行政院國家科學委員會專題研究計畫　成果報告

實資選擇權投資學在資源開發投資問題互動性策略彈性評估的應用

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1. Procedures of Research Undertaken

In this research, we take the multi-commodity flow requirements, aggregated underlying demand and linear concave non-decreasing cost funtion characterized by the Optimum Network Problem as inputs, and extend the framework of Pindyck (1988) to model the capacity expansion problem in consideration. Limitations of this model and propose possible implementations to further develop it are also discussed.

The main question to ask, when characterizing the capacity investment problem of a telecommunications operator, is not one of how to value simply the timing option (the option to invest), but rather the simultaneous valuation of two different types of options. Once the option to invest is exercised, the firm obtains a set of claims contingent on future production that provide a stochastic non-negative stream of profits. The option to produce for each period is exercised when the price is above variable cost. However price can drop below cost, there always exists the probability that future production will not be carried out. The value of claims in those periods hence can end up being worthless. Several interesting features of the shutdown option have been identified in our research. Like a financial option, it's value increases as the probability of it being exercised increases. Therefore, the shutdown option's value is positively related to uncertainty. Its value is negatively related to price when the unit of capacity is operating but positively related to price when the unit is shut down. Therefore. the added flexibility to shut down tends to temper the effect that price changes have on the value of operating capacity.

In order to establish the value of the option to wait, it is necessary to develop the investment decision criteria. In our model, we take the capacity level K as exogenous and valued the option to shut down based upon the stochastic movement of S in relation to the strike price $\hat{S}$. Since each unit of newly installed capacity affects the value of all previously installed units, the concept of a strike price for our investment problem is more complex. The problem of finding the strike price, in our terminology is to identify the optimal demand threshold S^*, such that the investment continues as long as $S \geq S^*$. And in order for the problem to be bounded, S^* has to be a monotonic increasing function of installed capacity. The value maximizing level of capacity is determined when the condition $S = S^*$ is met.

2. Results

As a result of the research undertaken, we derive the partial differential equation which governs the value of the option to invest in an incremental unit of capacity, $P(K,S)$, as follows:

$$\frac{1}{2}\sigma^2 S^2 P_{SS}(K,S) + (r-\delta) P_S(K,S) - rP(K,S) = 0 \tag{2.1}$$

subject to:

$$P(K,0) \to 0 \tag{2.2}$$

$$P(K,S^*) = F(K,S^*) - \overline{d} \tag{2.3}$$

$$P_S(K,S^*) = F_S(K,S^*) \tag{2.4}$$

The solution form for the above partial differentia equation is given by:

$$P(K,S) = l_1 S^{x^+} + l_2 S^{x^-} \tag{2.5}$$

where the powers x^+ and x^- are roots of the characteristic equation.

Boundary condition 2.2 is an absorbing barrier of the stochastic process. Applying it to 2.5 eliminates the constant l_2:

$$P(K,S) = l_1 S^{x^+} \tag{2.6}$$

The second boundary condition, 2.3, corresponds to the optimality statement. The third boundary condition, 2.4, i.e. the high contract or smooth pasting condition, is required to ensure the optimal capacity found is indeed the global optimal.

From the smooth pasting condition, we set derivative F_S and P_S equal to each other to get:

$$(S^*)^{x^+-1} x^+ l_1 = (S^*)^{x^--1} x^- \Phi_2 + \frac{1}{\delta}\left(\frac{\partial}{\partial K}\varepsilon(K) - \frac{\partial}{\partial K}\zeta(K)\right) \tag{2.7}$$

Then, rearranging to solve for l_1 :

$$l_1 = \frac{(S^*)^{x^- - x^+} x^- \Phi_2}{x^+} + \frac{(S^*)^{1-x^+}}{\delta x^+}\left(\frac{\partial}{\partial K}\varepsilon(K) - \frac{\partial}{\partial K}\zeta(K)\right) \tag{2.8}$$

Then the optimal threshold demand curve $S^\star$ is derived by substituting l_1 into boundary condition 2.3, the firm's optimal investment rule is the solution to:

(2.9)

$$(S^*)^{x^-} \Phi_2 \left[\frac{x^+ - x^-}{x^+}\right] + \frac{S^*}{\delta}\left(\frac{\partial}{\partial K}\varepsilon(K) - \frac{\partial}{\partial K}\zeta(K)\right)\left[\frac{x^+ - 1}{x^+}\right] - \frac{1}{r}\frac{\partial}{\partial K}\eta(K) - \bar{d} = 0$$

When $S > S^*(K)$ then investment will occur. However, as K increases, so will S^*. At optimal K^*, $S = S^*$ and invest will cease.

The value of the option to invest, $P(K,S)$ is then calculated by substituting l_1 (as obtained from 2.8) and S^* (as obtained from 2.9) into (2.6) and (2.7):

$$P(K,S^*) = \begin{cases} l_1 S^{x^+} & \text{for } S \le S^* \\ F(K,S) - \bar{d} & \text{for } S \ge S^* \end{cases} \tag{2.10}$$

2.1. **Value of The Installed Capacity.**

(2.11)

Option Components\Perturbed Parameters	r	δ	σ^2	$\frac{S}{\delta}\frac{\partial}{\partial K}\zeta(K) + \frac{1}{r}\frac{\partial}{\partial K}\eta(K)$
x^+	-	+	-	n/a
x^-	-	+	+	n/a
Φ_1	+	-	+	+
Φ_2	-	+	+	+

(2.12)

Option Components\Perturbed Parameters	r	δ	σ^2	$\frac{S}{\delta}\frac{\partial}{\partial K}\zeta(K) + \frac{1}{r}\frac{\partial}{\partial K}\eta(K)$
Incremental Capaciy: $F(K,S)$ at $S \le \hat{S}$	+	-	+	-
Incremental Capacity: $F(K,S)$ at $S \ge \hat{S}$	+	-	+	-
Shutdown Option: $\Phi_2 S^{x^-}$	-	+	+	+
$\frac{S}{\delta}\frac{\partial}{\partial K}\varepsilon(K) - \left(\frac{S}{\delta}\frac{\partial}{\partial K}\zeta(K) + \frac{1}{r}\frac{\partial}{\partial K}\eta(K)\right)$	+	-	n/a	-

For a given S, the model's parameter determines $F(K.S)$. The first row of 2.12 is the value of capacity when it is shut down. The next row is the total value of a producing unit. The last two rows are the components of the value of a producing unit; first the shut down option value followed by the discounted operating cash flow.

All parameters have the same directional effect on the marginal unit of installed capacity whether it is shut down or whether it is active. When the risk free rate increases, the value of capacity increases. For a producing unit, raising the risk free rate has the effect of reducing the present value of future costs and causes the discounted operating cash flow component to increase. The shutdown component declines in value because, with higher cash flows, the likelihood of using the option to shut down diminishes. When the unit is not producing, an increase in the risk free rate increases the likelihood that the option to start production will be exercised. Thus, the value of a shut down unit increases.

The results for an increase in the required rate of return δ are completely the opposite to that of an increase in the risk free rate. The reasoning, however is much the same. The discounted operating cash flow value is lower because the present value of revenues net of operational costs are now lower. This increases the likelihood of shutting down raising that component for an operating unit. If the unit is shut down, it will now be more likely to remain shut down and the option is worth less.

The directional effects of the last column of table 2.12 are also opposite. The reasoning again follows a similar logic. Increasing operational costs reduces the discounted cash flow component which produces the expected impact on the respective shutdown options.

Increasing the variance rate causes the value of installed capacity to increase. This result comes directly from the shutdown option. When shutting down becomes more likely through higher uncertainty, then the option value must increase. Likewise, increasing uncertainty increases the likelihood that a unit of capacity not in operation will be used in the future. As a result, the shutdown option increases for a unit that is shut down. Because the standard discounted operating cash flow term only considers the first moment of a distribution, its value is not affected by different variances. The positive relationship between $F(K,S)$ and σ^2 implies capacity should rise. However, as we will see in the following, rising uncertainty also raises the value of future incremental growth by more than the change in $F(K,S)$. As a result, capacity declines as uncertainty rises[1].

2.2. **Value of Future Growth.** Equation 2.9 has several important implications. Note that apart from the two terms in brackets and the installation cost $\bar{d}$, equation 2.9 is exactly as the lower path equation for the value of an operating incremental capacity unit. Thus, the term in brackets is a scaling factor that re-weights the original equation of net incremental value to include the impact of the option to wait. The first term is a re-weighted version of the shutdown option. The second and third terms are the re-weighted value of the investment's discounted operating cash flow. Ignoring the second and third terms for the moment, with a little work, we will be able to derive the expected positive investment-uncertainty relationship created by the shutdown option. First, using the equation for Φ_2 we rewrite the first term and express the equilibrium as:

$$\frac{r - x^{+}(r-\delta)}{x^{+}r\delta}\left(\frac{\partial}{\partial K}\varepsilon(K) - \frac{\partial}{\partial K}\zeta(K)\right)\left(\hat{S}\right)^{1-x^{-}}(S^{*})^{x^{-}} = \bar{d}$$

[1]When investments are assumed reversible, the standard shutdown option leads to the conclusion that investment is positive related to uncertainty. This is the result fom Caballero (1991); Abel (1983); and Hartman (1972). When firms have the option to wait, however, it grows with uncertainty and reverse the result presented in these papers.

By taking logs and rearranging, we obtain:

$$\ln\left\{\frac{1}{\delta x^{+}}+\left(\frac{\partial}{\partial K}\varepsilon\left(K\right)-\frac{\partial}{\partial K}\zeta\left(K\right)\right)-\frac{r-\delta}{r\delta}\right\}$$

$$+\ln\left(\hat{S}\right)+x^{-}\left(\ln\left(S^{*}\right)-\ln\left(\hat{S}\right)\right)=\ln\bar{d} \tag{2.13}$$

From Table 2.11, we see that an increase in the variance rate σ^2 causes x^+ to drop but x^- to rise. Hence it can easily be verified that the value of the first term in equation 2.13 will rise in response to an increase in the variance rate.

Since the firm would never optimally build capacity knowing it will be shut down, S^* should therefore always be greater than $\hat{S}$; this in turn implies: $\ln\left(S^*\right)-\ln\left(\hat{S}\right)>0$. The positive relationship between x^- and the variance rate also ensures an increase in $x^-\left(\ln\left(S^*\right)-\ln\left(\hat{S}\right)\right)$ as the variance rate increases. This illustrates the positive relationship between the shutdown option and the variance rate.

Finally, we show the dominance of the negative investment-uncertainty relationship by the option to wait over the positive relationship by the shutdown option, so that investment declines as uncertainty rises.

To make the investment-uncertainty relationship more easily seen, we simplify equation 2.9 by assuming $\bar{d}=0$, $\frac{\partial}{\partial K}\varepsilon\left(K\right)=1$, $\frac{\partial}{\partial K}\zeta\left(K\right)=0$, and $\frac{\partial}{\partial K}\eta\left(K\right)=K$. After rearranging (all terms are divided by $\frac{S^*}{r}$ then multiplied by $\frac{S^*}{K}$), we get:

$$\left(\frac{S^*}{K}\right)^{x^-}\left(\frac{r}{\delta x^+}-\frac{r-\delta}{\delta}\right)+\left(\frac{x^+-1}{x^+}\right)\left(\frac{r}{\delta}\right)\left(\frac{S^*}{K}\right)-1=0 \tag{2.14}$$

For an increased variance rate, $\left(\frac{r}{\delta x^+}-\frac{r-\delta}{\delta}\right)$ increases due to a decline in x^+ according to table 2.11. For a unit of capacity in operation, $\frac{S^*}{K}>1$ always, hence an increase in the exponential factor x^- will cause $\frac{S^*}{K}$ to rise. And since the exponential term x^- is negative, lowering K will cause the whole term to decline. Increasing variance rate induces a declined x^+, which in turn causes the second term $\left(\frac{x^+-1}{x^+}\right)\left(\frac{r}{\delta}\right)\left(\frac{S^*}{K}\right)$ to be lower. And since the exponential factor of $\left(\frac{S^*}{K}\right)$ in the second term is equal to one, a decline in K will cause the whole term to rise. Therefore, to restore the equation back to equilibrium after an rise in the variance rate, the effect on the option to wait outweights the changes in the value of the option to shut down. Hence, optimal capacity declines as uncertainty rises.

2.3. **Illustrating the Effect of Uncertainy on Network Value.** Assuming an un-capacitated nine node network (with initially no arcs built) based on the geographical locations of cities in United Kingdom. We examine the effect of uncertainty over the levels of future customer demand and it's impact on network investment. During the hypothetical expansion process, no construction lags or depreciation of installed capacity are considered.

Table 2.15 summaries the sesitivity of the total value of network over uncertainty in customer demand. The table gives results for the installation of units low capacity cables q_1 on the built arc $(1,2)$ in the nine node network for demand volatility values $\sigma=0.1$ and $\sigma=0.4$. For example, from Table 2.15, given the current network capacity configuration, and aggregate customer demand of at least 172,894 circuits is required to justify installation of three low capacity cables on arc$(1,2)$. The value of these three low capacity units, with respect to the overall network capacity, is

\$783,304, and the value of the option to invest in these three units is \$606,204. This option value is the equilibrium market value of the right to hold the incremental unit.

For both Tables 2.15 and 2.16, it can seen that as demand rises, both the value of the incremental unit of capacity, and the value of its option to invest also rise. The net value of each installed capacity unit on the arc is the sum of its marginal value, plus the option to invest. The net value of the arc increases as the number of units installed on it increases. For a low customer demand volatility $(\sigma = 0.1)$, the value of the options to increase capacity on the arc form a smaller fraction of the overall net value. For a higher demand volatility $(\sigma = 0.04)$, the value of these growth options increase and exceed the value of the installed capacity units on the arc. Thus higher demand uncertainty lead to the value of growth options forming a much higher proportion of the overall net value of the arc.

1. $\sigma = 0.10$, $r = 0.10$, $\delta = 0.02$

(2.15)

$K_{q_1(1,2)}$	$S^*\left(K_{q_1(1,2)}\right)$ $(\times 100)$	$F\left(K_{q_1(1,2)}, S^*\right)$ $(\times 100)$	$P\left(K_{q_1(1,2)}, S^*\right)$ $(\times 100)$	Net Value of Each Unit Installed : $F + P$
1	1719.16	7806.02	5995.02	13801.04
2	1724.34	7839.53	6028.53	13868.06
3	1728.94	7873.04	6062.04	13935.08
4	1733.57	7906.55	6095.55	14002.10
5	1738.23	7940.06	6192.06	14069.12

2. $\sigma = 0.40$, $r = 0.10$, $\delta = 0.02$

(2.16)

$K_{q_1(1,2)}$	$S^*\left(K_{q_1(1,2)}\right)$ $(\times 100)$	$F\left(K_{q_1(1,2)}, S^*\right)$ $(\times 100)$	$P\left(K_{q_1(1,2)}, S^*\right)$ $(\times 100)$	Net Value of Each Unit Installed : $F + P$
1	3186.40	18449.65	18638.65	37088.30
2	3194.89	18511.74	18700.74	37212.48
3	3203.42	18573.83	18762.83	37336.66
4	3212.00	18653.91	18824.91	37460.82
5	3220.62	18698.00	18887.00	37585.00

2.4. **Illustrating the Effect of Uncertainty on the Optimal Demand Thresholds.** The sensitivity of the network investment decisions towards the parameters σ and δ can be seen in Table 2.17. This table shows the decision to invest in one unit of low capacity on arc$(1, 2)$ changes in response to changes in demand volatility, σ, and the opportunity cost of deferring investment, δ. Notice that the demand threshold value increases as σ is increased; i.e. greater uncertainty induces the network operator to reduce investment, and to hold a smaller amount of network capacity. An increase in the value of δ also raises the demand threshold values. Since an increase in $\delta\,(= \mu - \alpha)$ implies a reduction in the expected growth rate, α, of customer demand, the expected value of any future options to expand capacity falls. This decrease leads to a smaller marginal value for the installation of additional units of capacity on the arc, thus requiring a higher demand threshold to justify any further investment.

(2.17)

	$\delta = 0.02$	$\delta = 0.04$	$\delta = 0.08$
$\sigma = 0.10$	1691.35	1719.76	1849.47
$\sigma = 0.20$	1973.21	2055.07	2305.70
$\sigma = 0.40$	3018.20	3186.40	3574.18
$\sigma = 0.60$	4670.92	4890.64	5356.42

where values in the table represent demand threshold S^* $(\times 100)$.

3. Conclusion

In this research, we have developed a model that extends the real options framework of Pindyck (1988) to characterize the investment strategies available for the quantity and timing of capacity expansion investments for a telecommunications network operator. Such investment decisions are determined under the assumption of a stochastic aggregate customer demand for telecommunication services following a geometric Brownian motion.

Our model relies on diminishing profitability of incremental additions to total capacity to bound the firm size and to define the conditions for optimal capacity, i.e. capacity expansion continues until the value of the last incremental unit of installed capacity is equal to the cost of installation plus the cost of the incremental future cost option. The latter option is equivalent to an opportunity cost of irreversibly exercising the option to invest in the unit, rather than waiting and keeping the option alive.

Each cable type has its own individual operating cost and contributes an immediate marginal flow of profits in perpetuity. It was shown that the optimal demand thresholds, S^*, which justify the investment, increase with increasing uncertainty over future customer demand. The effects of demand uncertainty with respect to the value of installed capacity and the opportunity cost of deferring an investment were also investigated. We found that increasing the variance rate causes the value of installed capacity to increase. There is a positive relationship between the value of installed capacity and uncertainty, and this in turn implies that the network capacity rises. However, as we have shown in section 2.2, rising uncertainty also raises the value of future incremental growth by more than the change in the value of installed capacity. As a result, in a market with a volatile demand, the network should hold less capacity, than if the capacity installed were reversible.

4. References

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