed to be one. Therefore, the variable costs can be treated as the wage rate. Finally, it is assumed the exchange rate *R*, expressed in units of home currency per foreign currency, follows an exogenously geometric Brownian motion[20]

$$\frac{dR}{R} = \mu \cdot dt + \sigma \cdot dz . \tag{2-1}$$

Here, μ is the growth rate of the exchange rate; σ is the volatility of the exchange rate; *t* is the time path and *z* is a Wiener process.[21]

The objective of the firm is assumed to obtain maximum expected utility in terms of its home country's currency. In order to introduce the concept of risk aversion and associate it with the motives of the firm, the following mean-variance expected utility function[22] is adopted:

$$EU(\pi) = E[\pi] - \frac{1}{2} a_p \mathrm{Var}(\pi) \tag{2-2}$$

where EU (.) is the expected utility; π shows profits of the firm; a_p is Arrow-Pratt's absolute risk aversion coefficient; Var(.) is the variance. Note that $\mathrm{Var}(\pi) = f(.)R_0^2 e^{2\mu t}(e^{\sigma^2 t} - 1)$, where R_0 is initial value of exchange rate and $f(.)>0$ is a function of π. It is obvious that a rise in σ or μ will increase the variance of the profits.

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[20] The subscript *t* of *R* and π is suppressed in this section for simplicity.

[21] Investigating the real exchange rates of the major industrialized countries' currencies, Frankel and Rose (1995), and Sarno and Taylor (2002) indicate that it generally cannot reject the random walk hypothesis. Furthermore, De Grauwe (1996) and Sarantis (1999) show that real bilateral exchange rates for the major industrial countries exhibited very long cycles and substantial drifts. Therefore, the specification of the stochastic process of the exchange rate as a Brownian motion with drift seems consistent with previous empirical evidence.

[22] This functional form is used by Kawai (1984) and Qin (2000). Moreover, Cushman (1985) and Goldberg and Kolstad (1995) also use a similar setting to analyze the impact of exchange rate uncertainty on FDI.

3. The effects of exchange rate uncertainty on export-substituting FDI

In the case of export-substituting FDI, it is assumed that there are two possible entry modes for the domestic firm to serve a foreign country: Export versus FDI. Suppose that prior to implementing FDI, a firm produces goods at its home country and exports products to a foreign country. Hence, its profit flows in terms of home country's currency per period are

$$\pi^0 = P_f R - W_d \qquad \text{(2-3a)}$$

where π is the profit function; the superscript 0 represents the pre-FDI state; P_f is foreign market price in terms of foreign currency and W_d is domestic wage rate. After the firm invests to produce abroad and serve the foreign country from its foreign subsidiary[23], its profit flows become

$$\pi^1 = P_f R - W_f R \qquad \text{(2-3b)}$$

where superscript 1 represents the post-FDI state; W_f is foreign wage rate in terms of foreign currency. From Equations (2-3a) and (2-3b), it is obvious that if the firm substitutes FDI for exports, the exposure of its profits to exchange rate risk will be reduced due to the fact that $(P_f R - W_f R) < P_f R$.

From Equations (2-2) and (2-3), the change in expected utility, $\Delta EU(\pi)$, from substituting FDI for exports can be derived as follows:

$$\begin{aligned}\Delta EU\left(\pi\right) &= EU\left(\pi^1\right) - EU\left(\pi^0\right) \\ &= W_d - W_f E\left[R\right] - a\left[\left(P_f - W_f\right)^2 - P_f^2\right] e^{2\mu t}\left(e^{\sigma^2 t} - 1\right), \qquad \text{(2-4)}\end{aligned}$$

where $a = \frac{1}{2} a_p R_0^2$. From Equation (2-4), if the firm invests to produce at the foreign

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[23] We assume that the total output of the foreign subsidiary is sold in the foreign country. The reverse-import case is excluded in the model.

country and stays in the market forever[24], the change in expected present utility, ξ_E, becomes

$$\xi_E(R) = \int_0^{\infty} \Delta EU(\pi) e^{-\rho t} dt$$
$$= -\frac{W_f R}{\rho - \mu} + \frac{W_d}{\rho} - a\left[\left(P_f - W_f\right)^2 - P_f^2\right]\gamma, \qquad (2\text{-}5)$$

where ρ is the firm's discount rate; $\gamma = \sigma^2 / [(\rho - 2\mu - \sigma^2)(\rho - 2\mu)]$; subscript E represents an export-substituting firm. For the purpose of convergence, we assume $\rho > 2\mu + \sigma^2$. It is obvious from Equation (2-5) that a depreciation of the home country's currency (i.e., increase in R) causes a reduction in $\xi_E(R)$, thus deterring its firms' FDI activity.

The decision problem of the firm is to choose an optimal time to enter the foreign market. At time *t*, the firm can either produce in the host country after investing a lump sum *k* and gets the extra expected present utility as shown in Equation (2-5), or stays in the original state and keeps the right to invest in the next period. Hence, in each period the firm faces a binary decision problem as follows:

$$V(R) = \max\left\{\xi_E(R) - k, \frac{1}{1+\Delta t \rho} E\left[V(R')\middle|R\right]\right\} \qquad (2\text{-}6)$$

where V is the optimal expected net present value; Δt is the time interval; k is the sunk costs expressed in the home country's currency[25]; R' is the exchange rate in period *t*+1. The former term on the right-hand side, $\xi_E(R_t) - k$, is the net entry value, and the latter term, $(1+\Delta t \rho)^{-1} E[V(R') \mid R]$, is the value of the option to wait.

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[24] The following results are not changed if we allow the firm to have an option to exit after it enters the market.

[25] To simplify the following analysis, in this thesis the sunk costs *k* are expressed in the home country's currency instead of foreign currencies, in contrast with Dixit (1989a) and other studies. Nevertheless, our results are not changed if the sunk costs are expressed in foreign currencies. This is because the initial exchange rate is exogenous and thus does not influence the firm's value of the option to wait.

Since the utility function in this case is a decreasing function of R, there is a cutoff point, R_E, at which if $R<R_E$, then net entry value $\xi_E(R)-k$ is greater than the value of the option to wait. Thus, the firm's optimal decision is to carry out FDI.[26] Using value-matching and smooth-pasting conditions, we have[27]

$$R_E = \frac{\rho-\mu}{W_f}\frac{\alpha}{1+\alpha}\left[\frac{W_d}{\rho} + a\left[P_f^2 - \left(P_f - W_f\right)^2\right]\gamma - k\right] \tag{2-7}$$

where $\alpha = \sigma^{-2}[(\mu-0.5\sigma^2)+\sqrt{(\mu-0.5\sigma^2)^2+2\sigma^2\rho}]>0$.[28] The higher the value of R_E is, the higher the probability will be that R is smaller than R_E. Hence, the firm has higher incentive to invest earlier. To ensure that there is a possibility for a risk-neutral firm to undertake FDI, we assume that $W_d/\rho - k > 0$.

In the following, before we discuss the general case presented in Propositions 2-4 and 2-5, we first derive the results of two special cases which correspond to the specifications in previous studies. The first special case is an investment decision of a risk-averse firm that has to be taken now or never; that is, there is no option to delay the investment. The other special case is an investment decision of a risk-neutral firm with an option to delay the investment.

Proposition 2-1 In the case of export-substituting FDI, an increase in exchange rate volatility will stimulate FDI activity of a firm if the firm cannot delay its investment.

Proof. From Equation (2-5), it can be shown that

$$\frac{\partial \xi_E}{\partial \sigma} = a\left[P_f^2 - \left(P_f - W_f\right)^2\right]\frac{2\sigma}{(\rho-2\mu-\sigma^2)^2} > 0,$$

which implies that the firm has a higher incentive to substitute FDI for exports if the

[26] See Dixit and Pindyck (1994), p.128.
[27] See Appendix 2-1 for proof.
[28] See Dixit (1989b), p.626, for the proof of $\alpha > 0$.

exchange rate uncertainty rises. ■

The economic rationale behind Proposition 2-1 is straightforward. If the firm cannot delay its investment, then the risk attitude is the only channel through which exchange rate uncertainty affects FDI. Substituting FDI for exports reduces the firm's exposure to exchange rate risk, and this gain from risk reduction is larger if the exchange rate is more volatile. Consequently, an increase in exchange rate volatility stimulates the firm's FDI activity. This result is the same as that found in Goldberg and Kolstad (1995) and Cushman (1985).

Proposition 2-2 **A** risk-neutral export-substituting firm will delay its FDI activity when the exchange rate volatility rises; that is, $\partial R_E / \partial \sigma |_{a_p=0} < 0$.

Proof. From Equation (2-7), we have

$$\left.\frac{\partial R_L}{\partial \sigma}\right|_{a_p=0} = \frac{R_E}{\alpha(1+\alpha)}\frac{\partial \alpha}{\partial \sigma} < 0$$

where $\dfrac{\partial \alpha}{\partial \sigma} = -\dfrac{\alpha}{\sigma}[1 + \dfrac{2\mu + \sigma^2}{2\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}}] < 0$. ■

The economic intuition of Proposition 2-2 is as follows. If the firm is risk-neutral, then the option value of investment flexibility is the only channel through which exchange rate uncertainty affects FDI. An investment is like a call option whose value rises if the underlying uncertainty increases. Hence, facing an irreversible investment and uncertain future, a potential entrant has more incentive to delay its investment so as to get extra information.

Proposition 2-3 **A** risk-neutral export-substituting firm will delay its FDI activity under a rising exchange rate trend; that is, $\partial R_E / \partial \mu |_{a_p=0} < 0$.

Proof. From Equation (2-7), we have

$$\left.\frac{\partial R_E}{\partial \mu}\right|_{a_p=0} = \frac{R_E \cdot \psi}{(1+\alpha)(\rho-\mu)\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}} < 0,$$

where $\psi = \rho - \mu - (1+\alpha)\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2} < 0$.[29]

The reason for μ to be negatively related to the FDI activity of a risk-neutral firm is due to the fact that μ represents the expected future level of exchange rate. Moreover, a greater level of μ implies that the probability of future level of exchange rate being less than R_E is smaller. Hence, the risk-neutral firm will delay its investment as μ rises.

Lemma 2-1 $\partial^2 R_E / \partial\sigma\partial a_p > 0$ and $\partial^2 R_E / \partial\mu\partial a_p > 0$.

Proof. See Appendix 2-3.

Proposition 2-4 In the case of export-substituting FDI, the effect of exchange rate volatility on the timing of FDI is ambiguous. However, there exists a threshold in the degree of risk aversion $\tilde{a}$ such that this effect is positive (negative) if the firm's risk-aversion coefficient a_p is greater (smaller) than $\tilde{a}$.

Proof. From Equation (2-7), we have

$$\frac{\partial R_E}{\partial \sigma} = \Gamma_1 + a_p \Gamma_2 \qquad \text{(2-8a)}$$

where

[29] See Appendix 2-2 for the proof of $\psi < 0$.

$$\Gamma_1 = \frac{\rho-\mu}{W_f}\frac{1}{(1+\alpha)^2}\left[\frac{W_d}{\rho}-k\right]\frac{\partial\alpha}{\partial\sigma}$$

and

$$\Gamma_2 = \frac{R_0^2}{2}\frac{\rho-\mu}{W_f}\frac{\alpha}{1+\alpha}\left[P_f^2-\left(P_f-W_f\right)^2\right]\left[\frac{\gamma}{\alpha(1+\alpha)}\frac{\partial\alpha}{\partial\sigma}+\frac{2\sigma}{(\rho-2\mu-\sigma^2)^2}\right].$$

Given $\partial R_E/\partial\sigma\,|_{a_p=0}=\Gamma_1<0$ (Proposition 2-2) and $\partial^2 R_E/\partial\sigma\partial a_p=\Gamma_2>0$ (Lemma 2-1), since $\partial R_E/\partial\sigma$ is a linear function of a_p, there must exist a critical value, $\tilde{a}$, at which $\partial R_E/\partial\sigma>0$ if $a_p>\tilde{a}$, and $\partial R_E/\partial\sigma<0$ if $a_p<\tilde{a}$. ■

The economic intuition of Proposition 2-4 is as follows. As shown in Propositions 2-1 and 2-2, exchange rate volatility σ affects the FDI through two channels: the risk attitude of a firm and the option value of investment flexibility. These two channels have opposite effects on the FDI activity of an export-substituting firm. Therefore, the effect of exchange rate volatility on FDI is ambiguous. However, given the negative effect on FDI activity from the option value of investment flexibility, if the positive effect resulting from the risk aversion as well as the change in the exposure to exchange risk resulting from the firm's FDI becomes large (small) enough, the net effect will be positive (negative).

Proposition 2-5 In the case of export-substituting FDI, the effect of the exchange rate trend on the timing of FDI is ambiguous. However, there exists a threshold in the degree of risk aversion $\hat{a}$ such that this effect is positive (negative) if the firm's risk-aversion coefficient a_p is greater (smaller) than $\hat{a}$.

Proof. From Equation (2-7), we have

$$\frac{\partial R_E}{\partial \mu} = \Upsilon_1 + a_p \Upsilon_2 \tag{2-8b}$$

where

$$\Upsilon_1 = \frac{\psi\alpha}{(1+\alpha)^2 W_f \sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}} \left[\frac{W_d}{\rho} - k \right],$$

and

$$\Upsilon_2 = \frac{R_0^2}{2} \frac{(\rho-\mu)\alpha}{W_f(1+\alpha)} \left[P_f^2 - \left(P_f - W_f\right)^2 \right] \left[\frac{\psi\gamma}{(1+\alpha)(\rho-\mu)\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}} + \frac{\partial \gamma}{\partial \mu} \right],$$

and

$$\frac{\partial \gamma}{\partial \mu} = \frac{2\sigma^2(2\rho - 4\mu - \sigma^2)}{(\rho - 2\mu - \sigma^2)^2(\rho - 2\mu)^2} > 0 .$$

Given $\partial R_E / \partial \mu |_{a_p=0} = \Upsilon_1 < 0$ (Proposition 2-3) and $\partial^2 R_E / \partial \mu \partial a_p = \Upsilon_2 > 0$ (Lemma 2-1), since $\partial R_E / \partial \mu$ is a linear function of a_p, there must exist a critical value, $\hat{a}$, at which $\partial R_E / \partial \mu > 0$ if $a_p > \hat{a}$, and $\partial R_E / \partial \mu < 0$ if $a_p < \hat{a}$. ■

Regarding the effect of exchange rate trend μ, on the one hand, because μ represents the expected future level of exchange rate, an increase in μ decreases the probability of future level of exchange rate being smaller than R_E. Hence, the firm will delay its investment, as shown in Proposition 2-3. On the other hand, an increase in μ will raise the variance of a firm's profits as mentioned above, and thus stimulate the FDI activity of a risk-averse export-substituting firm. Therefore, the effect of exchange rate trend on FDI is also ambiguous. However, the latter effect is larger for a higher risk-averse firm, as shown in Lemma 2-1. Thus, when the degree of risk aversion exceeds a critical level, the latter positive effect will dominate the former negative effect, meaning an increasing trend in exchange rate will stimulate FDI activity of an export-substituting firm, and vice versa.

4. The effects of exchange rate uncertainty on market-seeking FDI

Since we assume a market-seeking firm has not served the foreign market via exports prior to undertaking FDI, the profits from exports are zero in state 0. The change in net present utility, ξ_M , from FDI can be shown as follows[30]:

$$\xi_M(R) = \frac{(P_f - W_f)R}{\rho - \mu} - a(P_f - W_f)^2 \gamma \tag{2-9}$$

where subscript *M* represents the market-seeking firms. It is obvious from Equation (2-9) that a depreciation of the home country's currency (i.e., an increase in R) will raise the value of $\xi_M(R)$, thus stimulating its firms' FDI activity.

The binary decision problem for the firm in each period is

$$V(R) = \max\left\{\xi_M(R) - k, \frac{1}{1+\Delta t\rho} E\left[V(R')|R\right]\right\} \tag{2-10}$$

There is an entry threshold rate R_H at which a potential entrant enters if $R>R_H$. In other words, the lower the value of R_H is, the higher the incentive will be for the firm to enter the market. Using value-matching and smooth-pasting conditions, we have

$$R_H = \frac{\rho - \mu}{P_f - W_f} \frac{\beta}{\beta - 1}\left[a(P_f - W_f)^2 \gamma + k\right] \tag{2-11}$$

where $\beta = \sigma^{-2}[-(\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2\rho}] > 1$.[31]

Proposition 2-6 In the case of market-seeking FDI, an increase in exchange rate volatility will delay the FDI activity of the firm; that is $\partial R_H / \partial \sigma > 0$.

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[30] The derivation of this equation is similar to the case of the export-substituting FDI.
[31] See Dixit (1989b), p.626, for the proof of $\beta > 1$

Proof. From Equation (2-11), we have

$$\frac{\partial R_H}{\partial \sigma} = \frac{-R_H}{\beta(\beta-1)}\frac{\partial \beta}{\partial \sigma} + \frac{\rho-\mu}{P_f - W_f}\frac{\beta a}{\beta-1}\left(P_f - W_f\right)^2 \frac{2\sigma}{(\rho-2\mu-\sigma^2)^2} > 0, \qquad \text{(2-12a)}$$

where $\frac{\partial \beta}{\partial \sigma} = \frac{\beta}{\sigma}[-1 + \frac{2\mu+\sigma^2}{2\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}}] < 0$, given the assumption

$\rho > 2\mu + \sigma^2$. ■

The economic intuition of Proposition 2-6 is as follows. In this case, FDI activity will make the firm's exposure to exchange rate risk increase. Thus, an increase in exchange rate volatility will reduce the expected utility gain from this activity for a risk-aversion firm. At the same time, an increase in exchange rate volatility will increase the option value of delaying the investment so as to deter the FDI activity further.

Proposition 2-7 A risk-neutral market-seeking firm will accelerate its FDI activity when the exchange rate trend rises, that is, $\partial R_H / \partial \mu |_{a_p=0} < 0$.

Proof. From Equation (2-11), it can be shown that

$$\left.\frac{\partial R_H}{\partial \mu}\right|_{a_p=0} = \frac{R_H \cdot \phi}{(\beta-1)(\rho-\mu)\sqrt{2\rho\sigma^2 + \left(\mu - 0.5\sigma^2\right)^2}} < 0$$

where $\phi = \rho - \mu - (\beta-1)\sqrt{2\rho\sigma^2 + (\mu-0.5\sigma^2)^2} < 0$.[32] ■

Similar to Proposition 2-3, the economic intuition of Proposition 2-7 is straightforward. An increase in μ raises the probability of future exchange rate level

[32] See Appendix 2-2 for the proof of $\phi < 0$.

to be larger than R_H, and thus stimulates the risk-neutral firm to invest earlier.

Lemma 2-2 $\partial^2 R_H / \partial\mu\partial a_p > 0$.

Proof. See Appendix 2-3.

Proposition 2-8 In the case of market-seeking FDI, the effect of exchange rate trend on the timing of FDI is ambiguous. However, there exists a threshold in the degree of risk aversion $\bar{a}$ such that this effect is negative (positive) if the firm's risk-aversion coefficient a_p is greater (smaller) than $\bar{a}$.

Proof. From Equation (2-11), we have

$$\frac{\partial R_H}{\partial\mu} = \Lambda_1 + \Lambda_2 a_p, \tag{2-12b}$$

where

$$\Lambda_1 = \frac{\phi\beta k}{(\beta-1)^2(P_f - W_f)\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}},$$

and

$$\Lambda_2 = \frac{R_0^2}{2}\frac{\rho-\mu}{P_f - W_f}\frac{\beta}{\beta-1}\left(P_f - W_f\right)^2\left[\frac{\phi\gamma}{(\beta-1)(\rho-\mu)\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)}} + \frac{\partial\gamma}{\partial\mu}\right].$$

Here $\partial R_H / \partial\mu$ is a linear function of a_p. In addition, $\partial R_H / \partial\mu |_{a_p=0} = \Lambda_1 < 0$ (Proposition 2-7) and $\partial^2 R_H / \partial\mu\partial a_p = \Lambda_2 > 0$ (Lemma 2-2). Thus, there must exist a trigger value $\bar{a}$ at which $\partial R_H / \partial\mu > (<) 0$ if $a_p > (<) \bar{a}$. ■

The reasoning regarding the effect of exchange rate trend in the market-seeking case is similar to what we have found in the export-substituting case. An increase in μ affects FDI through two channels: the probability of R being greater R_H and the variance of the firm's profits. The effect of an increase μ on FDI activity from the first channel is positive (Proposition 2-7), and the effect from the second channel is negative, its total effect is ambiguous. However, when the degree of risk aversion exceeds a trigger level, the second effect will dominate the first effect, thus an increasing trend in exchange rate will defer FDI activity of a market-seeking firm, and vice versa.

5. The effects of exchange rate uncertainty on reverse-importing FDI

Reverse-importing FDI refers to the situation in which a firm sets up a foreign subsidiary to produce and exports its output back to the home country.[33] It is assumed that the firm wholly exports output of its foreign subsidiary back to the home country. Thus, the profit flows after investing in the host country, π_R, per period can be expressed as

$$\pi_R(R) = P_d - W_f R .$$

where P_d is the domestic market price in domestic currency. The subscript of *R* refers to a reverse-importing firm hereafter. The change in net present utility, ξ_R, from FDI can be shown as follows[34]:

$$\xi_R(R) = \frac{P_d}{\rho} - \frac{W_f R}{\rho - \mu} - aW_f^2\gamma . \tag{2-13}$$

It is obvious from Equation (2-13) that a depreciation of the home country's currency (i.e., an increase in R) will reduce the value of $\xi_R(R)$, thus deterring its firms' FDI activity. Therefore, there is an entry threshold rate R_L at which a potential

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[33] This phenomenon is referred to as "reverse imports" in the literature. Liu and Lin (2002) find that the reverse imports of Taiwanese multinational firms in the electronics & electric appliances, metal products and textile industries account for more than 30% of total revenue in their foreign subsidiaries.
[34] The derivation of this equation is similar to the case of the export-substituting FDI.

entrant enters if $R < R_L$. In other words, the higher the value of R_L is, the higher the incentive will be for the firm to enter the market. The firm faces a binary decision problem each period as follows:

$$V(R) = \max\left\{\xi_R(R) - k, \frac{1}{1+\Delta t\rho}\mathrm{E}\left[V(R')\middle| R\right]\right\}. \quad (2\text{-}14)$$

Using value-matching and smooth-pasting conditions, we have[35]

$$R_L = \frac{\rho-\mu}{W_f}\frac{\alpha}{\alpha+1}\left(\frac{P_d}{\rho} - k - aW_f^2\gamma\right). \quad (2\text{-}15)$$

To ensure that there is a possibility for a reverse-importing firm to undertake FDI, we assume that $P_d/\rho - k - aW_f^2\gamma > 0$.

Proposition 2-9 In the case of reverse-importing FDI, an increase in exchange rate volatility will delay the FDI activity of the firm; that is $\partial R_L/\partial\sigma < 0$.

Proof. From Equation (2-15), we have

$$\frac{\partial R_L}{\partial\sigma} = \frac{R_R}{\alpha(1+\alpha)}\frac{\partial\alpha}{\partial\sigma} - \frac{\rho-\mu}{W_f}\frac{\alpha}{1+\alpha}aW_f^2\frac{2\sigma}{(\rho-2\mu-\sigma^2)^2} < 0, \quad (2\text{-}16a)$$

where $\frac{\partial\alpha}{\partial\sigma} = -\frac{\alpha}{\sigma}[1+\frac{2\mu+\sigma^2}{2\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}] < 0$. ■

The economic intuition of Proposition 2-9 is similar to that of Proposition 2-6. In this case, FDI activity will make the firm's exposure to exchange rate risk increase. Thus, an increase in exchange rate volatility will reduce the expected utility gain from this activity for a risk-averse firm. At the same time, an increase in exchange rate

[35] Since the derivation of this entry threshold rate is similar to that in the case of the export-substituting FDI, it is omitted.

volatility will increase the option value of delaying the investment so as to deter the FDI activity further.

Proposition 2-10 A reverse-importing firm will delay its FDI activity when the exchange rate trend rises; that is, $\partial R_L / \partial \mu < 0$.

Proof. From Equation (2-15), we have

$$\frac{\partial R_L}{\partial \mu} = \frac{R_L \cdot \psi}{(1+\alpha)(\rho-\mu)\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}} - \frac{\rho-\mu}{W_f}\frac{\alpha}{1+\alpha}aW_f^2\frac{\partial\gamma}{\partial\mu} < 0, \quad \text{(2-16b)}$$

where $\psi = \rho - \mu - (1+\alpha)\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2} < 0$ [36] and $\partial\gamma/\partial\mu > 0$ [37]. ■

In the case of reverse-importing firms, an increase in μ also affects FDI through two channels: the probability of R being lower than R_L and the variance of the firm's profits. The effect of an increase in μ on FDI activity from the first channel is negative, and the effect from the second channel is negative, thus its total effect is negative. Therefore, an increasing trend in exchange rate will deter FDI activity of a reverse-importing firm.

6. The effects of other determinants of FDI

Proposition 2-11 Sunk costs are negatively related to the FDI activity of export-substituting firms, market-seeking firms, and reverse-importing firms; that is, $\partial R_E/\partial k < 0$, $\partial R_H/\partial k > 0$, and $\partial R_L/\partial k < 0$.

Proof. The proofs are straightforward, and thus are omitted.

[36] See Appendix 2-1 for the proof of $\psi < 0$.

[37] See Proposition 2-5 for the proof of $\partial\gamma/\partial\mu > 0$.

The economic intuition of Proposition 2-11 is clear. The higher the entry costs are, the higher the revenues or the lower the variable costs will be that are requested to compensate the opportunity loss. Therefore, high sunk costs will deter the FDI activity of those firms with a view to either seeking new market or reducing production costs.

Proposition 2-12 The FDI activity of export-substituting firms is negatively related to foreign wage rate and positively related to domestic wage rate; that is, $\partial R_L / \partial W_f < 0$, $\partial R_L / \partial W_d > 0$.

Proof. From Equation (2-7), it can be shown that

$$\frac{\partial R_L}{\partial W_d} = \frac{\rho - \mu}{W_f} \frac{\alpha}{1+\alpha} \frac{1}{\rho} > 0,$$

and

$$\frac{\partial R_L}{\partial W_f} = -\frac{\rho - \mu}{{W_f}^2} \frac{\alpha}{1+\alpha} \left(\frac{W_d}{\rho} + a{W_f}^2 - k \right) < 0.$$

The second inequality holds due to the assumption of $W_d / \rho - k > 0$. ■

Proposition 2-13 The foreign wage rate is negatively related to the FDI activity of a risk-neutral market-seeking firm. However, the relationship between foreign wage rate and the FDI activity for a risk-averse market-seeking firm is ambiguous.

Proof. From Equation (2-11), we have

$$\frac{\partial R_H}{\partial W_f} = \frac{\rho - \mu}{(P_f - W_f)^2} \frac{\beta}{\beta - 1} \left[k - a(P_f - W_f)^2 \gamma \right].$$

It is obvious that the sign of $\partial R_H / \partial W_f$ is ambiguous, depending on the relative magnitude of risk aversion, a_p, and the sunk costs, k. However, if a_p=0, then

$\partial R_H / \partial W_f > 0$. ■

Proposition 2-14 The FDI activity of reverse-importing firms is negatively related to foreign wage rate; that is, $\partial R_L / \partial W_f < 0$.

Proof. From Equation (15), it can be shown that

$$\frac{\partial R_L}{\partial W_f} = -\frac{\rho-\mu}{W_f^2}\frac{\alpha}{1+\alpha}\left(\frac{P_d}{\rho} - k + aW_f^{\,2}\gamma\right) < 0 .$$

The inequality holds due to the assumption of $P_d/\rho - k - aW_f^2\gamma > 0$. ■

An increase in domestic wage will make the variable production costs in the foreign country relatively cheap, and thus cause export-substituting FDI to increase. Therefore, the relation between domestic wage and export-substituting FDI is positive. As for the reason why the effect of foreign wage on the FDI activity of export-substituting firms and reverse-importing firms is negative while its effect on the FDI activity of market-seeking is ambiguous is due to the following fact: on the one hand, the higher the foreign wage rate is, the higher the variable costs will be that are involved in foreign production which reduces the profitability of all types of FDI. On the other hand, an increase in the foreign wage rate will raise the exposure of the profits of an export-substituting FDI and a reverse-importing FDI, but reduce the exposure of the profits of a market-seeking FDI. Therefore, the total effects of foreign wage rate on the FDI activity of export-substituting firms and reverse-importing firms are negative, while its effect on the FDI activity of market-seeking firms is ambiguous. Nevertheless, as obvious from the proof of Proposition 2-13, if the sunk costs are high, or the degree of risk aversion is low enough, then the total effect of foreign wage rate on the FDI activity of market-seeking firms is also negative.

7. Conclusion

This chapter theoretically examines how exchange rate uncertainty influences the timing of FDI. An integrated model of FDI under exchange rate uncertainty is developed to illustrate the impact of exchange rate volatility on the FDI activity of an export-substituting firm, a market-seeking firm, and a reverse-importing firm. It shows that while exchange rate uncertainty tends to delay the FDI activity of a market-seeking firm and reverse-importing firm, it actually may accelerate the FDI activity of an export-substituting firm if the degree of risk aversion of the firm is high enough. The rationale behind these results is that market-seeking FDI and reverse-importing FDI might increase the exposure of the firm's profits to exchange rate risk, while export-substituting FDI might reduce it.

In addition, it also shows that previous studies with either a real options model (such as Dixit (1989a, b), or a traditional risk-aversion model (such as Itagaki (1981), Goldberg and Kolstad (1995) and Cushman (1985)), are equivalent to the special cases of our model. In other words, the relationship between exchange rate uncertainty and FDI is generally indeterminate, depending on the FDI types. The theoretical model in this chapter can be viewed as a synthesis of many previous studies on this issue. The comparative statics results are summarized in Table 2-1.

The theoretical results in this chapter demonstrate the importance in considering the diversity of investing motives when examining the relationship between exchange rate movements and FDI activity. Without considering this fact in an empirical study, its testing results might suffer from aggregation bias. In the following two chapters, the panel data on Taiwanese firms will be used to test the validity of the theoretical results developed in this chapter.

Table 2-1. Expected signs of the determinants of FDI

Types \ Variables		Exchange Rate Level (R)	Exchange Rate Trend (μ)	Exchange Rate Volatility (σ)	Sunk Costs (k)	Foreign Wage Rate (W_f)	Domestic Wage Rate (W_d)
Export-substituting FDI	risk neutrality	-	-	-	-	-	+
	risk aversion	-	?	?	-	-	+
Market-seeking FDI	risk neutrality	+	+	-	-	-	
	risk aversion	+	?	-	-	?	
Reverse-importing FDI	risk neutrality	-	-	-	-	-	
	risk aversion	-	-	-	-	-	

Appendix 2-1. The Derivation of Equations (2-7) and (2-11)

The derivation of Equation (2-7)

Recall that

$$\xi_E(R) = -\frac{W_f R}{\rho - \mu} + \frac{W_d}{\rho} - a\left[\left(P_f - W_f\right)^2 - P_f^2\right]\gamma \tag{2-5}$$

and

$$V(R) = \max\left\{\xi_E(R) - k, \frac{1}{1+\Delta t\rho} E\left[V(R')\middle|R\right]\right\}. \tag{2-6}$$

Following the procedure in Dixit and Pindyck (1994, Chapter 4), the optimum entry threshold, R_E, is solved as follows: According to the criterion in Dixit and Pindyck (1994, p.128), since $\xi_E'(R) < 0$, there is a cutoff point, R_E, at which if $R<R_E$, then the firm's optimal decision is termination (to carry out FDI). In the continuation region, the Bellman equation is

$$\rho V(R)dt = E\left[dV(R)\right]. \tag{2A-1}$$

This equation implies that over a time interval dt, the total expected utility of FDI, $\rho V(R)dt$, is equal to its expected rate of capital appreciation. Using Ito's Lemma, the Bellman equation becomes

$$\frac{1}{2}\sigma^2 R^2 V''(R) + \mu R V'(R) - \rho V(R) = 0. \tag{2A-2}$$

The second order differential equation in Equation (2A-2) can be solve as

$$V(R) = A_1 R^{-\alpha} + A_2 R^{\beta}, \tag{2A-3}$$

where A_1 and A_2 are undetermined coefficients,

$$\alpha = \frac{(\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2\rho}}{\sigma^2} > 0$$

and

$$\beta = \frac{-(\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2\rho}}{\sigma^2} > 1 .$$

This equation also represents the value of the option to implement FDI at the optimal time (see Dixit 1989b, p.626). In addition, since the equilibrium will be in the continuation region when R approximate to infinite, the boundary condition should be $\lim_{R\to\infty} V(R) = 0$. Hence, $A_2 = 0$ and Equation (2A-3) can be rewrite as

$$V(R) = A_1 R^{-\alpha} . \qquad \text{(2A-4)}$$

Using value-matching and smooth-pasting conditions (see Dixit and Pindyck 1994, p.130), we have

$$-\frac{W_f R_E}{\rho - \mu} + \frac{W_d}{\rho} - a\left[\left(P_f - W_f\right)^2 - P_f^2\right]\gamma - k = A_1 R_E^{-\alpha} \qquad \text{(2A-5a)}$$

and

$$-\frac{W_f}{\rho - \mu} = -\alpha A_1 R_E^{-\alpha - 1} . \qquad \text{(2A-5b)}$$

It is easy to show that the optimum entry threshold is

$$R_E = \frac{\rho - \mu}{W_f} \frac{\alpha}{1 + \alpha} \left[\frac{W_d}{\rho} + a\left[P_f^2 - \left(P_f - W_f\right)^2\right]\gamma - k \right]$$

and A_1 is

$$A_1 = \frac{W_f}{\rho - \mu} \frac{1}{\alpha} R_E^{1+\alpha} .$$

The derivation of Equation (2-11)

Recall that

$$\xi_M(R) = \frac{\left(P_f - W_f\right)R}{\rho - \mu} - a\left(P_f - W_f\right)^2 \gamma \qquad \text{(2-9)}$$

and

$$V(R)=\max\left\{\xi_M(R)-k,\frac{1}{1+\Delta t\rho}E\left[V(R')|R\right]\right\}. \tag{2-10}$$

Following the similar procedure above, since $\xi_M'(R)>0$, there is a cutoff point, R_H, at which if $R>R_H$, then the firm's optimal decision is to implement FDI. The general solution of Equation (2-10) is the same as Equation (2A-3), but the boundary condition becomes $\lim_{R\to 0}V(R)=0$. Thus, $A_1=0$ and the value function becomes

$$V(R)=A_2R^{\beta}. \tag{2A-6}$$

Using value-matching and smooth-pasting conditions, we have

$$\frac{(P_f-W_f)R_H}{\rho-\mu}-a(P_f-W_f)^2\gamma-k=A_2R_H^{\beta} \tag{2A-7a}$$

and

$$\frac{P_f-W_f}{\rho-\mu}=\beta A_2R_H^{\beta-1}. \tag{2A-7b}$$

It is easy to show that the optimum entry threshold is

$$R_H=\frac{\rho-\mu}{P_f-W_f}\frac{\beta}{\beta-1}\left[a(P_f-W_f)^2\gamma+k\right]$$

and A_2 is

$$A_2=\frac{P_f-W_f}{\rho-\mu}\frac{1}{\beta}R_H^{1-\beta}.$$

Appendix 2-2. The Derivation of $\phi < 0$ and $\psi < 0$

Recall that in the main text,

$$\beta = \sigma^{-2}[-(\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2\rho}] > 1, \quad \text{(2A-8)}$$

and

$$\alpha = \sigma^{-2}[(\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2\sigma^2\rho}] > 0, \quad \text{(2A-9)}$$

and

$$\frac{\partial\beta}{\partial\sigma} = \frac{\beta}{\sigma}[-1 + \frac{2\mu + \sigma^2}{2\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}}] < 0, \quad \text{(2A-10)}$$

and

$$\phi = \rho - \mu - (\beta - 1)\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}, \quad \text{(2A-11)}$$

and

$$\psi = \rho - \mu - (1 + \alpha)\sqrt{2\rho\sigma^2 + (\mu - 0.5\sigma^2)^2}. \quad \text{(2A-12)}$$

Proof of $\phi < 0$

Since $\partial\phi/\partial\mu = -\sigma(\partial\beta/\partial\sigma) > 0$, ϕ is a strictly increasing function of μ. Moreover, $\phi = 0$ when $\mu = \rho$, and thus we have $\phi < 0$. ■

Proof of $\psi < 0$

Since $\rho > 0$ and $\psi|_{\rho=0} = -2\mu^2/\sigma^2 < 0$, thus $\psi < 0$ if $\partial\psi/\partial\rho < 0$. Note that

$$\frac{\partial\psi}{\partial\rho} = \frac{-\mu - 0.5\sigma^2 - \sqrt{2\rho\sigma^2 + \left(\mu - 0.5\sigma^2\right)^2}}{\sqrt{2\rho\sigma^2 + \left(\mu - 0.5\sigma^2\right)^2}}. \quad \text{(2A-13)}$$

Because μ is the growth rate of the exchange rate, we have $\mu > -1$.[38] Therefore, $\partial\psi/\partial\rho < 0$, if $\partial\psi/\partial\rho|_{\mu=-1} \le 0$ and $\partial^2\psi/\partial\mu\partial\rho < 0$. From Equation (2A-13), we

[38] Since an exchange rate, *R*, has to be positive and μ represents the growth rate of *R*, thus μ has to be greater than -1; otherwise the value of *R* will become negative.

have

$$\frac{\partial^2\psi}{\partial\mu\partial\rho} = \frac{-\sigma^2\left(4\rho-2\mu+\sigma^2\right)}{2\left[2\rho\sigma^2+\left(\mu-0.5\sigma^2\right)^2\right]^{3/2}} < 0,$$

and

$$\frac{\partial}{\partial\sigma^2}\left(\left.\frac{\partial\psi}{\partial\rho}\right|_{\mu=-1}\right) = \frac{-2-\rho\left(2+\sigma^2\right)-\sigma^2}{2\sqrt{\left(1+\sigma^2+2\rho\sigma^2+0.25\sigma^4\right)^3}} < 0,$$

which implies that $\left.\partial\psi/\partial\rho\right|_{\mu=-1}$ is a monotone function of σ^2. Moreover, $\left.\partial\psi/\partial\rho\right|_{\mu=-1,\sigma^2=0} = 0$ and $\left.\partial\psi/\partial\rho\right|_{\mu=-1,\sigma^2=\infty} = -2 < 0$, and thus $\left.\partial\psi/\partial\rho\right|_{\mu=-1} \le 0$. We have now completed the proof of $\psi < 0$. ■

Appendix 2-3. Proofs of Lemmas

Proof of Lemma 2-1

Using Equations (2-8a) and (2-8b), it can be shown that

$$\frac{\partial^2 R_E}{\partial\mu\partial a_p}=\Psi\left[\frac{\rho-\mu}{\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}+(1+\alpha)\left(\frac{\rho-2\mu-0.5\sigma^2}{\rho-2\mu-\sigma^2}\frac{4(\rho-\mu)}{\rho-2\mu}-1\right)\right] \quad \text{(2A-14)}$$

and

$$\frac{\partial^2 R_E}{\partial\sigma\partial a_p}=\Phi\left[-\frac{\sigma}{\rho-2\mu}\left(1+\frac{2\mu+\sigma^2}{2\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}\right)+\frac{2\sigma}{\rho-2\mu-\sigma^2}(1+\alpha)\right] \quad \text{(2A-15)}$$

where $\Phi=\frac{R_0^2\alpha(\rho-\mu)[P_f^2-(P_f-W_f)^2]}{2W_f(1+\alpha)^2(\rho-2\mu-\sigma^2)}>0$; $\Psi=\frac{R_0^2\alpha\gamma[P_f^2-(P_f-W_f)^2]}{2W_f(1+\alpha)^2}>0$. Since we assume $\rho>2\mu+\sigma^2$, $\frac{\rho-2\mu-0.5\sigma^2}{\rho-2\mu-\sigma^2}>1$ and $\frac{4(\rho-\mu)}{\rho-2\mu}>1$. Thus, $\partial^2 R_E/\partial\mu\partial a_p>0$.

Regarding Equation (2A-15), since $\frac{\sigma}{\rho-2\mu}<\frac{2\sigma}{\rho-2\mu-\sigma^2}$, $\partial^2 R_L/\partial\sigma\partial a_p>0$ if $\alpha>\frac{2\mu+\sigma^2}{2\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}$. To prove $\alpha>\frac{2\mu+\sigma^2}{2\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}$, note that

$$\alpha-\frac{2\mu+\sigma^2}{2\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}$$
$$=\frac{\alpha}{4\rho}\left[\left(\rho-2\mu-\sigma^2\right)+\frac{3\rho\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}+2(\mu+0.5\sigma^2)(\mu-0.5\sigma^2)}{\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}\right].$$

Given that $3\rho>2(\mu+0.5\sigma^2)$ and $\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}>|\mu-0.5\sigma^2|$ under the assumption $\rho>2\mu+\sigma^2$, thus we obtain $\alpha>\frac{2\mu+\sigma^2}{2\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}}$. Consequently, we have proved that $\partial^2 R_E/\partial\sigma\partial a_p>0$. ■

Proof of Lemma 2-2

From Equation (2-12b), we have

$$\frac{\partial^2 R_H}{\partial\mu\partial a_p} = \Gamma\left[\frac{\rho-\mu}{\sqrt{2\rho\sigma^2+(\mu-0.5\sigma^2)^2}} + (\beta-1)\left(\frac{\rho-2\mu-0.5\sigma^2}{\rho-2\mu-\sigma^2}\frac{4(\rho-\mu)}{\rho-2\mu}-1\right)\right] \text{(2A-16)}$$

where $\Gamma = \frac{R_0^2\beta\gamma(P_f - W_f)}{2(\beta-1)^2} > 0$. With a similar reasoning as that in proving Equation (2A-14), it can be shown that $\partial^2 R_H / \partial\mu\partial a_p > 0$.